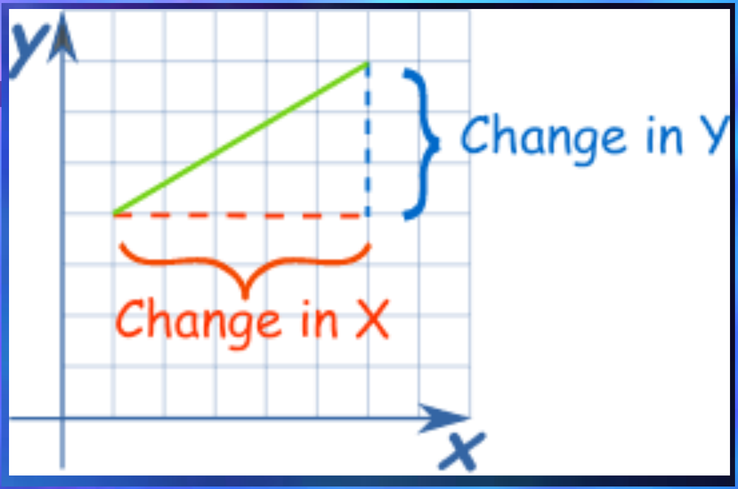
Calculus Summary



$$\frac{dx^n}{dx} = nx^{n-1}$$

$$\frac{d\sin x}{dx} = \cos x$$

$$\frac{d\cos x}{dx} = -\sin x$$

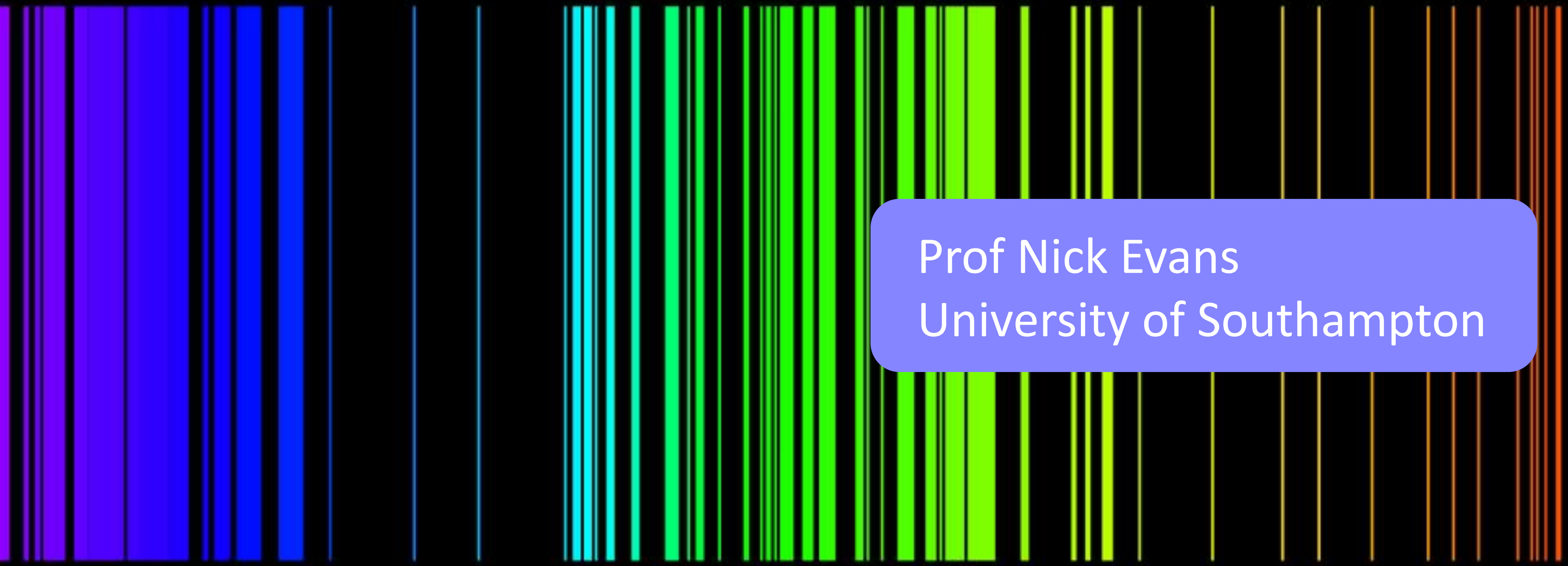
$$\frac{d}{dx}\sin(kx) = k\cos(kx)$$

$$u = kx \qquad \frac{d}{dx}\sin u = \frac{du}{dx}\frac{d}{du}\sin u.$$

$$\frac{d(e^x)}{dx} = e^x$$

An Introduction to Quantum Physics:

When common sense broke!

