① GAUGE THEORY

QED

MINIMAL SUBSTITUTION

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

Orsted + Ampère 1820
↓
1892 for Lorentz to
derive

$$S = \int L\,dt \qquad L = \tfrac{1}{2}m|\dot{\vec{x}}|^2 + q\dot{\vec{x}}.\vec{A} - q\phi$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla}\phi \quad , \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Remember: $\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{\vec{x}}.\vec{\nabla}$

$$\dot{\vec{x}} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\dot{\vec{x}}.\vec{A}) - (\dot{\vec{x}}.\vec{\nabla})\vec{A}$$

$$\vec{p}_{gen} = \frac{\partial L}{\partial \dot{\vec{x}}} = m\dot{\vec{x}} + q\vec{A}$$ "minimal sub"

IN 4-VECTOR FORM

$$A^{\mu} = (\phi, \vec{A}) \qquad F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu} \sim \begin{pmatrix} 0 & E & E & E \\ -E & 0 & B & -B \\ -E & -B & 0 & B \\ -E & B & -B & 0 \end{pmatrix}$$

$$f^{\mu} = q\underbrace{u_{\nu}}_{\gamma(c,\vec{v})}F^{\mu\nu} \quad \& \quad p^{\mu} = mu^{\mu} + qA^{\mu}$$

$$\xrightarrow{QM} i(\partial^{\mu} - iqA^{\mu})$$

"adhoc rule"

ONE NOTES GAUGE INVARIANCE OF, $\vec{E}, \vec{B}$ & $F^{\mu\nu}$

$$A^{\mu} \rightarrow A^{\mu} + \frac{1}{q}\partial_{\mu}\omega(x^{\mu})$$

GLOBAL TRANSFORMATIONS OF DIRAC ACTION

$$\mathcal{L} = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi$$

$$\Psi \to e^{i\omega}\Psi \qquad = (1 + i\omega + \ldots)\Psi \qquad \delta\Psi = i\omega\Psi$$

$$\bar{\Psi} \to e^{-i\omega}\underset{\psi^\dagger\gamma^0}{\underset{\wr}{\bar{\Psi}}} \qquad = (1 - i\omega + \ldots)\bar{\Psi} \qquad \delta\bar{\Psi} = -i\omega\bar{\Psi}$$

If ω is independent of x^μ then this is an invariance of the action.

NB $\quad e^{i\alpha}e^{i\beta} = e^{i(\alpha+\beta)} = e^{i\beta}e^{i\alpha}$

"abelian" transformations

GAUGE TRANSFORMATIONS

$$\Psi \to e^{i\alpha(x^\mu)}\Psi \qquad \text{(local or } \underline{\text{gauge}}\text{)}$$

Get extra term $\quad \delta\mathcal{L} = -\bar{\Psi}\gamma^\mu(\partial_\mu\omega)\Psi$

In full QED it does work though

$$\mathcal{L} = \bar{\Psi}\left[i\gamma^\mu(\partial_\mu - iqA_\mu)\right]\Psi$$

$$\Psi \to e^{i\omega}\Psi \quad \& \quad A^\mu \to A^\mu + \underbrace{\frac{1}{q}\partial_\mu\omega}_{\delta A^\mu}$$

$$\delta\mathcal{L} = -\bar{\Psi}\gamma^\mu(\partial_\mu\omega)\Psi + \bar{\Psi}\gamma^\mu\frac{q}{q}(\partial_\mu\omega)\Psi$$

$$= 0 \ \checkmark$$

Now reverse the logic: the symmetry requires an A^μ with the gauge invariance transformation. QED appears by magic!

