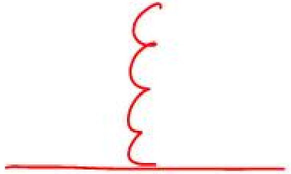
NAGT Renormalization

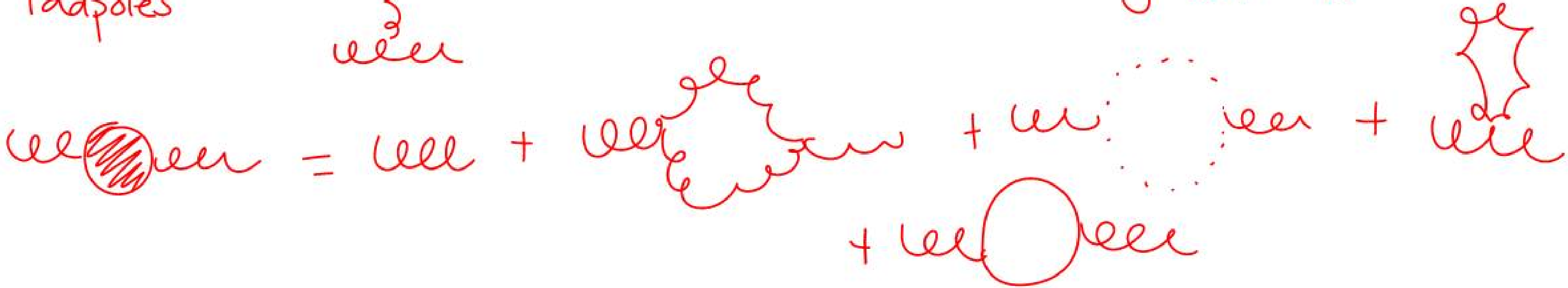


$$g_R = \frac{Z_3^{1/2} Z_2}{Z_1} g_0$$

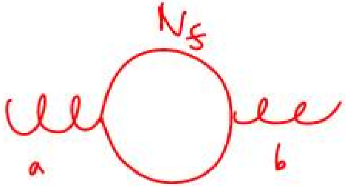
Gluon Propagator

"Tadpoles"

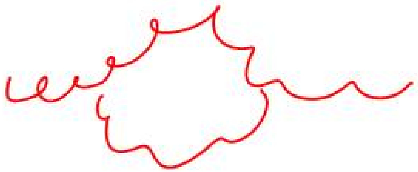
these are all zero ... here $\int \frac{d^4k}{(2\pi)^4} \frac{k^\mu}{k^2} \overset{\text{odd}}{=} 0$



Keeping the poles: $\Pi^{ab}_{\mu\nu} = \delta^{ab} \frac{g^2}{16\pi^2 \epsilon} \left(g_{\mu\nu} p^2 + p_\mu p_\nu \right) \left(5 - \frac{2N_f}{3} \right)$

Group Theory: 

$\mathrm{Tr}\, T^a T^b = \frac{1}{2} \delta^{ab}$

 $+$  $\sim f^{acd} f^{dcb}$

$\sim \Lambda^c_{ad} \Lambda^c_{db}$ Adjoint rep generators

$= C_A \mathbb{1}_{ab}$ quadratic Casimir in adjoint rep

$= N_c$ for $SU(N_c)$ $N_c = 3$

$$\Rightarrow Z_3 = 1 + \frac{g^2}{16\pi^2 \epsilon} \left(5 - \frac{2N_f}{3} \right)$$

-nergy

$$\sim (T^c)_{ad}\,(T^c)_{db} = C_2(F)\,\mathbb{1}_{ab}$$

Quadratic Casimir in fundamental rep

$$= \frac{N_c^2-1}{2N_c} \qquad SU(N_c)$$

$$\rightarrow \; Z_2 = 1 - \frac{g^2}{12\pi^2\epsilon}$$

Vertex Corrections

$\sim (C_F - C_A/2)$ $\quad$ $\sim \frac{C_A}{2}$

$$\Lambda^a_\mu = \frac{g^3}{16\pi^2\epsilon}\,\frac{13}{3}\,\gamma_\mu T^a$$

$$Z_1 = 1 - \frac{g^2}{16\pi^2\epsilon}\,\frac{13}{3}$$

Now $g_R = \frac{Z_3^{1/2} Z_2}{Z_1} \mu^{-\epsilon} g_0$

$$= \mu^{-\epsilon} g_0 \left(1 + \frac{g_0^2}{16\pi^2\epsilon}\left[\frac{5}{3} - \frac{N_f}{3} - \frac{4}{3} + \frac{13}{3}\right] + \dots\right)$$