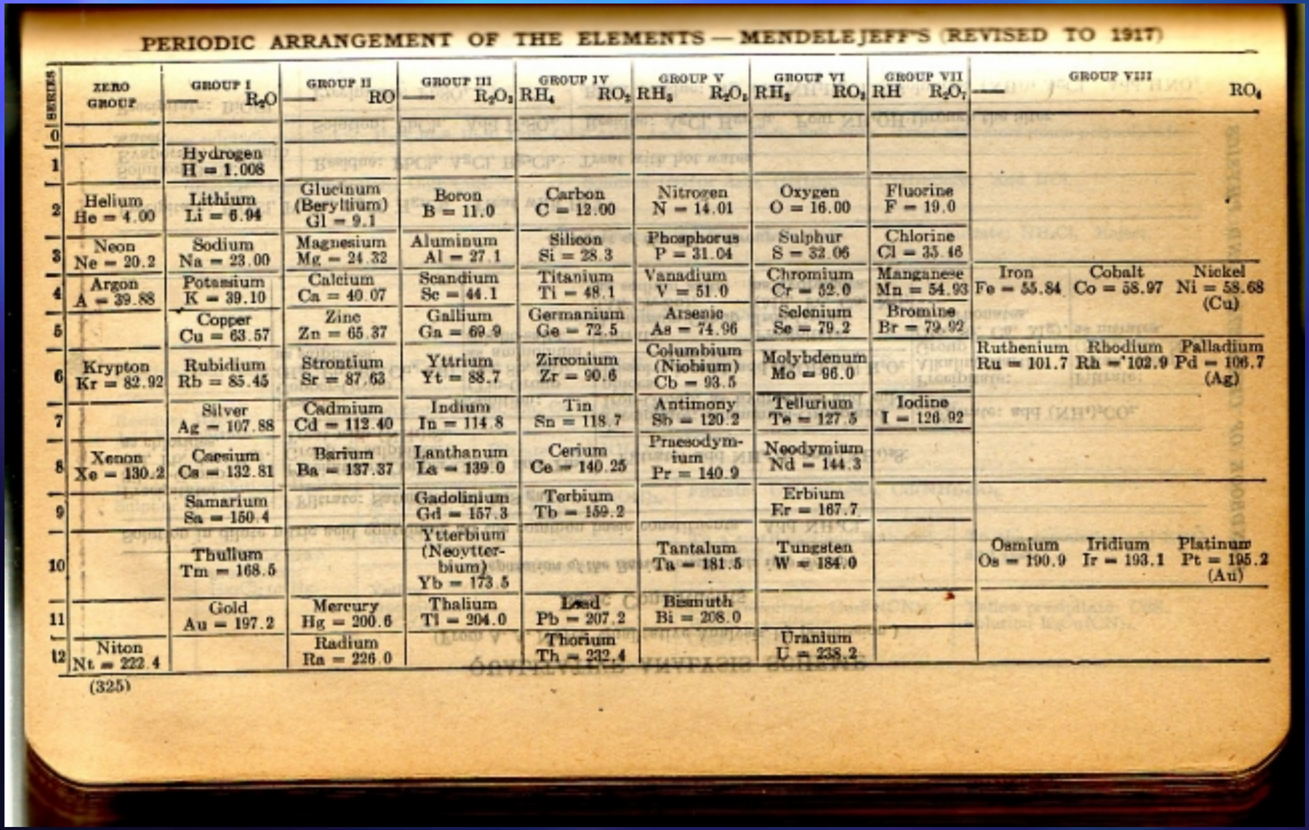


Prof Nick Evans
University of Southampton

Where did quantum mechanics come from? What is it?

Why is it weird and should we do anything about it?

Physics in the 1900s



PERIODIC ARRANGEMENT OF THE ELEMENTS — MENDELEJEFF'S (REVISED TO 1917)

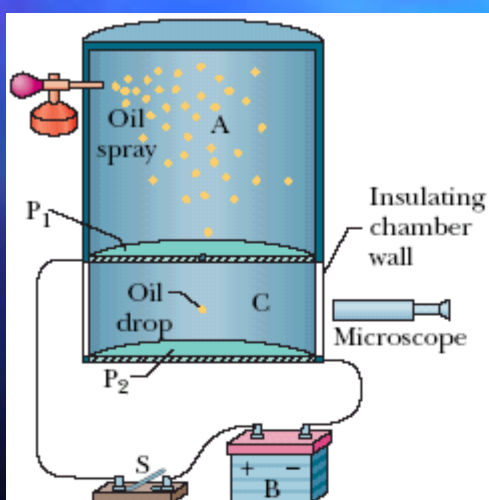
SERIES	ZERO GROUP	GROUP I — R_2O	GROUP II — RO	GROUP III — R_2O_3	GROUP IV RH_4 RO_2	GROUP V RH_3 R_2O_5	GROUP VI RH_2 RO_3	GROUP VII RH R_2O_7	GROUP VIII — RO_4		
0											
1		Hydrogen H = 1.008									
2	Helium He = 4.00	Lithium Li = 6.94	Glucinum (Beryllium) Gl = 9.1	Boron B = 11.0	Carbon C = 12.00	Nitrogen N = 14.01	Oxygen O = 16.00	Fluorine F = 19.0			
3	Neon Ne = 20.2	Sodium Na = 23.00	Magnesium Mg = 24.32	Aluminum Al = 27.1	Silicon Si = 28.3	Phosphorus P = 31.04	Sulphur S = 32.06	Chlorine Cl = 35.46			
4	Argon A = 39.88	Potassium K = 39.10	Calcium Ca = 40.07	Scandium Sc = 44.1	Titanium Ti = 48.1	Vanadium V = 51.0	Chromium Cr = 52.0	Manganese Mn = 54.93	Iron Fe = 55.84	Cobalt Co = 58.97	Nickel Ni = 58.68 (Cu)
5		Copper Cu = 63.57	Zinc Zn = 65.37	Gallium Ga = 69.9	Germanium Ge = 72.5	Arsenic As = 74.96	Selenium Se = 79.2	Bromine Br = 79.92			
6	Krypton Kr = 82.92	Rubidium Rb = 85.45	Strontium Sr = 87.63	Yttrium Yt = 88.7	Zirconium Zr = 90.6	Columbium (Niobium) Cb = 93.5	Molybdenum Mo = 96.0		Ruthenium Ru = 101.7	Rhodium Rh = 102.9	Palladium Pd = 106.7 (Ag)
7		Silver Ag = 107.88	Cadmium Cd = 112.40	Indium In = 114.8	Tin Sn = 118.7	Antimony Sb = 120.2	Tellurium Te = 127.5	Iodine I = 126.92			
8	Xenon Xe = 130.2	Caesium Cs = 132.81	Barium Ba = 137.37	Lanthanum La = 139.0	Cerium Ce = 140.25	Praesodymium Pr = 140.9	Neodymium Nd = 144.3				
9		Samarium Sa = 150.4		Gadolinium Gd = 157.3	Terbium Tb = 159.2		Erbium Er = 167.7				
10		Thulium Tm = 168.5		Ytterbium (Neoytterbium) Yb = 173.5		Tantalum Ta = 181.5	Tungsten W = 184.0		Osmium Os = 190.9	Iridium Ir = 193.1	Platinum Pt = 195.2 (Au)
11		Gold Au = 197.2	Mercury Hg = 200.6	Thalium Tl = 204.0	Lead Pb = 207.2	Bismuth Bi = 208.0					
12	Niton Nt = 222.4		Radium Ra = 226.0		Thorium Th = 232.4		Uranium U = 238.2				

(325)

Materials were classified into groups in the periodic table of Chemistry based on their like interactions..