We must write down all gauge invariant terms - an extra Lorentz invariant term in $F^{\mu\nu}$

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

(normalization is conventional normalizati as q

Up to dimension 4 terms we uniquely have

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\not{D} - m)\psi$$

$$D_\mu = \partial_\mu - iqA_\mu$$

"Covariant derivative"

NB/ We have predicted $M_\gamma = 0$

$M_\gamma^2 A^\mu A_\mu$ is not gauge invariant.

$$F^{op}_{\mu\nu} = +\frac{i}{q}[D_\mu, D_\nu] = +\frac{i}{q}\cancel{[\partial_\mu, \partial_\nu]} + [\partial_\mu, A_\nu] + [A_\mu, \partial_\nu] + ie\cancel{[A_\mu, A_\nu]}$$

$$= \partial_\mu A_\nu \underbrace{- A_\nu \partial_\mu + A_\mu \partial_\nu}_{0 \text{ when act on right}} - \partial_\nu A_\mu$$

NON-ABELIAN GAUGE INVARIANCE

If there are 2 or more identical particles then QMly we can change basis

eg u & d look the same to the strong force

so $\psi = \frac{1}{\sqrt{2}}(u \pm d)$ are good basis vectors too.

Transform generally:

$$\underset{\begin{pmatrix}u\\d\end{pmatrix}\ \ \overset{\uparrow}{1\text{ or }2}}{\psi_i} \longrightarrow \underbrace{\left(e^{i\,\underset{\text{real parameters}}{\omega^a}\,\underset{\text{generators}}{T^a}}\right)^j{}_i}_{U^j{}_i} \psi_j$$

eg for SU(2) $T^1 = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $T^2 = \frac{1}{2}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$

$T^3 = 1/2\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $(T^i = T^{i\dagger})$

Note $[T^a, T^b] = i f^{abc} T^c$

$$\& \quad \underset{1 + i\omega_1^a T^a + \dots}{\left(e^{i\omega_1^a T^a}\right)^k{}_i} \left(e^{i\omega_2^b T^b}\right)^j{}_k \neq \left(e^{i\omega_2^b T^b}\right)^k{}_i \left(e^{i\omega_1^a T^a}\right)^j{}_k$$

The transformations do not commute : "non-abelian"

Global versions of these transformations are an invariance of

$$\mathcal{L} = \overset{U^\dagger = U^{-1}}{\bar{\psi}^i}\left(i\gamma^\mu \partial_\mu - m\right)\overset{U}{\psi_i}$$

$$U^\dagger U = 1$$

If we make symmetry local get a term

$$\bar{\Psi}\, U^{-1}\, i\gamma^\mu (\partial_\mu U)\, \Psi$$

↑ note there are 3 $\omega^a(x)$ in here for SU(2). We need 3 $A^{\mu a}$'s to get rid of these terms.

Introduce $A^\mu = A^{a\mu} T^a$ ← summed over generators

We are going to add terms like

$$-\underset{U^{-1}}{\bar{\Psi}}\, \gamma^\mu g\, \underbrace{\underset{U}{T^a}\, \underset{U^{-1}}{A^a_\mu}}_{\text{need}}\, \underset{U}{\Psi}$$

For this term alone to be invariant we need

$$A_\mu \to U A_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)\, U^{-1}$$

this extra term allows us to cancel the unwanted terms above.

note this places A_μ in the adjoint rep since $N \otimes \bar{N} = \text{Adj} + 1$

So our gauge invariant theory is

$$\mathcal{L} = \bar{\Psi}^i \left(i\not{D} - m\mathbb{1}\right)^j{}_i\, \Psi_j$$

flavour diagonal

$$D_\mu = \partial_\mu \mathbb{1} + i g\, T^a A^a_\mu$$

sign is convention on g

Note under GT $D_\mu \to U D_\mu U^{-1}$

We then construct

$$F^{\mu\nu} = \frac{-i}{g}\left[D_\mu , D_\nu\right]$$

$$= \frac{-i}{g}\cancel{\left[\partial_\mu , \partial_\nu\right]} + \left[\partial_\mu , A_\nu\right] + \left[A_\mu , \partial_\nu\right] + ig\left[A_\mu , A_\nu\right]$$

$$F^a_{\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g f_{abc} A^b_\mu A^c_\mu$$

