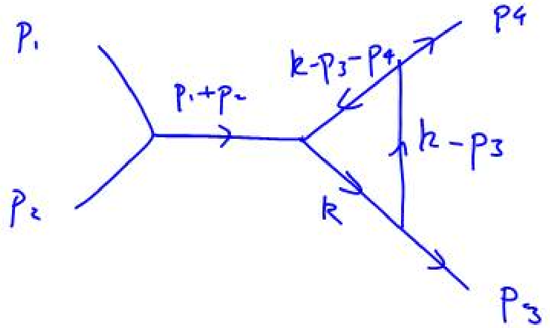
ϕ^3 THEORY : $\delta\mathcal{L} = -\frac{\lambda}{3!}\phi^3$

VERTEX CORRECTION



$(-i)^4 = 1 \qquad (i)^4 = 1$

$$i\mathcal{M} = \frac{\lambda^4}{(p_1+p_2)^2 - m^2}\int \frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-m^2)}\frac{1}{[(k-p_3)^2-m^2]}\frac{1}{[(k-p_3-p_4)^2-m^2]}$$

$p_i^2 = m^2 \qquad (p_1+p_2)^2 = (p_3+p_4)^2 = s$

$$= \frac{\lambda^?}{s-m^2} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)} \frac{1}{(k^2-2p_3\cdot k)} \frac{1}{(k^2+s-2k\cdot(p_3+p_4)-m^2)}$$

We want form $\int d^4k \frac{1}{(k^2-A^2)^p}$ so Feynman parameterize

$$I = 2\int \frac{d^4k}{(2\pi)^4} d\alpha\, d\beta\, d\gamma \frac{\delta(1-\alpha-\beta-\gamma)}{\left[\alpha(k^2-2k\cdot p_3 \underset{+m^2-m^2}{}) + \beta(k^2-2k\cdot(p_3+p_4)+s-m^2) + \gamma(k^2-m^2)\right]^3}$$

Do γ integral to set $\alpha+\beta+\gamma=1$

$$= 2\int \frac{d^4k}{(2\pi)^4} \int_0^1\int_0^{1-\alpha} d\alpha\, d\beta \frac{1}{\left[k^2-m^2-2k\cdot(p_3(\alpha+\beta)+p_4\beta)+s\beta+\alpha m^2\right]^3}$$

Now shift k to remove cross term $\quad k^\mu \to k'^\mu = k^\mu - p_3^\mu(\alpha+\beta) - \beta p_4^\mu$

$$D = k'^2 - m^2 - \left(p_3(\alpha+\beta) + \beta p_4\right)^2 + s\beta + \alpha m^2$$

$\uparrow$ two terms from k^2 & $k.D$ contribute $+1$ & -2 !

$$= k'^2 - m^2 - (\alpha+\beta)^2 m^2 - \beta^2 m^2 - 2\beta(\alpha+\beta)\underbrace{p_3\cdot p_4}_{\frac{1}{2}(s-2m^2)} + s\beta + \alpha m^2$$

$$s = (p_3+p_4)^2 = 2m^2 + 2p_3\cdot p_4$$

$$= k'^2 + \underbrace{s\beta(1-\cancel{\alpha}-\beta) - m^2\left(1-\alpha(1-\alpha)\right)}_{-A^2}$$

So

$$i\mathcal{M} = \frac{2\lambda^4}{s-m^2}\int \frac{d^4k'}{(2\pi)^4}\, d\alpha\, d\beta\, \frac{1}{(k'^2-A^2)^3}$$

Now Wick rotate : ga.. i in dk_0 & $(k'^2 - A^2)^3 \longrightarrow (-k_E'^2 - A^2)^3 \rightarrow -(k_E'^2 + A^2)^3$