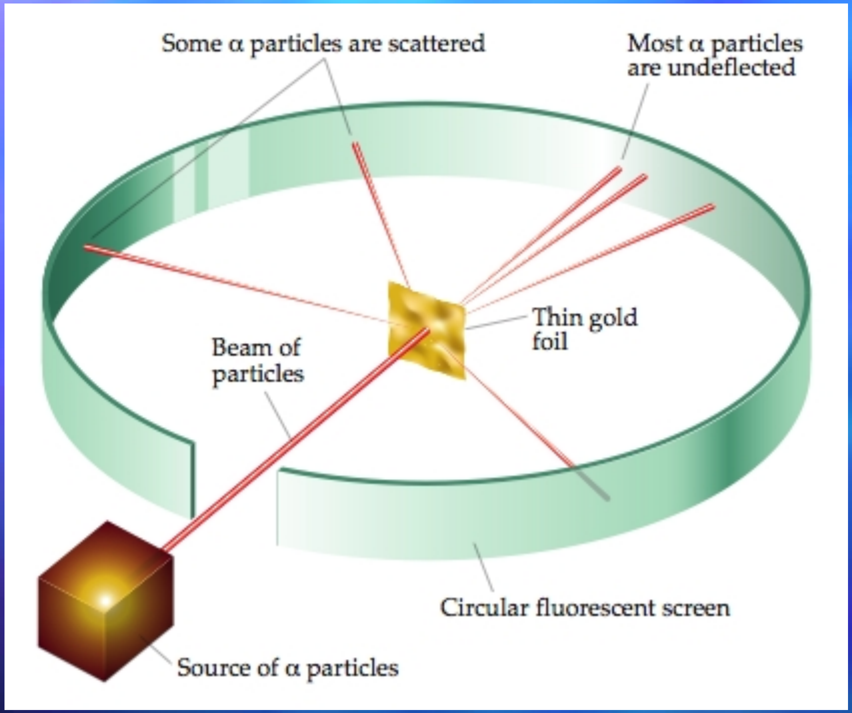
Electrons

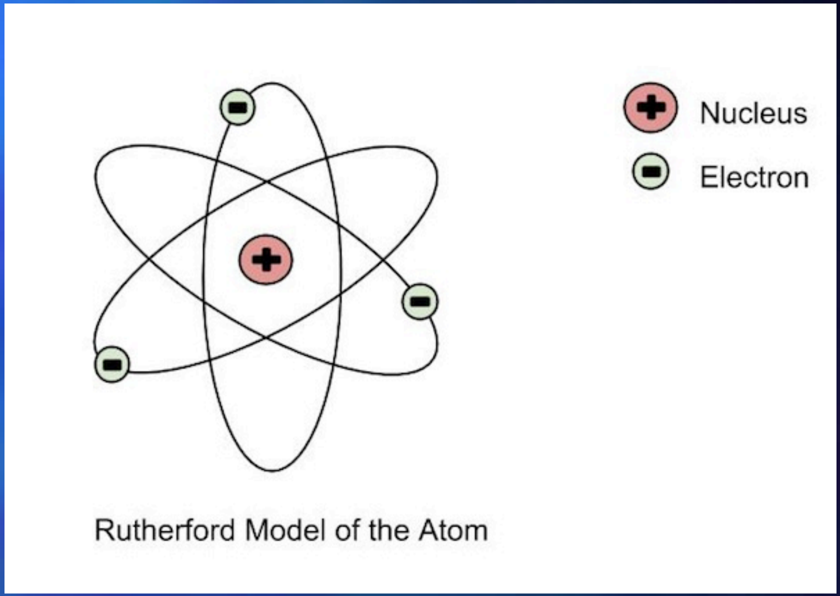
You can strip electrons out of atoms quite easily and gather them together and make them flow – electricity (upto 1880s).



In 1909 Millikan had worked out the smallest charge you could add to an oil drop – one electron.. He knew it's charge and mass.

Rutherford's Solar Systems



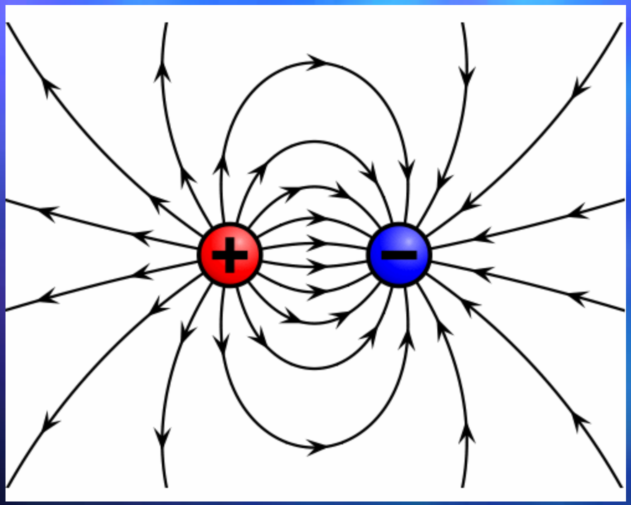


1911

Electric Fields

The forces between charges were well understood (Maxwell's Equations)….

A charge fills space around it with an electric field



Originally a note keeping device – "what force would another charge here feel"…

They became real since energy is transferred through them

Light is an electromagnetic wave that can exist independently of charges…

Technical Hitches

Planets can be moved to orbits closer and closer to the sun..

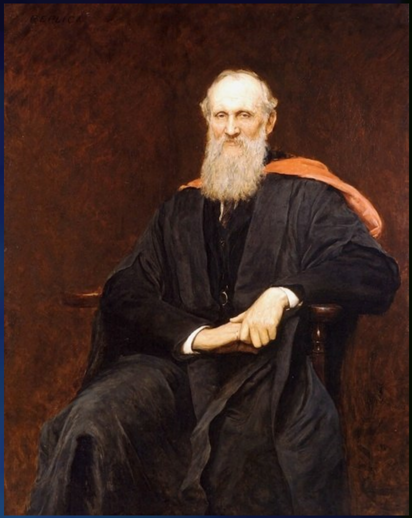
Electrons in hydrogen atoms can’t be moved arbitrarily close to the proton…

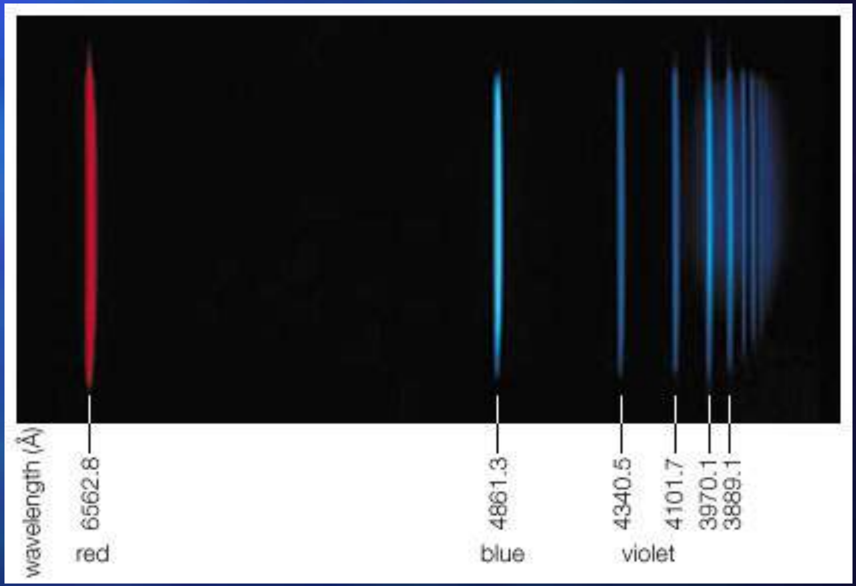
In principle it seemed you could get an infinite amount of energy out of an atom this way…

In fact you get discrete spectra out (and a maximum energy)…

There is nothing new to be discovered in physics now. All that remains is more and more precise measurement.

Lord Kelvin (never said this!)