Note $\quad F^{\mu\nu} \xrightarrow{GT} U F^{\mu\nu} U^{-1}$

To make an invariant

$$\mathcal{L} = -\frac{1}{2} \operatorname{Tr} F^{\mu\nu} F_{\mu\nu}$$

$$= -\frac{1}{4} F^{a\mu\nu} F_{a\mu\nu}$$

cyclic

$\operatorname{Tr} U^{-1} B U$

$\operatorname{Tr} T^a T^b = \frac{1}{2}\delta^{ab}$

$$\boxed{\mathcal{L}_{NAGT} = -\frac{1}{2} \operatorname{Tr} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}\left(i\not{D} - m\mathbb{1}\right)\psi}$$

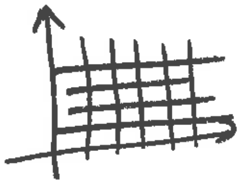
② GAUGE FIXING

The Fadeev-Popov procedure is quite subtle - here I try a first introduction - but it's well understood.

A (Dumb?) Warm Up

Consider the area integral

$$\int_0^L \int_0^L dx\, dy = L^2$$



We can turn it into a line integral by inserting $\delta(y)$

$$\int_0^L \int_0^L \delta(y)\, dx\, dy = \int_0^L dx = L$$

Now let's turn to polar coords

$$r(x,y) \qquad \theta(x,y)$$

in general some parallelogram

area = $\vec{a} \times \vec{b}$

The area changes by a "Jacobian" factor (a determinant)

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix} = r$$

$$\iint dx\, dy \rightarrow \iint J\, dr\, d\theta \rightarrow \iint r\, dr\, d\theta$$

Now you might try to get the line integral along x by inserting $\delta(\theta)$ but...

$$\iint \delta\theta \, r \, dr \, d\theta = \int_0^L x \, dx = L^2/2$$

To correct this we need to change the normalization of the δ-fn when we change coordinates

$$\delta(y) \rightarrow \underset{\substack{\uparrow \\ \text{this removes} \\ \text{the } r}}{\frac{1}{J}\Big|_{\theta=\theta_0}} \; \underset{\substack{\uparrow \\ \text{need to remove } x \text{ at } \theta=0 \\ \text{or } y \text{ at } \theta=\pi}}{\delta(\theta-\theta_0)}$$

NAGT

Gauge freedom $\quad A^\mu \rightarrow U A^\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}$

For ∞-imal transforms $\quad U = 1 + i\omega^a T^a$

$$\delta A^a{}_\mu = f^{abc} A^b_\mu \omega^c - \frac{1}{g}(\partial_\mu \omega^a)$$

This is a redundancy of description - you change A^μ but $\vec{E}$ & $\vec{B}$ are the same.... this over counting causes trouble in the path integral

$$S = \int d^4x \; -\frac{1}{4} F^{a\mu\nu} \underbrace{F^a{}_{\mu\nu}}_{\partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \dots}$$

$$= \int d^4x \; -\frac{1}{4}\Big(\overbrace{\partial^\mu A^{a\nu}}^{-} \partial_\mu A^a_\nu - \overbrace{\partial^\mu A^{a\nu}}^{-} \partial_\nu A^a_\mu$$

$$- \underbrace{\partial^\nu A^{a\mu}}_{-} \partial_\mu A^a_{\nu} + \underbrace{\partial^\nu A^{a\mu}}_{-} \partial_\nu A^a_\mu$$

$$= \int d^4x \; \frac{1}{2} A^a_\mu \underset{\substack{\uparrow \\ (g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\delta_{ab}}}{O^{\mu\nu}_{ab}} A^b_\nu$$