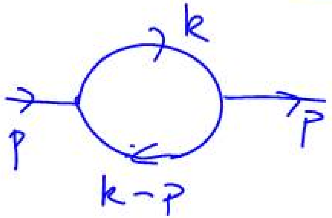
$$i\mathcal{M} = \frac{-i\lambda^4}{(s-m^2)} \, 2 \int \underbrace{\frac{d^4k_E}{(2\pi)^4}}_{\frac{k_E^2 \, dk_E^2}{16\pi^2}} \, d\alpha \, d\beta \, \frac{1}{(k_E^2 + A^2)^3}$$

$$\sim \int^{\Lambda} \frac{dk_E^2}{k_E^4}$$

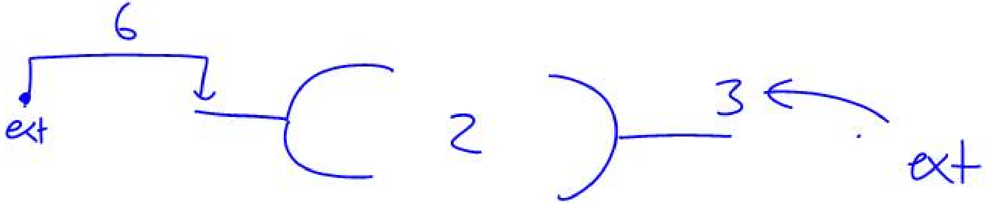
the integration is not divergent .... but it does depend on s...

$$= \frac{-i\lambda^4}{(s-m^2)} \, \Delta F(s)$$

SELF ENERGY

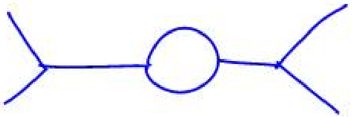


$$e^{i\int d^4x\,\delta\mathcal{L}} \to 1 + \dots + \frac{1}{2!}\left(\frac{\lambda}{3!}\right)^2 \int d^4x\,\phi^3 \int d^4y\,\phi^3$$



combinatorics: $\dfrac{6\times3\times2}{2\cdot3\cdot2\cdot3\cdot2} = \dfrac{1}{2}$

$$-i\Sigma(p^2) = \frac{1}{2}(i)^2(-i)^2\,\lambda^2\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{(k^2-m^2)(k^2-2k\cdot p+p^2-m^2)}$$

Don't set $p^2 = m^2$ since might be 

Feynman parameterize

$$\Sigma(p^2) = \frac{i}{2}\lambda^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\alpha\, d\beta \, \frac{\delta(1-\alpha-\beta)}{\left[\beta(k^2-m^2) + \alpha(k^2 - 2k\cdot p + p^2 - m^2)\right]^2}$$

$$= \frac{i}{2}\lambda^2 \int \frac{d^4k}{(2\pi)^4} \int_0^1 d\alpha \, \frac{1}{\left(k^2 - m^2 - 2\alpha k\cdot p + \alpha p^2\right)^2}$$

Shift $k^\mu = k'^\mu + \alpha p^\mu$

$$= \frac{i}{2}\lambda^2 \int \frac{d^4k'}{(2\pi)^4} \int_0^1 d\alpha \, \frac{1}{\left(k'^2 - m^2 - \alpha^2 p^2 + \alpha p^2\right)^2}$$

Wick rotate $\quad i\,(-1)^2$

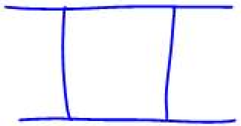
$$= \frac{-\lambda^2}{32\pi^2}\int_0^1 d\alpha \int \frac{dk_E^2\, k_E^2 + A^2 - A^2}{(k_E^2 + A^2)^2}$$

$$\ln \int_0^{\Lambda^2} dk_E^2 \left[\frac{1}{k_E^2 + A^2} - \frac{A}{(k_E^2+A^2)^2}\right]$$

$$\Sigma(p^2) = \frac{-\lambda^2}{32\pi^2}\int_0^1 d\alpha\, \ln\left[\frac{\Lambda^2}{m^2 - \alpha(1-\alpha)p^2}\right] + \text{finite} \ldots$$

This is divergent

Box diagrams



$$\sim \int_0^{\Lambda} k^2\, dk^2 \left(\frac{1}{k^2}\right)^4 \sim \frac{1}{\Lambda^4}$$

not divergent ...

$$\frac{dk^2}{k^6} \sim \frac{dx}{x^3} \sim \frac{1}{x^2} \sim \frac{1}{\Lambda^4} \checkmark$$