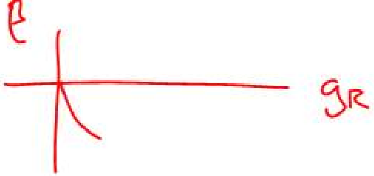
$$= g_0\mu^{-\epsilon}\left(1 - \frac{g_0^2}{32\pi^2\epsilon}\left(-11 + \frac{2N_f}{3}\right)\right)$$

Now compute β as in QED ...

$$\beta \sim \mu \frac{dg_R}{d\mu} = \frac{g_R^3}{16\pi^2}\overbrace{\left(-11+\frac{2N_f}{3}\right)}^{b}$$

$$\alpha_R(\mu^2) = \frac{g_R^2(\mu^2)}{4\pi} = \frac{\alpha_0}{1+\frac{\alpha_0}{4\pi}\, b \ln \mu^2/\mu_0^2}$$

$N_f < \frac{33}{2}$



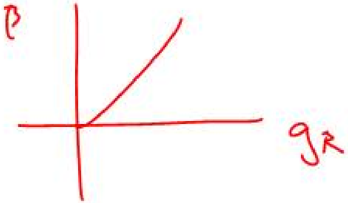
where $\alpha \to \infty$

asymptotic freedom

→ quark confinement

→ jet production

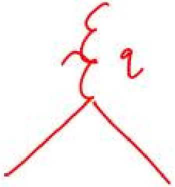
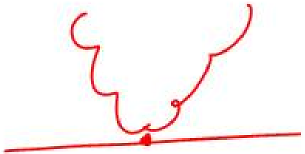
$N_f > \frac{33}{2}$

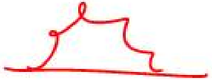
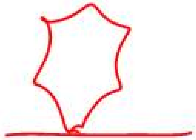


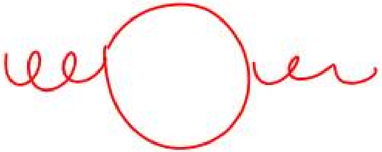
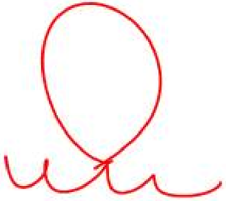
→ QED-like

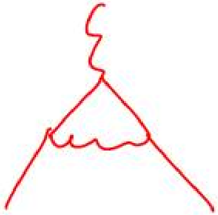
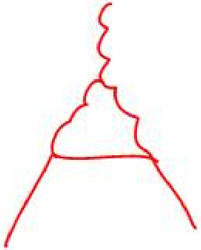
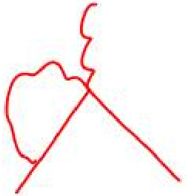
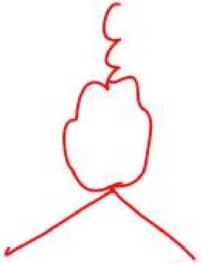
Thresholds: quarks basically contribute when $q^2 > m^2$ where they can be pair produced.

SCALAR PROBLEM HELP: OK to take $m_s = 0$

scalar F.R.:  $igT^a q^\mu$  $ig^2(T^aT^b+T^bT^a)g^{\mu\nu}$

scalar self energy:  + 

gauge boson self energy: usual NAGT +  + 

vertex:  +  +  + 

$$Z_1 = \frac{1}{(4\pi)^2}\left(4C_F - 2C_A\right)$$

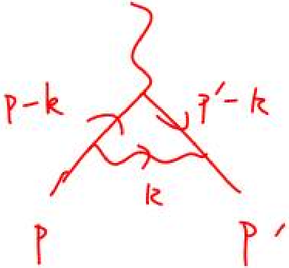
$$Z_2 = \frac{4C_F}{(4\pi)^2}$$

$$Z_3 = \frac{1}{(4\pi)^2}\left[\frac{10}{3}C_A - \frac{1}{3}N_f\right]$$

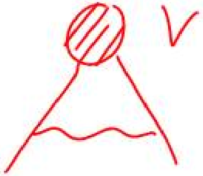
IR DIVERGENCES IN QED

There are extra divergences as $k \to 0$

eg



or



$$\int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2} (-ie\gamma^\mu) \frac{i}{\not p - \not k - m} V \frac{i}{\not p' - \not k - m} (-ie\gamma_\mu)$$

Take $k \to 0$ "Eikonal approximation"

$$\frac{1}{(p-k)^2 - m^2} = \frac{1}{-2p\cdot k}$$

since $k^2 \simeq 0$, $p^2 = p'^2 = m^2$

$$-ie^2 \int \frac{d^4k}{(2\pi)^4} \; \frac{\gamma^\mu (\not p + m) \, V \, (\not p + m) \gamma_\mu}{k^2 \quad 2p\cdot k \quad 2p'\cdot k}$$

k integration: $\sim \int_0^\Lambda \frac{k^2 \, dk^2}{k^4} \sim \ln \Lambda^2/0^2$!

