# NAGT

$$\not{D} = \partial + ig A^a_\mu T^a$$

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu} + \bar\Psi\left(i\not{D} - m\right)\Psi + \frac{1}{(1-\ \ )}\left(\partial^\mu A_\mu\right)^2 + \bar\eta\, \partial_\mu D^\mu\, \eta$$

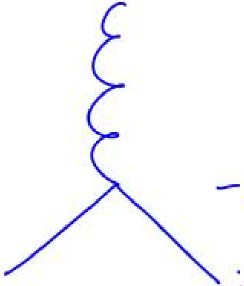
$$\xi = 0$$

## Tree Level

$a$  $b$ $\quad \dfrac{-i g_{\mu\nu}}{p^2}\,\delta^{ab}$

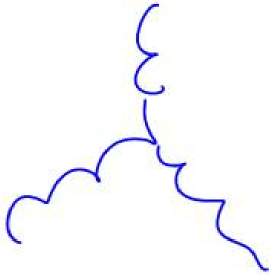
$i$  $j$ $\quad \dfrac{i}{\not{p} - m}\,\delta^{ij}$

$a$  $b$ $\quad \dfrac{i\,\delta_{ab}}{p^2}$

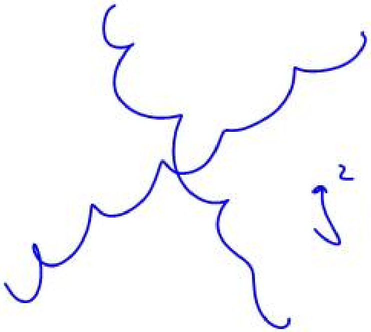


$j \qquad i \qquad -ig\left(T^a\right)_{ij}$

$$F^{\mu\nu} \sim \partial^\mu A^\nu - \partial^\nu A^\mu + ig[A^\mu, A^\nu]$$



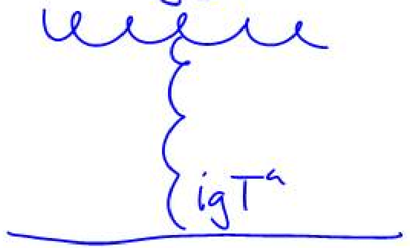
$ig\, p^\mu f_{abc}$



$g^2 f f$

e.g. NA Compton Scattering


$$( \ , \ , \ ) \begin{pmatrix} T^a T^b \\ T^b T^a \end{pmatrix} \begin{pmatrix} \vdots \end{pmatrix}$$

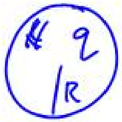


# BEYOND TREE LEVEL

Beyond tree level we meet self interactions of pts – which is tricky even in classical physics.

eg a charge in $\vec{E}$ $\qquad \vec{F} = m\vec{a} = q\vec{E}$

BUT:




$$E = \frac{3}{5}\, \frac{e^2}{4\pi\varepsilon_0 R}$$

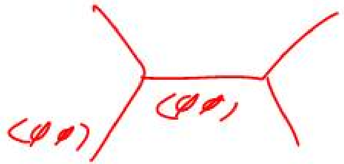
As $R \to 0$ $\quad E \to \infty$, $\ m \to \infty$

This is rubbish – classical physics doesn't describe the UV where $m_e$ is "derived". We will see the same process in QFT.

# $\phi^4$ THEORY

real scalar

$$\mathcal{L} = \tfrac{1}{2}\partial^\mu\phi\,\partial_\mu\phi - \tfrac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$




Consider the 2-pt fn

$$G^2(x,0) = \langle 0|T\,\phi(x)\,\phi(0)|0\rangle$$

$$= \Delta_0(x) + \frac{1}{Z}\int D[\phi]\,e^{iS_0}\left(\frac{-i\lambda}{4!}\right)\int d^4z\,\phi^4(z)\,\phi(x)\phi(0) + O(\lambda^2)$$