Balmer Spectrum of H 1880s

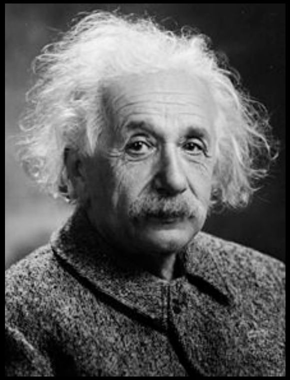
… and breath…

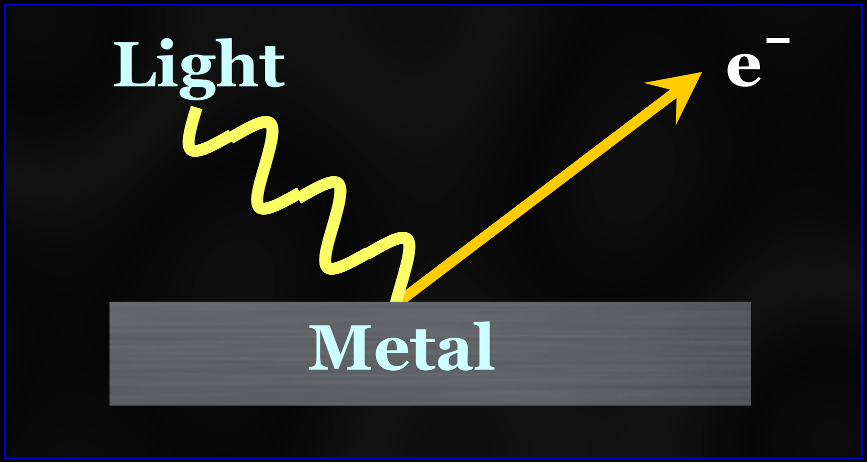
Q7 for 10 minutes…

Light and Matter

The Photo-Electric Effect

Light can provide energy to kick electrons out of a metal





1905

If light intensity is lowered so there is less energy, we expect the evicted electrons to have less energy… *but they don't*… we just see fewer electrons of the same energy…

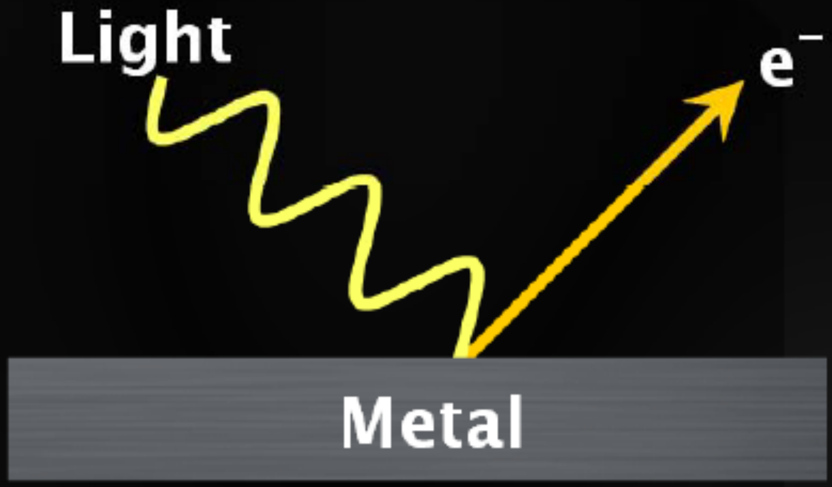
The energy in light comes in lumps!

The Photo-Electric Effect

- The size of the energy lumps depends on the frequency of the light

$$E = hf$$

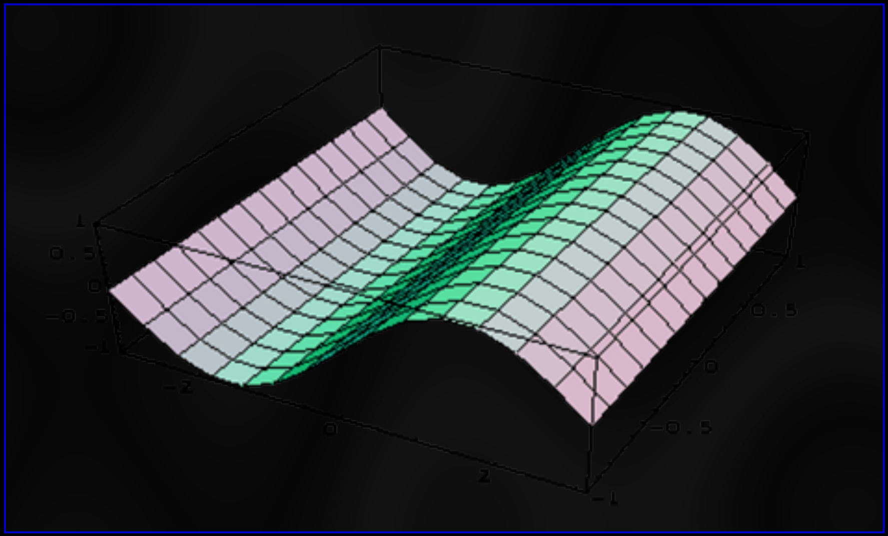
$$h = 6.6\times10^{-34}\,\text{Js}$$



- If we reduce the frequency too much then the electrons don’t receive enough energy to escape the metal surface.

Light Quanta

The energy in light comes in lumps



We can think of light as particles in our detector – **photons**

The wave describes the photons probability distribution!

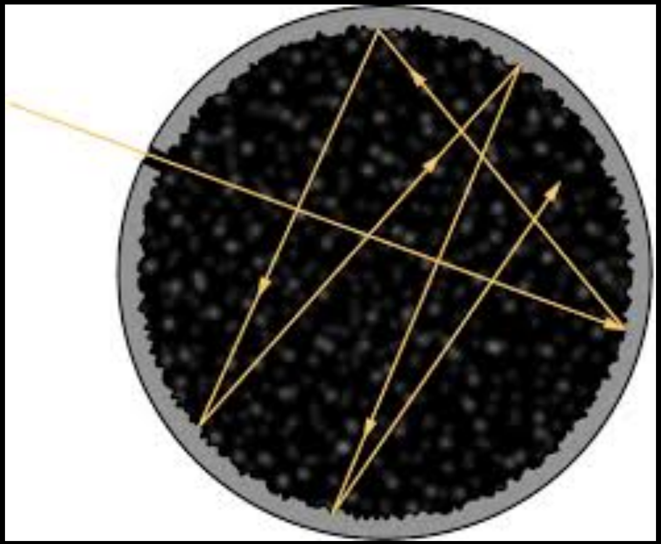
Black Body Radiation

What is a black body?