The propagator is the inverse of $O^{\mu\nu}$ but here there is no inverse because there's a zero eigenvector

$$O^{\mu\nu} \, \partial_\nu \omega(x) = 0$$

So we should restrict to solutions in some fixed gauge

$$D[A_\mu] \rightarrow D[A_\mu^{\omega=0}] \; D[\omega] \; \delta(\omega)$$

But we normally fix a gauge through a condition like

$$\partial^\mu A_\mu = f(x)$$

so we're tempted to include

$$\delta[\partial^\mu A_\mu - f]$$

which we can as long as we include a "functional determinant" J

$$Z = \int D[A_\mu] \, \delta(\partial^\mu A_\mu - f) \; J\big|_{\omega=0} \; e^{iS}$$

↑ this means we only need ∞-itesimal transformation

Now $$\delta(\partial^\mu A^a_\mu) = f^{abc} \, \partial^\mu A^b_\mu \, \omega^c - \frac{1}{g} \partial^\mu \partial_\mu \omega^a$$

$$\Rightarrow \frac{\delta(\partial^\mu A^a_\mu)(x)}{\delta \omega^b(y)} = f^{abc} \, \partial^\mu A_\mu^c \, \delta^4(x-y) - \frac{1}{g} \partial^\mu \partial_\mu \delta^4(x-y) \delta^a$$

$$= -\frac{1}{g} \delta^4(x-y) \; \partial^\mu D^{ab}_\mu \; (x)$$

$$D^{ab}_\mu = \partial_\mu \delta^{ab} - g f^{abc} A_{c\mu}$$

since f^{abc} are adjoint generators.

We want the determinant of this.... here's a trick

$$\det M = \int D[\eta]\, D[\bar\eta]\, e^{-\int d^4x\, \bar\eta(x)\, M\, \eta(x)}$$

Grassman fields

write η in eigenstates of M $\quad M\phi_n = \lambda_n \phi_n$

$$\eta = \sum_n c_n \phi_n$$

Grassmanian $\int dc_n 1 = 0$

$\int dc_n c_n = 1$

$$= \int d\bar c_n\, dc_n\, e^{-\Sigma \lambda_n \bar c_n c_n}$$

by completeness

$$= \Pi \lambda_n = \det M \checkmark$$

So we enact J by adding to the action

$$iS = i\int d^4x\, \bar\eta^a\, \partial^\mu D_\mu^{ab}\, \eta^b$$

($-\frac{1}{g}$ absorbed into norms of η & $\bar\eta$)

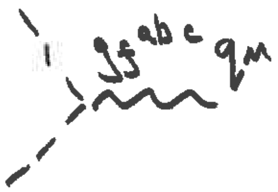
The η & $\bar\eta$ fields are "Fadeev-Popov" ghosts

They are not physical so don't occur as in or out states in Feynman diagrams - only in loops.

They are Grassmanian scalars - they get a minus sign on their loops.

ghost prop

&



One final trick.... any choice of $f(x)$ gives the same answer... pick two & add the answers & you'll get double Z... why not pick a smeared distribution?

$$Z = \int D[f]\, e^{i\int d^4x \, \frac{1}{2(1-\xi)} f^2(x)} \int D[A_\mu]\, \delta[\partial.A - f]\, J\, e^{iS}$$

OK upto a constant... do the f integral...

$$Z = \int D[A_\mu]\, J\, e^{iS}\, e^{i\int d^4x \, \frac{1}{2(1-\xi)} (\partial^\mu A_\mu)^2}$$

↳ integrate by parts

$-A_\mu \partial^\mu \partial^\nu A_\nu$

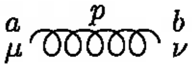
Now $$O^{\mu\nu}_{ab} = i\delta_{ab}\left(g^{\mu\nu}\partial^2 + \frac{\xi}{(1-\xi)}\partial^\mu\partial^\nu\right)$$

which has an inverse

$$-\delta_{ab}\left(g_{\mu\nu} - \xi\frac{p_\mu p_\nu}{p^2}\right)\frac{i}{p^2}$$