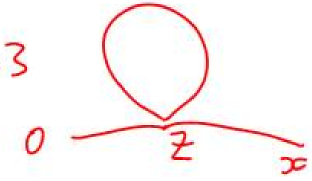
$x$ ——— $0$

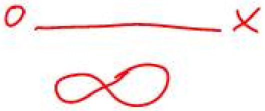
WICK CONTRACTIONS

$$\phi(z)\phi(z)\phi(z)\overbrace{\phi(z)\phi(x)}^{4}\phi(0)$$

3

4x3



(also  this vacuum diagram cancels $\frac{1}{Z}$ )

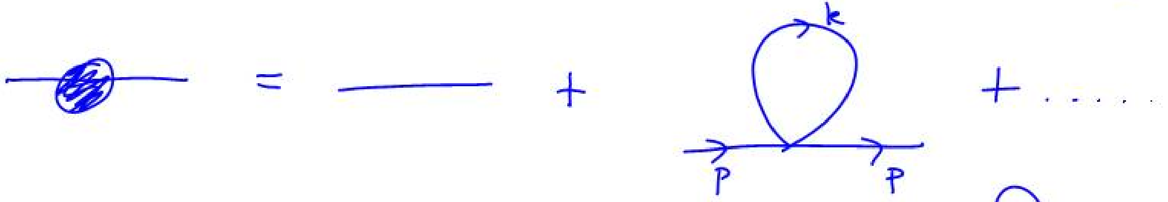
$$G^2(p) = \underbrace{\Delta_0(p^2)}_{\frac{i}{p^2-m^2}} - \frac{i\lambda}{2}\int d^4x\, e^{-ip.x}\int d^4z\, \underbrace{\Delta_0(x-z)}_{k_1}\,\underbrace{\Delta_0(z)}_{k_2}\,\underbrace{\Delta_0(0)}_{k}$$

$$-\frac{i\lambda}{2}\int d^4x\, e^{-ip.x}\int d^4z\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k}{(2\pi)^4}\,\frac{e^{ik_1(x-z)}\, i^3\, e^{ik_2 z}}{(k_1^2-m^2)(k_2^2-m^2)(k^2-m^2)}$$

$z$ integration: $(2\pi)^4\,\delta^4(k_1-k_2) \longrightarrow k_2$ integral just sets $k_1=k_2$

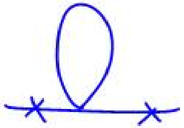
$x$ integration: $(2\pi)^4\,\delta^4(k_1-p) \longrightarrow k_1$ " " $k_1=k_2=p$

$$G^2(p) = \Delta_0(p) - \frac{i\lambda}{2}\,\frac{i}{p^2-m^2}\left[\int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}\right]\frac{i}{p^2-m^2} + \cdots$$




integral → allow all $k$ in the loop

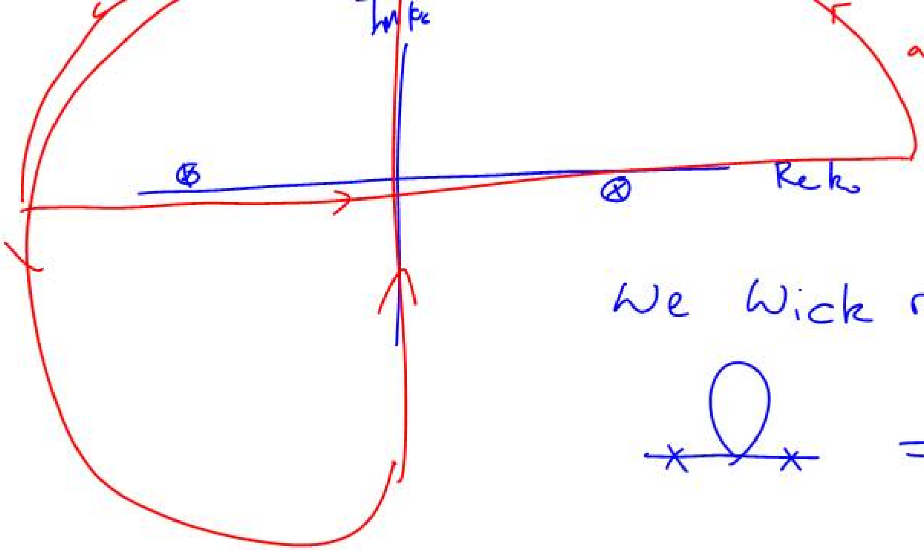
Now remove external propagators



$$= \frac{-i\lambda}{2}\int \frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$