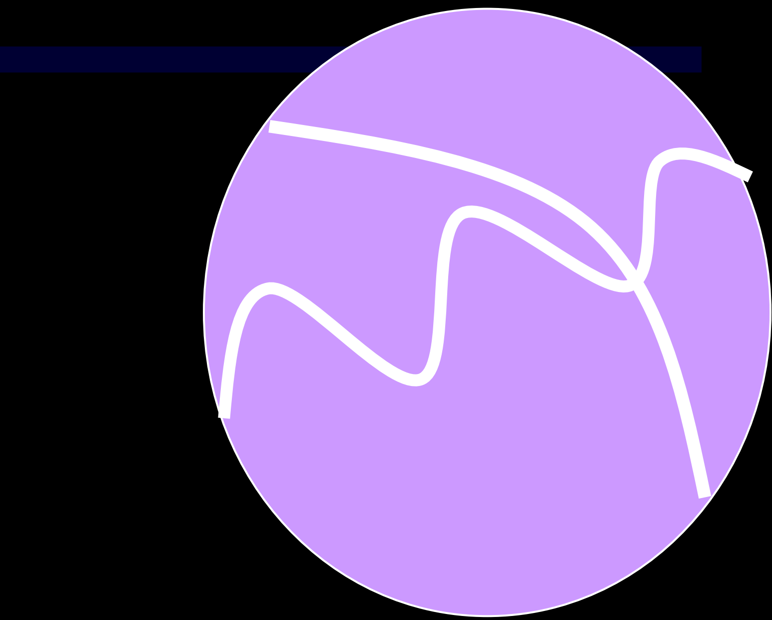
Something that absorbs and emits all light at all wave lengths

At low temperatures it looks black…

At high T it looks like the sun!....



An Infinite Number of Modes



Any number of oscillations provided vanishes at the edge…

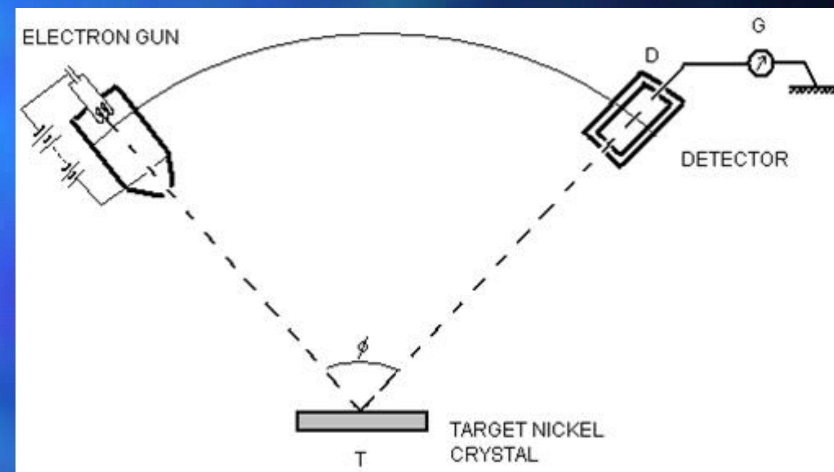
In classical physics at a temperature T each ”degree of freedom” has equal energy kT…

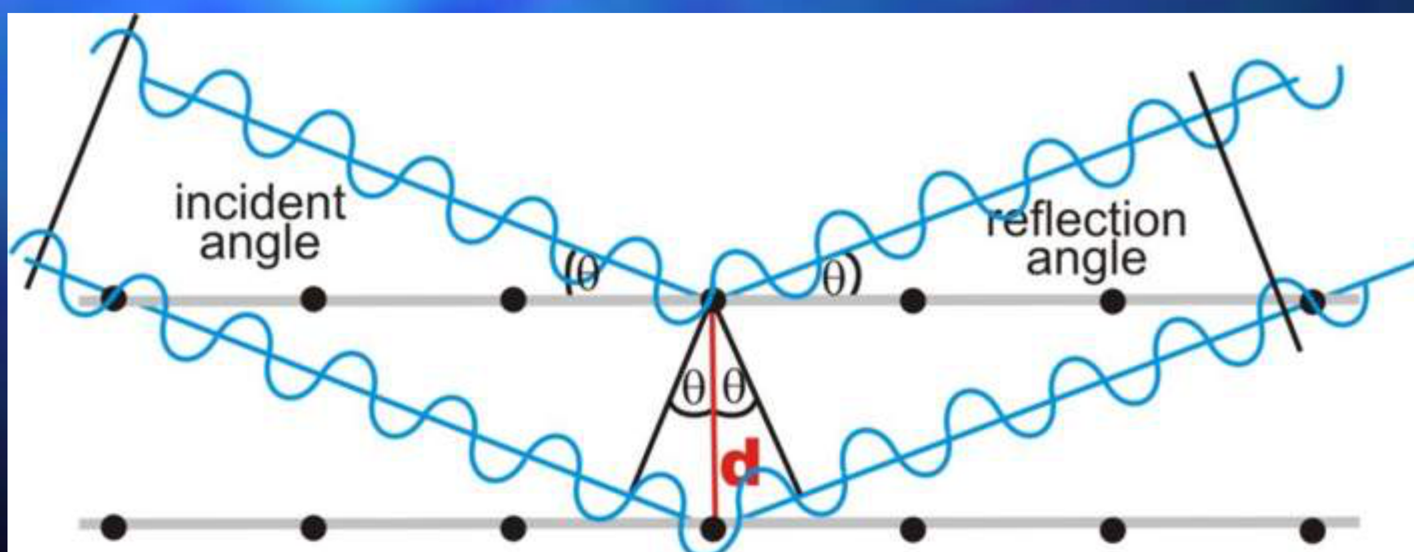
Planck proposed E = n h f so if k T < h f the state doesn’t contribute…

De Broglie (1924) guessed if light was particulate that electrons were wave-like

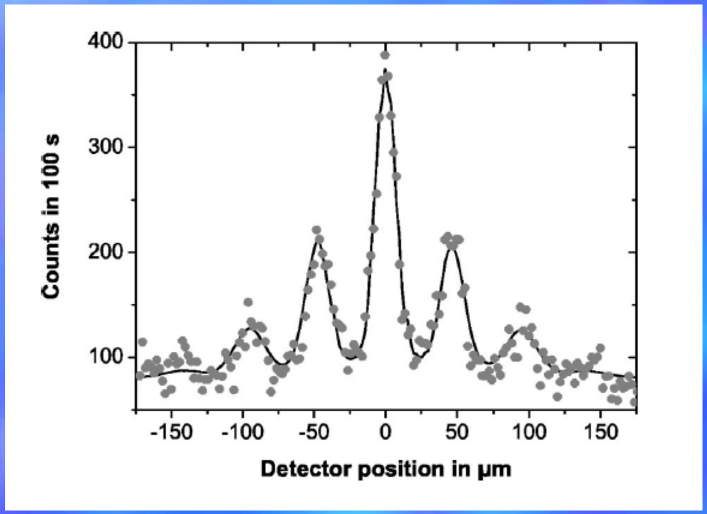
$$\lambda = h / p$$

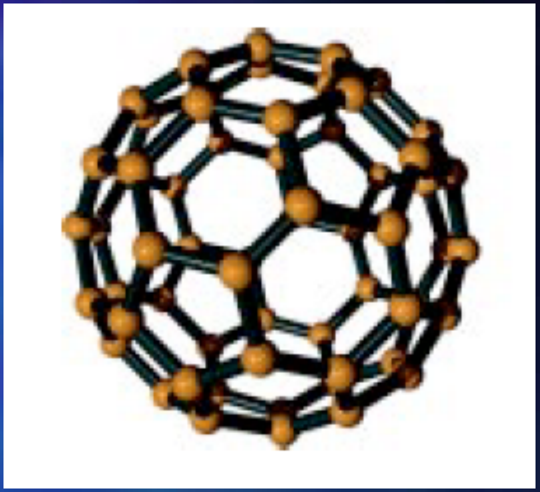
Davisson & Germer (1927) displayed this behaviour