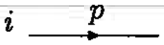
$\xi = 0$ is a good choice! "Feynman gauge"

Gluon (Feynman gauge)



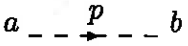
$-i\,\delta_{ab}g_{\mu\nu}/p^2$

Fermion



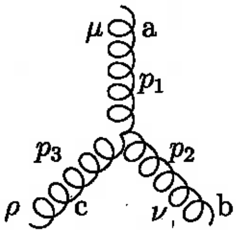
$i\,\delta_{ij}(\gamma^\mu p_\mu + m)/(p^2 - m^2)$

Faddeev-Popov ghost

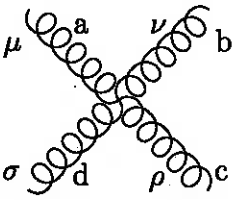


$i\,\delta_{ab}/p^2$

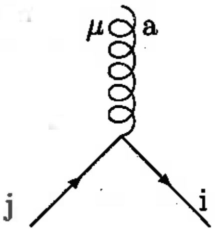
Vertices:
(all momenta are flowing into the vertex).



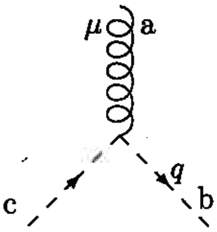
$$-g\,f_{abc}\left(g_{\mu\nu}\left(p_1-p_2\right)_\rho + g_{\nu\rho}\left(p_2-p_3\right)_\mu + g_{\rho\mu}\left(p_3-p_1\right)_\nu\right)$$



$$\begin{aligned}&-i\,g^2 f_{eab}f_{ecd}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)\\&-i\,g^2 f_{eac}f_{ebd}\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)\\&-i\,g^2 f_{ead}f_{ebc}\left(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}\right)\end{aligned}$$

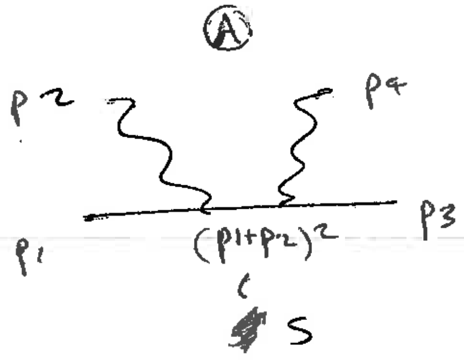


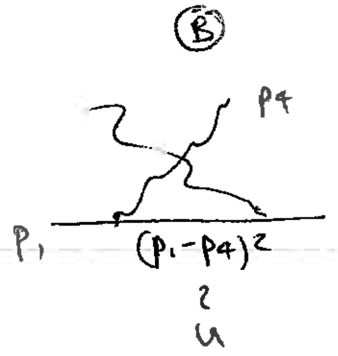
$$-i\,g\,\gamma^\mu\left(T^a\right)_{ij}$$

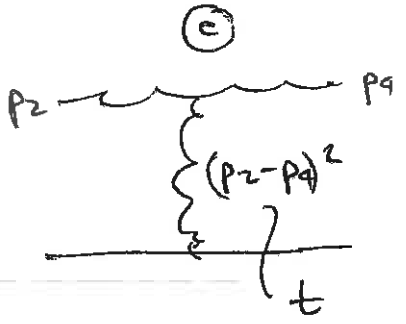


$$g\,f_{abc}\,q_\mu$$

EG NA COMPTON SCATTERING







Assume $m_e = 0$.