The $k_0$ integral contains poles... $i\varepsilon$ prescription important...

$$\frac{1}{k^2-m^2+i\varepsilon} = \frac{1}{k_0^2-(|\vec{k}|^2+m^2)+i\varepsilon} = \frac{1}{\left(k_0-\sqrt{|k|^2+m^2}+i\varepsilon'\right)\left(k_0+\sqrt{|k|^2+m^2}-i\varepsilon'\right)}$$



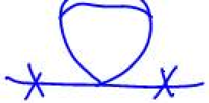

at $k_0 \to \infty \quad \frac{1}{k_0^2} \to 0$

We Wick rotate and instead solve

$$= \frac{\lambda}{2} \int_{-i\infty}^{i\infty} \frac{dk_0}{2\pi} \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{1}{k_0^2 - \underline{k}^2 - m^2 + i\varepsilon}$$

Let $l_0 = -ik_0$, $dk_0 = i\,dl_0$

$$= -\frac{i\lambda}{2} \int_{-\infty}^{\infty} \frac{dl_0}{2\pi} \int \frac{d^3\underline{k}}{(2\pi)^3} \frac{1}{+l_0^2 + \underline{k}^2 + m^2 + i\varepsilon}$$



$$\int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2-M^2} \xrightarrow[\text{Rotate}]{\text{W.ck}} -\int \frac{d^4k_E}{(2\pi)^4} \frac{1}{k_E^2+M^2}$$

$\vec{k}_E = (k_0, \underline{k})$

Euclidean 4-vector

Integration: 1D $\int dk$ 3D $\int k^2 \sin\theta \, d\theta \, d\phi \, dk$

2D $\int k \, dk$

4D: $$\int \frac{d^4k}{(2\pi)^4} = \int \frac{k^3}{(2\pi)^4} \sin^2\theta \, \sin\chi \; \underbrace{dk}_{0\ldots\Lambda} \; \underbrace{d\theta}_{0\ldots\pi} \; \underbrace{d\chi}_{0\ldots\pi} \; \underbrace{d\phi}_{0\ldots 2\pi}$$

$$\frac{dk^2}{dk} = 2k$$

$$= \int \frac{k^2 \, dk^2}{2^5 \pi^4} \times \underbrace{\text{ang integrals}}_{2\pi^2} = \int \frac{k^2 \, dk^2}{16\pi^2}$$

when no angles in integrand

$$k_E'^2 = k_E^2 + m^2 - m^2$$

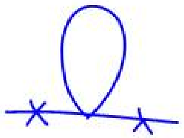
$$-\frac{i\lambda}{2}\int_0^{\Lambda^2} \frac{k_E^2\, dk_E^2}{16\pi^2}\,\frac{1}{k_E^2+m^2}$$

$$= -\frac{i\lambda}{2}\int_0^{\Lambda^2}\frac{dk_E^2}{16\pi^2} + \frac{i\lambda}{2}\int_0^{\Lambda^2}\frac{m^2}{k_E^2+m^2}\,dk_E^2$$

$$= -\frac{i\lambda}{2}\Lambda^2/16\pi^2 + \ldots \text{logs} \ldots$$

As $\Lambda \to \infty$....