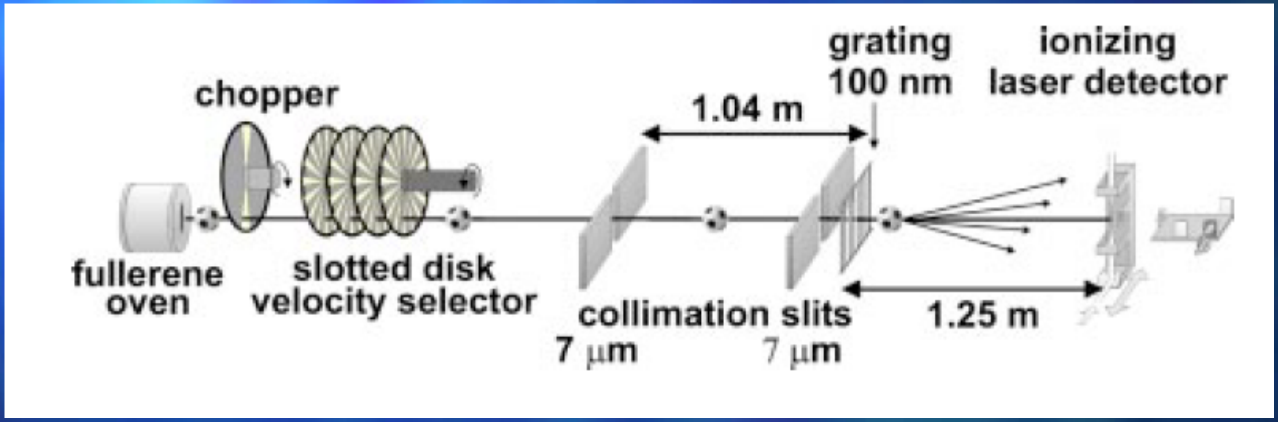




All Particles Behave As Waves - Buckyballs







Quantum interference experiments with large molecules

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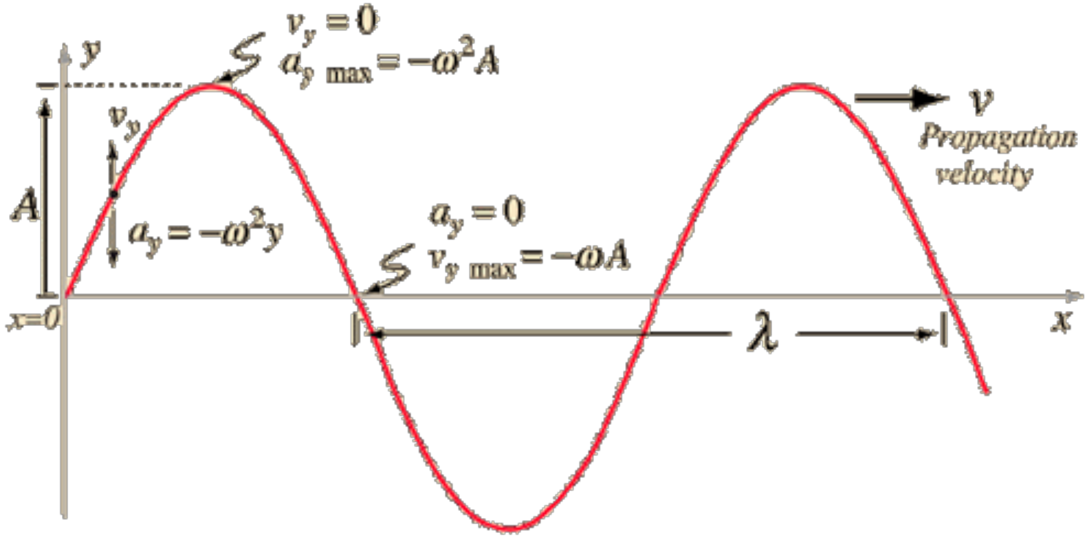
(Received 27 June 2002; accepted 30 October 2002)



Description of the transverse motion.

$$\frac{2\pi v}{\lambda} = 2\pi f = \omega$$

$$v = f\lambda$$

$$y(x,t) = A\sin\frac{2\pi}{\lambda}(x - vt)$$

$$v_y(x,t) = \frac{dy}{dt} = \omega A\cos\frac{2\pi}{\lambda}(x - vt)$$

$$a_y(x,t) = \frac{d^2y}{dt^2} = -\omega^2 y = -\omega^2 A\sin\frac{2\pi}{\lambda}(x - vt)$$

$$p = \frac{h}{\lambda},$$

$$E = hf = h\frac{v}{\lambda}$$

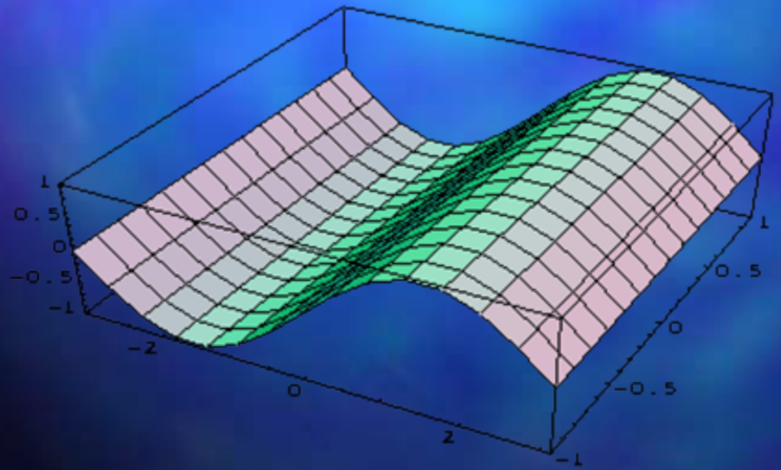
$$y = A\sin\left(\frac{2\pi p}{h}x - \frac{2\pi E}{h}t\right)$$

Fermions vs Bosons

Why do some materials seem wave like (light)

& others particle like (electrons)?