Ⓐ $\epsilon_\mu(\lambda_2)\,\mathcal{E}_\nu(\lambda_4)\,\bar{u}^j(p_3)\left(-ig\gamma^{\nu}_{\mu}(T^b)^k{}_j\right)\left(\frac{i\,\not{p}_2+\not{p}_1}{s}\right)$
$\left(-ig\gamma^{\mu}(T^a)^i{}_k\right)$

$$= -\frac{ig^2}{s}\,\epsilon_\mu(\lambda_2)\epsilon_\nu(\lambda_4)\,\bar{u}(p_3)\left(\gamma^\nu(\not{p}_1+\not{p}_2)\gamma^\mu\right)\left(T^bT^a\right)u(p_1)$$

Ⓑ $$= -\frac{ig^2}{u}\,\epsilon_\mu(\lambda_2)\,\epsilon_\nu(\lambda_4)\,\bar{u}(p_3)\left(\gamma^\nu(\not{p}_1-\not{p}_4)\gamma^\mu\right)\left(T^aT^b\right)u(p_1)$$

ⓒ p_4 out is $-p_4$ flowing into the 3 pt vertex ; p_4-p_2 flows in ; p_2 flows in

$$-g f_{abc}\left[g_{\mu\nu}(p_2+p_4)_\rho + g_{\rho\nu}(p_2-2p_4)_\mu + g_{\mu\rho}(p_4-2p_2)_\nu\right.$$

$$\times\ \epsilon^\mu(\lambda_2)\,\epsilon^\nu(\lambda_4)\,\bar{u}^j(p_3)\left(ig\gamma_\sigma(T^c)^i{}_j\right)u_i(p_1)\left(-\frac{ig^{\rho\sigma}}{t}\right)$$

$$\boxed{if_{abc}T^c = [T^a,T^b]}$$

$$= -\frac{ig^2}{t}\,\epsilon^\mu(\lambda_2)\epsilon^\nu(\lambda_4)\,\bar{u}(p_3)\,[T^a,T^b]\,\gamma^\rho u(p_1)$$

$$\left(g_{\mu\nu}(p_2+p_4)_\rho - 2p_{4\mu}g_{\nu\rho} - 2p_{2\nu}g_{\mu\rho}\right)$$

$\wr$

eg $p_2\cdot\epsilon(\lambda_2) = 0$

$p_4\cdot\epsilon(\lambda_4) = 0$

③ BEYOND TREE LEVEL

Our main job now is to go beyond tree level in perturb. theory which will involve self interactions for particles. This is a tricky issue even in classical theory!

eg What acceleration does an e^- feel in an $\vec{E}$ field? $\vec{F} = m\vec{a} = q\vec{E}$

BUT

$$\rho = \frac{e}{4/3\pi R^3}$$

$$\delta E = \int_0^R \frac{4\pi r^2 dr\,\rho \cdot 4/3\pi r^3 \rho}{4\pi\epsilon_0 r}$$

$$= \frac{e^2}{(4/3\pi R^3)^2}\,\frac{1}{\epsilon_0}\,\frac{4}{3}\pi\int_0^R r^4 dr$$

$$= \frac{e^2}{(4/3\pi R^3)^2}\,\frac{1}{\epsilon_0}\,\frac{4}{3}\pi\,\frac{R^5}{5}$$

$$= 3/5\,\frac{e^2}{4\pi\epsilon_0 r}$$

Presumably $R \to 0$ ... so $E = mc^2 \to \infty$??

Well no... this isn't a correct theory of me - it doesn't apply at quantum scales... so we use the physical me & it works.... that's basically renormalization which we'll see in field theory

ϕ^4 THEORY (BRIEFLY)

$$\mathcal{L} = 1/2\,(\partial^\mu\phi)(\partial_\mu\phi) - 1/2\, m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$

Consider the 2-pt Green's ~~gunction~~ function

$$G^2(x,0) = \langle 0|T\,\phi(x)\,\phi(0)|0\rangle$$

$$= \Delta_0(x) + \frac{1}{Z}\int D[\phi]\,e^{iS_0}\left(\frac{-i\lambda}{4!}\right)\int d^4z\ \phi^4(z)\phi(x)$$

$$+\ O(\lambda^2)$$

consider the term

Wick contract:

$$\underbrace{\phi(z)\,\phi(z)}\,\underbrace{\phi(z)\,\overbrace{\phi(z)\,\phi(x)}^{4}\,\phi(0)}_{3}$$