NB/ divergonent term depends on $p \cdot p'$ so can't suck into couplings

Carry on rigorously: numerator: $N = \bar{u}(p) \gamma^\mu (\not p + m) \, V \, (\not p' + m) \gamma_\mu u(p')$

we use $(\not p' - m) u = 0$ after $\{\gamma^\mu, \gamma^\nu\} = 2 g^{\mu\nu}$

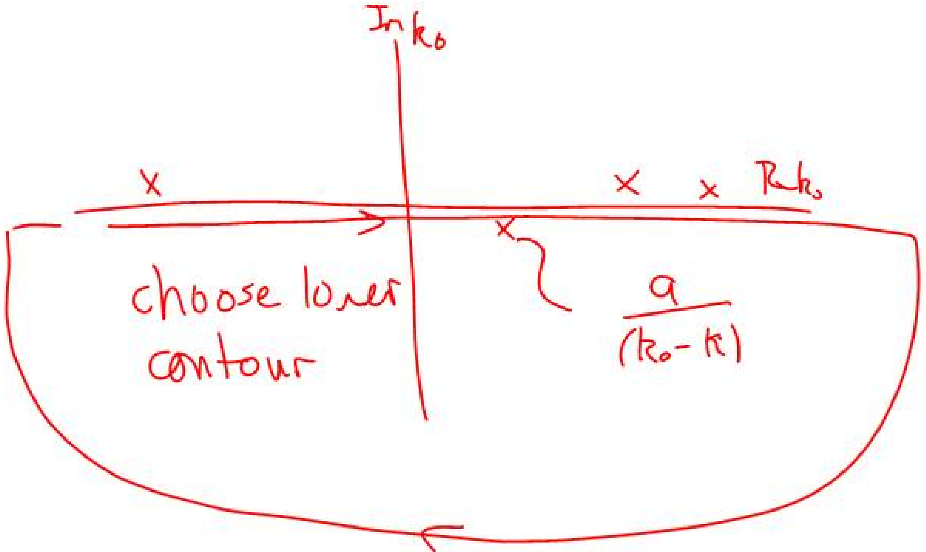
$$(\not p' + m) \gamma^\mu u(p') = \Big(\underbrace{-\gamma^\mu \not p' + \gamma^\mu m}_{\to 0} + 2 p'^\mu \Big) u(p')$$

$$\Rightarrow N = 4\, p\cdot p' \;\bar{u}(p)\, V\, u(p')$$

Full diagram: $$-ie^2\, p\cdot p'\, V \int \frac{d^4k}{(2\pi)^4}\, \frac{1}{k^2+i\epsilon}\, \frac{1}{p\cdot k - i\epsilon}\, \frac{1}{p'\cdot k - i\epsilon}$$

$$\frac{1}{(p-k)^2-m^2+i\epsilon}$$

Do the k_0 integral by contour integration



$$\int \frac{dk_0}{2\pi}\, \frac{1}{(k_0^2-|\underline{k}|^2+i\epsilon)(p_0k_0-\underline{p}\cdot\underline{k}-i\epsilon)} \to \frac{1}{-2p\cdot k+i\epsilon}$$

$$(p_0'k_0-\underline{p}'\cdot\underline{k}-i\epsilon) \to -\frac{1}{(2p\cdot k-i\epsilon)}$$

$$(k_0+|\underline{k}|-i\epsilon)(k_0-|\underline{k}|+i\epsilon)$$

$$2\pi i\, \frac{1}{2|\underline{k}|(p_0|\underline{k}|-\underline{p}\cdot\underline{k})(p_0'|\underline{k}|-\underline{p}'\cdot\underline{k})}$$

$$e^2\, p\cdot p'\, V \int \frac{d^3k}{(2\pi)^3} \frac{1}{2|k|} \frac{1}{(p^0|k| - p\cdot k)} \frac{1}{(p'^0|k| - p'\cdot k)}$$

$$= e^2\, p\cdot p'\, V \int_\lambda^{E_{UV}} \frac{k^2 \cdot dk}{(2\pi)^3} \frac{1}{2k^3} \frac{1}{(E - p\cos\theta)} \frac{1}{(E' - p'\cos\theta')}$$