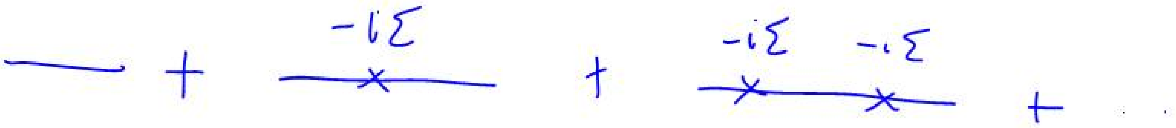
note here answer is independent of $p$ & this is a contribution to "self energy"

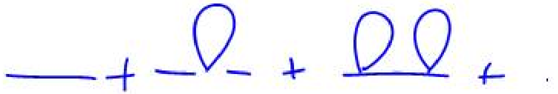
 $\equiv -i\Sigma$

it's a correction to the mass

$\equiv -im^2$

Consider the full propagator



$$\frac{i}{p^2-m^2} + \frac{i}{p^2-m^2}\,(-i\Sigma)\,\frac{i}{p^2-m^2} + \ldots$$

$$= \frac{i}{p^2-m^2}\left(1 + \frac{\Sigma}{p^2-m^2} + \ldots\right)$$

$\lambda\Lambda^2$

$$= \frac{i}{p^2-m^2}\left(1 - \frac{\Sigma}{p^2-m^2}\right)^{-1} = \frac{i}{p^2-m^2-\Sigma}$$

The true mass of the pt is

$$m_R^2 = m_{bare}^2 + \Sigma$$

# FEYNMAN PARAMETERIZATION

We will want to write

$$\frac{1}{a_1} = \int_0^1 d\alpha_1 \, \frac{\delta(1-\alpha_1)}{a_1 \alpha_1}$$

$$\frac{1}{a_1 a_2} = \int_0^1 \int_0^1 d\alpha_1 \, d\alpha_2 \, \frac{\delta(1-\alpha_1-\alpha_2)}{(a_1\alpha_1 + a_2\alpha_2)^2}$$

$$= \int_0^1 d\alpha_1 \, \frac{1}{(a_1\alpha_1 + (1-\alpha_1)a_2)^2} = \left[ \frac{-1}{a_1 - a_2} \cdot \frac{1}{\alpha_1 a_1 + a_2(1-\alpha_1)} \right]_0^1$$

$$= \frac{1}{a_1 - a_2}\left( \frac{-1}{a_1} + \frac{1}{a_2} \right) = \frac{1}{a_1 a_2}$$

In general

$$\frac{1}{a_1 a_2 \cdots a_n} = (n-1)! \int_0^1 d\alpha_1 \cdots d\alpha_n \frac{\delta(1-\Sigma\alpha_i)}{(\alpha_1 a_1 + \cdots \alpha_n a_n)^n}$$

subtlety

$$\frac{1}{a_1 a_2 a_3} = 2 \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \frac{\delta(1-\alpha_1-\alpha_2-\alpha_3)}{(a_1\alpha_1 + a_2\alpha_2 + a_3\alpha_3)^3}$$

we can do $\alpha_3$ integral $\alpha_3 = 1-\alpha_1-\alpha_2$. BUT as $\alpha_1$ & $\alpha_2$ vary it can happen that $\alpha_1+\alpha_2 > 1$ then the $\delta$ fn isn't in range 0..1 & you get 0

$$= 2\int_0^1 d\alpha_1 d\alpha_2 \frac{\theta(1-\alpha_1-\alpha_2)}{(a_1\alpha_1 + a_2\alpha_2 + a_3(1-\alpha_1-\alpha_2))^3}$$

$$= 2\int_0^1 d\alpha_1 \int_0^{1-\alpha_1} d\alpha_2 \frac{1}{\left[a_1\alpha_1 + a_2\alpha_2 + a_3(1-\alpha_1-\alpha_2)\right]^3}$$