(There's also

which is part og vacuum & cancels vs 1/Z)

There are 4×3 such contractions

$$\tilde{G}^2(p) = \overset{i/p^2-m^2}{\Delta_0(p)} - \frac{i\lambda}{2}\int d^4x\,e^{-ip.x}\int dz\ \underset{k_1}{\Delta_0(x-z)}\ \underset{k_2}{\Delta_0(z)}\ \underset{k}{\Delta_0(0)}$$

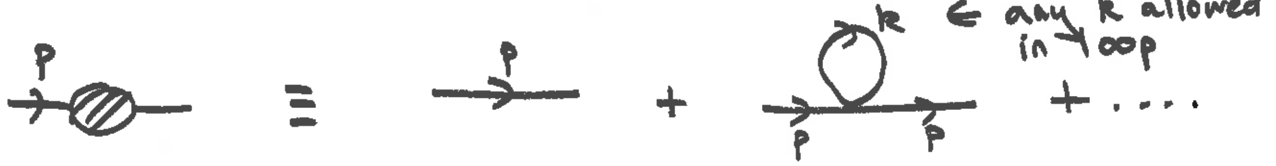
$$\frac{-i\lambda}{2}\int d^4x\,e^{-ip.x}\int d^4z\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4h}{(2\pi)^4}\,e^{ik_1(x-z)}\,e^{ik_2z}$$

$$\times\ \frac{i^3}{(k_1^2-m^2)(k_2^2-m^2)(k^2-m^2)}$$

z integration ⇒ $(2\pi)^4\,\delta^4(k_1-k_2)$ ⇒ k_2 integral easy

x integration ⇒ $(2\pi)^4\,\delta^4(k_1-p)$ ⇒ k_1 integral easy

$$G^2(p) = \Delta_0(p) - \frac{i\lambda}{2}\,\frac{i}{p^2-m^2}\left[\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}\right]\frac{i}{p^2-m^2} + \dots$$



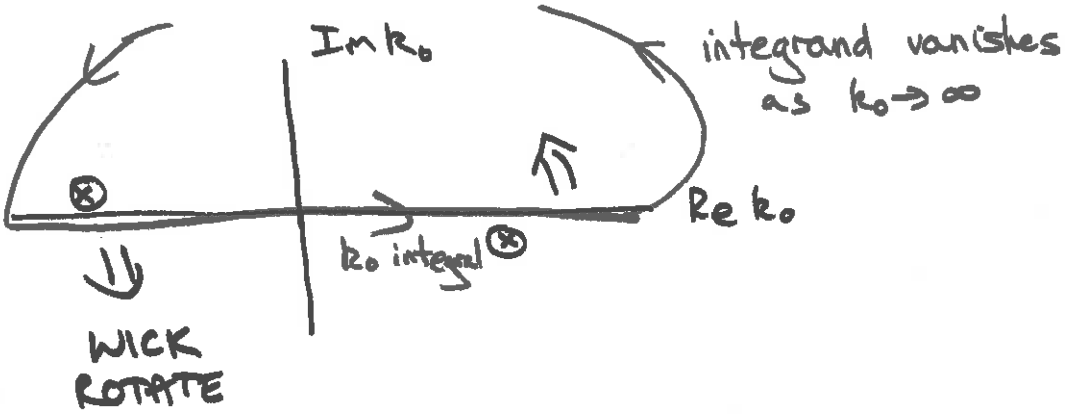
Now remove the external propagators 

$$= -\frac{i\lambda}{2}\int\frac{d^4k}{(2\pi)^4}\,\frac{i}{k^2-m^2}$$

The k_0 integral is subtle and we need to be more careful about poles → $+i\epsilon$ prescription

$$\frac{1}{k^2-m^2+i\epsilon} = \frac{1}{k_0^2-(|\underline{k}|^2+m^2)+i\epsilon}$$

$$= \frac{1}{(k_0-\omega_k+i\epsilon')(k_0+\omega_k-i\epsilon')} \qquad \omega_k=\sqrt{\underline{k}^2+m^2}$$



 $$= \frac{\lambda}{2}\int_{-i\infty}^{i\infty}\frac{dk_0}{2\pi}\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{k_0^2-\underline{k}^2-m^2+i\epsilon}$$