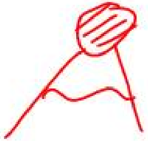
$$= e^2\, p\cdot p'\, V \ln\frac{E_{UV}}{\lambda} \underbrace{\int d\Omega \frac{1}{(E - p\cos\theta)} \frac{1}{(E' - p'\cos\theta')}}_{I_\Omega}$$

Again diverges as $\lambda \to 0$

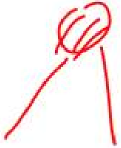
BLOCH-NORDSIECK THEOREM

The divergence is not present $|-i\mathcal{M}|^2$ for a physical process.

The leading term in prob from cross terms between

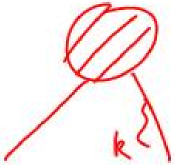


&



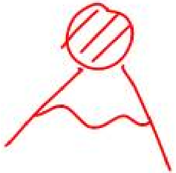
$\sim e^2$

The claim is it cancels vs cross term between



$\sim e^2$

As $k \to 0$ such soft photons become undetectable in the expt. so we must include...



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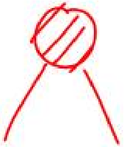
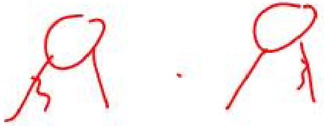


$$\sim \frac{e^2}{(2\pi)^3}\, p \cdot p' \, |V|^2 \ln E/\lambda \; I_\Omega$$

2 cross terms

So now look at 

$$\sim 2 \underbrace{\int \frac{d^3k}{(2\pi)^3}\frac{1}{2|\underline{k}|}}_{\text{sum over phase space of } k} \sum_{\varepsilon} \left[\varepsilon_\mu(-ie\gamma^\mu)\frac{i}{\not p - \not k - m} V\right]\left[\frac{-i}{\not p' - \not k - m}(ie\gamma^\nu)V^\dagger \varepsilon_\nu^*\right]$$

$k \to 0$ ... & use $\sum \varepsilon_\mu \varepsilon_\nu^* = -g_{\mu\nu}$

$$\sim -\int \frac{d^3k}{(2\pi)^3}\frac{1}{\cancel{2}|k|}\, e^2 \, \frac{\gamma^\mu(\not p+m)V(\not p'+m)\gamma_\mu V^\dagger}{(2p\cdot k)(2p'\cdot k)}$$

Numerator is precisely that of the loop diagram $\longrightarrow 4p\cdot p'|V|^2$ & $\underset{\text{on-shell!}}{k_0 = |\underline{k}|}$

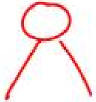
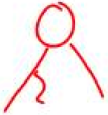
$$\sim -\int_\lambda^{\Delta E} e^2\, p\cdot p'\, |V|^2 \frac{|k|^2\, dk\, d\Omega}{(2\pi)^3 |k|^3} \frac{1}{(E-p\cos\theta)(E'-p'\cos\theta')}$$

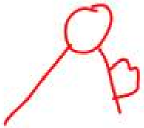
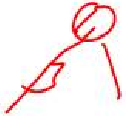
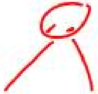
ΔE is detector resolution

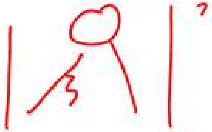
$$\sim -\frac{e^2}{(2\pi)^3}\, p\cdot p'\, |V|^2 \ln \Delta E/\lambda \; I_\Omega$$

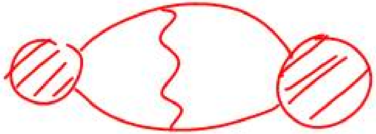
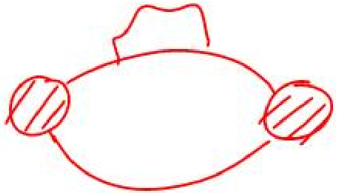
Adding the two contributions to the prob gives

$$-\frac{e^2}{(2\pi)^3}\, p\cdot p\, |V|^2 \ln \Delta E/E_{uv} \; I_\Omega$$

Conclusion:  &  cancels vs  & 

Likewise  is IR divergent... here interference of  & 

cancels vs 

At a deeper level note  & 

do not have IR divergences – we have got IR d.v. from cutting these diagrams in different ways!