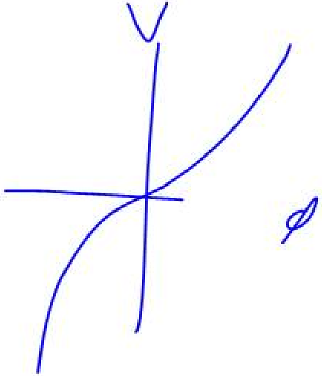
$\uparrow$ again forces $\alpha_1 + \alpha_2 \leq 1$

keep going to get $\dfrac{1}{a_1 a_2 a_3}$

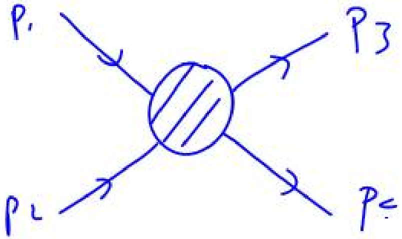
## $\phi^3$ THEORY

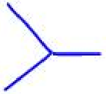
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - m^2\phi^2 - \frac{\lambda}{3!}\phi^3$$

real scalar




Consider

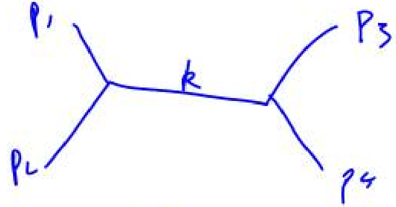


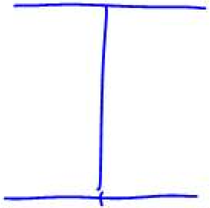



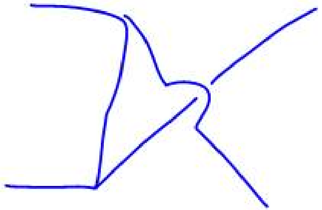
**Feynman rules:**

- Draw all graphs with n vertices
- $1/\sqrt{Z}$ for each external line
- $\dfrac{i}{k^2 - m^2 + i\epsilon}$ internal prop.
- symmetry factors!
- $-i\lambda$ at vertex
- $(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$ per vertex
- $\int \dfrac{d^4k}{(2\pi)^4}$ on internal momen

$O(\lambda^2)$



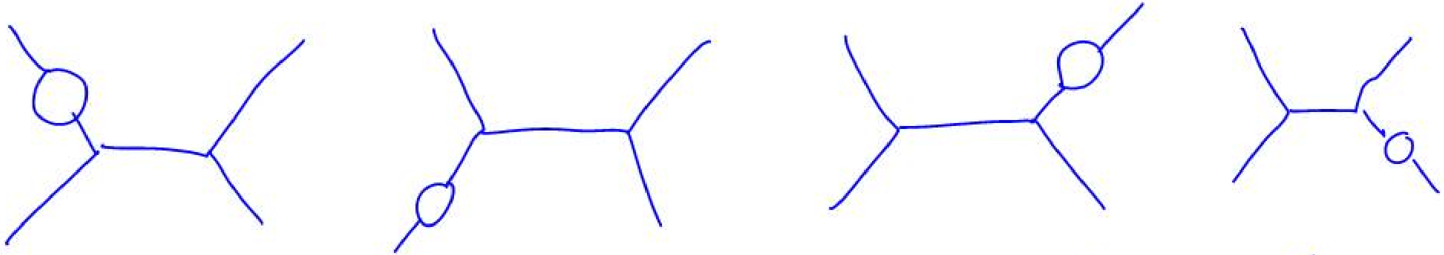


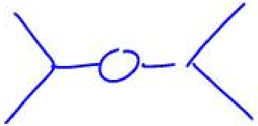
$$\int \frac{d^4k}{(2\pi)^4} (2\pi)^4 \delta^4(p^2+p^1-k) \, (2\pi)^4 (p^3+p^4-k)$$

$$\longrightarrow \quad (2\pi)^4 \delta^4(p_1+p_2-p_3-p_4)$$

$$\& \quad (-i\lambda)^2 \frac{i}{s-m^2}$$

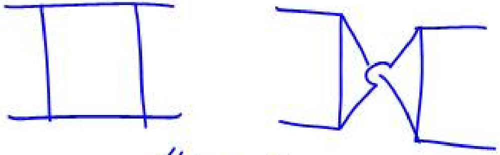
$O(\lambda^4)$





"self energy" diagrams

"vertex"



"box"

4 integrals, 4 vertex $\delta$ fn $\longrightarrow$ 1 $\delta$ fn on external momentum & 1 integral on the loop momentum