Let $l_0=-ik_0$: $$= \frac{i\lambda}{2}\int\frac{dl_0}{(2\pi)}\int\frac{d^3k}{(2\pi)^3}\,\frac{1}{-l_0^2-\underline{k}^2-m^2+i\epsilon}$$

$$= -\frac{i\lambda}{2}\int \frac{d^4k_E}{(2\pi)^4} \quad \frac{1}{(k_E^2+m^2)}$$

$\vec{k}_E = (l_0, \underline{k})$ is a Euclidean 4-vector

Integration d^4k :

$$\int \frac{d^4k}{(2\pi)^4} = \int \frac{k^3 \sin^2\theta}{(2\pi)^4} \sin\chi \; \underset{0\ldots\Lambda}{dk} \; \underset{0\ldots\pi}{d\theta} \; \underset{0\ldots\pi}{d\chi} \; \underset{0\ldots 2\pi}{d\phi}$$

1D: $\int dr$

2D: $\int r\,dr\,d\theta$

3D: $\int r^2 \sin\theta\, dr\, d\theta\, d\phi$

$$\frac{dk^2}{dk} = 2k$$

$$\int \frac{d^4k}{(2\pi)^4} = \int \frac{k^2\, dk^2}{2^5\pi^4} \int \underbrace{\sin^2\theta \,\sin\chi \,\overset{\pi/2}{\overset{\downarrow}{d\theta}}\, \overset{2}{\overset{\downarrow}{d\chi}}\, \overset{2\pi}{\overset{\downarrow}{d\phi}}}_{2\pi^2}$$

when integrating something angular independent

$$= \int \frac{k^2\, dk^2}{16\pi^2}$$

Back to ϕ^4:

$$= -\frac{i\lambda}{2}\int_0^{\Lambda^2} \frac{\overset{k_E^2+m^2-m^2}{\swarrow}}{} \frac{k_E^2\, dk_E^2}{16\pi^2} \quad \frac{1}{k_E^2+m^2}$$

$$= -\frac{i\lambda}{2}\int_0^{\Lambda^2} \frac{dk_E^2}{16\pi^2} + i\int_0^{\Lambda^2} \frac{m^2}{k_E^2+m^2} \; dk_E^2$$

$$= \frac{-i}{16\pi^2}\,\frac{\lambda}{2}\,\Lambda^2 + \text{logs.}$$

As $\Lambda^2 \to \infty$ .....

Here where there is no p dependence we have computed a contribution to the "self energy"

$\equiv -i\Sigma$

It's a correction to the mass

$\equiv -im^2$

For example consider the full propagator at all orders

$$= \frac{i}{p^2-m^2} + \frac{i}{p^2-m^2} -i\Sigma \frac{i}{p^2-m^2} + \cdots$$

$$= \frac{i}{p^2-m^2}\left(1 + \frac{\Sigma}{p^2-m^2} + \frac{\Sigma^2}{(p^2-m^2)^2} + \cdots\right)$$

$$= \frac{i}{p^2-m^2}\left(1 - \frac{\Sigma}{p^2-m^2}\right)^{-1}$$

$$= \frac{i}{p^2-m^2-\Sigma}$$

So the physical mass is $m^2 + \Sigma = M_R^2$

"bare mass" is $-\infty$?!!

("renormalized mass" is what we observe