We've come to learn that bosons can have any number of quanta in a particular state… so you can build up the wave…



For fermions there can only be one quanta in a given state so they always look bitty…

… and breath…

Q8, Q9 for 20 minutes….

The Schroedinger Equation

Schroedinger wrote down an equation which for a sine wave gives…

$$\frac{-\hbar^2}{2m}\nabla^2\Psi(\mathrm{r}) + V(r)\Psi(\mathrm{r}) = i\frac{\partial}{\partial t}\Psi$$

Kinetic Energy + *Potential Energy* = *Total Energy*

Deriving it (kind of)…

$$y = A\sin\left(\frac{2\pi p}{h}x - \frac{2\pi E}{h}t\right)$$

$$E^2\psi = -\left(\frac{h}{2\pi}\right)^2\frac{d^2\psi}{dt^2}$$

$$p^2\psi = -\left(\frac{h}{2\pi}\right)^2\frac{d^2\psi}{dx^2}$$

$$\frac{1}{2}mv^2 + V = E$$

$$\frac{1}{2}\frac{p^2}{m} + V = E$$

$$\frac{1}{2m}\left(-\frac{h^2}{(2\pi)^2}\right)\frac{d^2}{dx^2}\psi + V\psi = i\frac{h}{2\pi}\frac{d}{dt}\psi$$

We used the photon energy relation…

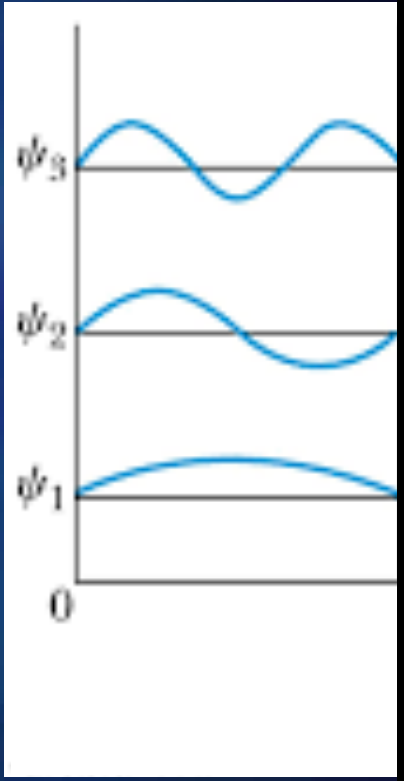
But magically this is right!

Let’s find a very simple solution

Let’s consider a particle trapped in a box by an infinite potential wall on either side…

We solve the Schroedinger equation with V=0

We require the wave to vanish at the edges x=0,L



$$\frac{1}{2m}\left(-\frac{h^2}{(2\pi)^2}\right)\frac{d^2}{dx^2}\psi = i\frac{h}{2\pi}\frac{d}{dt}\psi$$

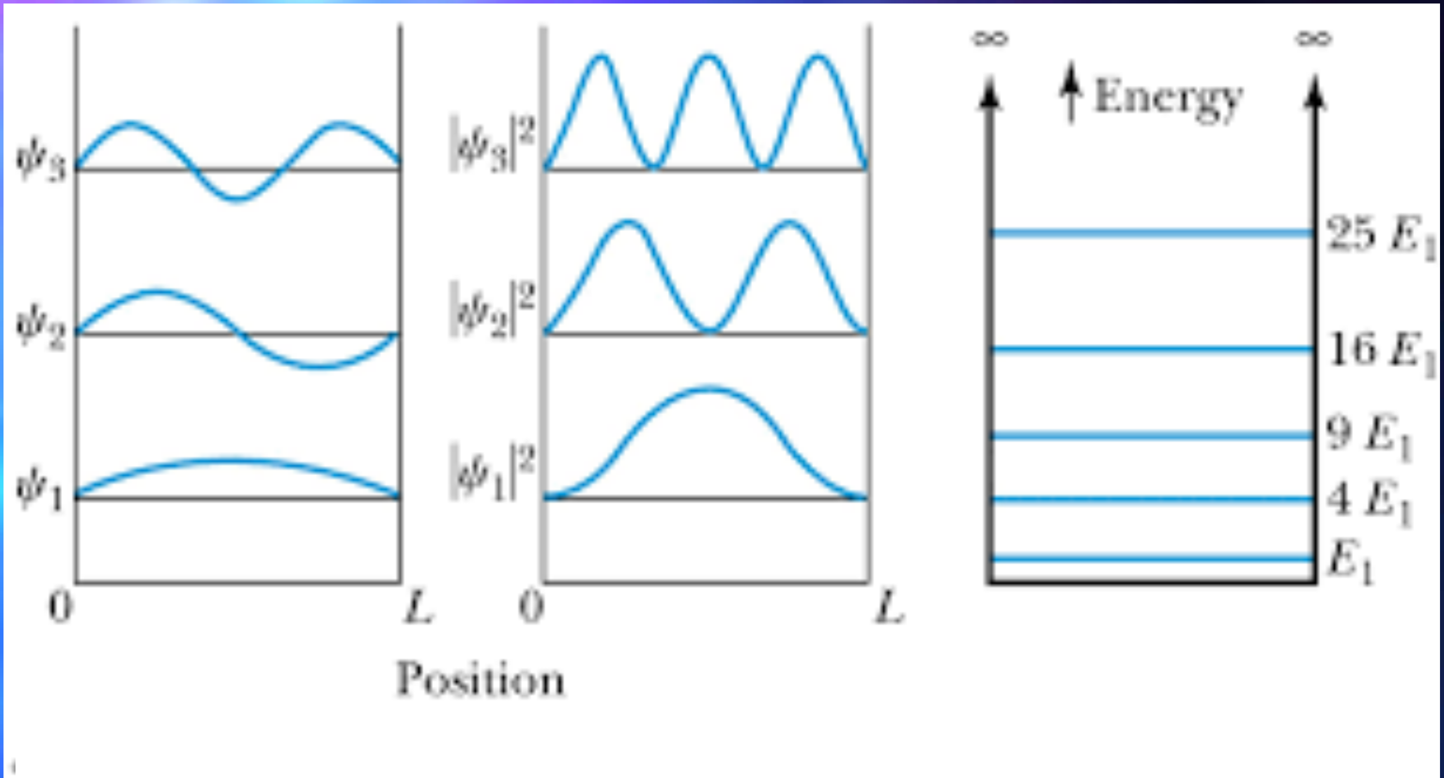
The solution is

$$\psi = \sqrt{\frac{2}{a}}\,\sin\left(\frac{n\pi x}{L}\right)\,e^{-i2\pi E_n t/h}$$

Substitute it in to check

$$\left(-\frac{h^2}{(2\pi)^2 2m}\right)\left(-\frac{n^2\pi^2}{L^2}\right)\psi = E_n\psi$$

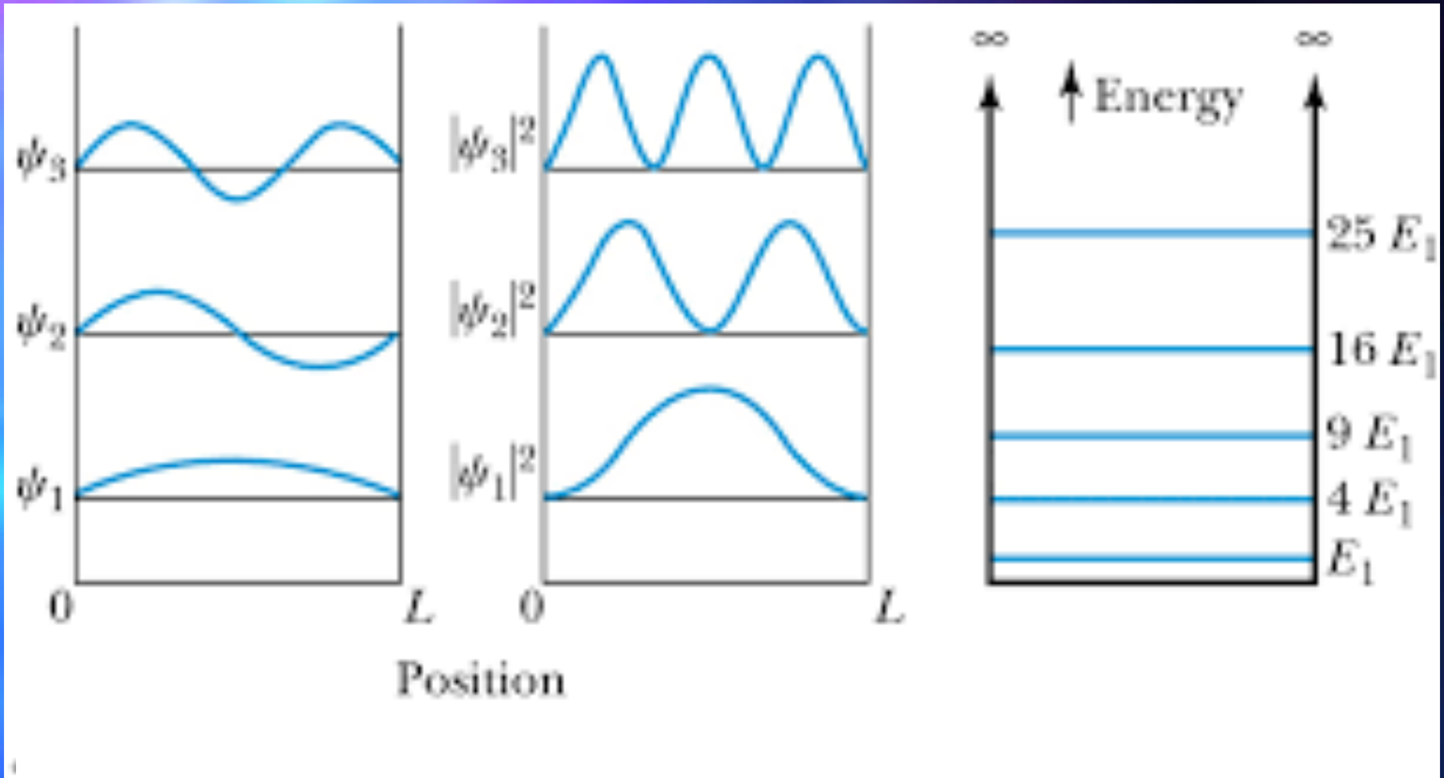
$$E_n = \frac{h^2 n^2}{8mL^2}$$



So these are our solutions plotted at t=0….

The big problem is that they are complex… the magnitude though is real..

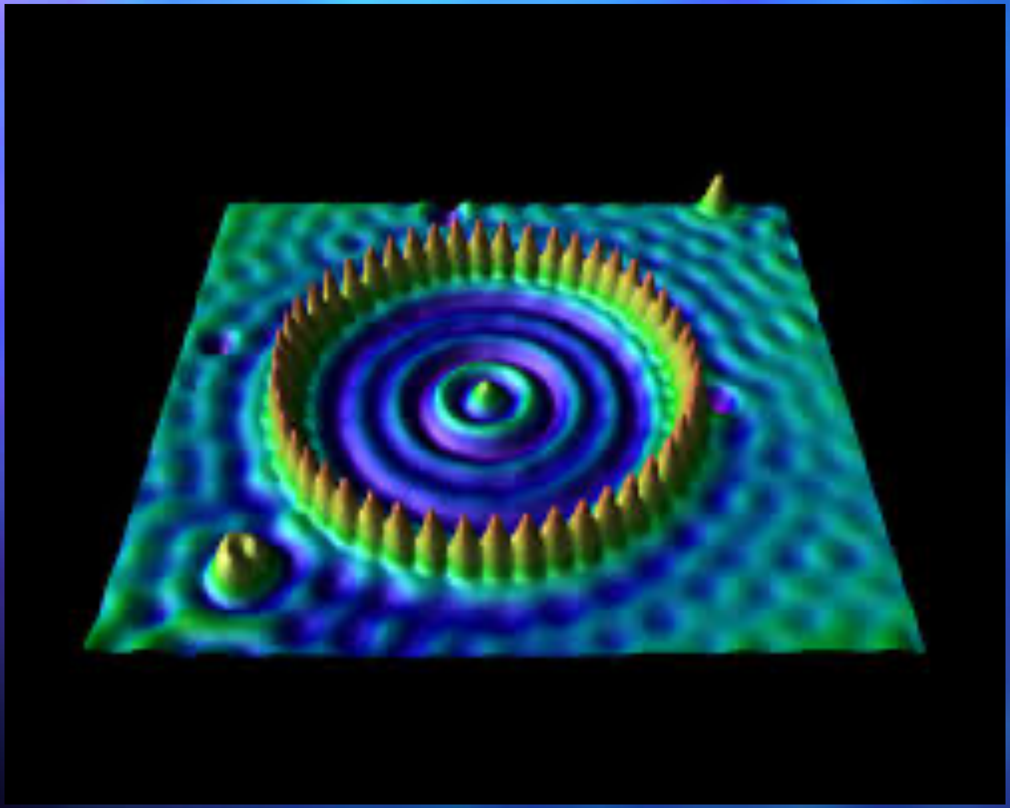
$$\psi^*\psi = (a-ib)\,(a+ib) = a^2 + b^2$$



We interpret the second plot as showing us the probability of the particle being at each point in space when we make an observation….

The theory predicts probabilities… and we'll return to this oddity next time…

Quantum Dots, Quantum Corrals



These days we can make quantum wells… and photograph the wave function using electron microscopy!

People engineer quantum wells (dots) to control the frequency/colour of light they emit….

… and breath…

Try Q10 & Q11…