Exchange Splitting of Backward Volume Spin Wave Configuration Dispersion Curves in a Permalloy Nano-stripe

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Abstract— Micromagnetic simulations of spin wave (SW) propagation in a permalloy film are used to extract the SW dispersion relation. The SWs are excited using the Oersted fields around a current carrying microstrip. Strong exchange interactions yield forward volume dispersion, despite using a backward volume configuration. Edge effects cause the formation of standing longitudinal waves, which manifest themselves as resonance peaks in the SW transmission spectrum. By studying the frequency separation between adjacent resonance peaks, we extract an effective exchange length for the SW excitations in the nano-stripe geometry. The interaction length is in good agreement with the values used in the simulation, thus validating the underlying theoretical approximations. The radiation resistance for the structure is also obtained from the dispersion curve. It shows a minimum at 217 GHz.

1. INTRODUCTION

The dispersion relation of SWs, in magnetic structures, has been a matter of investigation for quite some time [1, 2]. The dispersion relation, $\omega(k)$, is a valuable source of information for structures of various geometries and various materials [3, 4]. Traditionally, these relations have been obtained by experimental means, which often have to be run multiple times. Recent advancements in computational methods have enabled us to obtain these dispersion relations numerically [5–7].

The Landau-Lifshitz-Gilbert (LLG) equation [8] is the governing differential equation that describes the dynamics of the magnetization in a magnonic waveguide. There are various packages which can be used for this. We rely on the finite element method based package NMAG [9]. We compare simulation against analytically obtained results. We are also interested in a study of SW excitation due to a microstrip. This has important ramifications for actual device fabrication, and we model a current pulse, applied to a microstrip transducer above the permalloy stripe, to excite SWs. The chosen geometry allows us to demonstrate the use of micromagnetic simulations to extract the frequency dependence of the radiation resistance, paving the path for studies that involve the interaction of magnetic nano-structures coupled to external circuitry [10].

2. PROBLEM SPECIFICATIONS

The problem, which was considered, is a modification of the one which has been proposed as a standard problem for micromagnetic simulations [11]. The geometry of the problem (shown in Fig. 1(a)), was a permalloy stripe of dimension $(1000 \times 50 \times 1)\,\text{nm}^3$. Further specifications of the problem are given in Table 1.

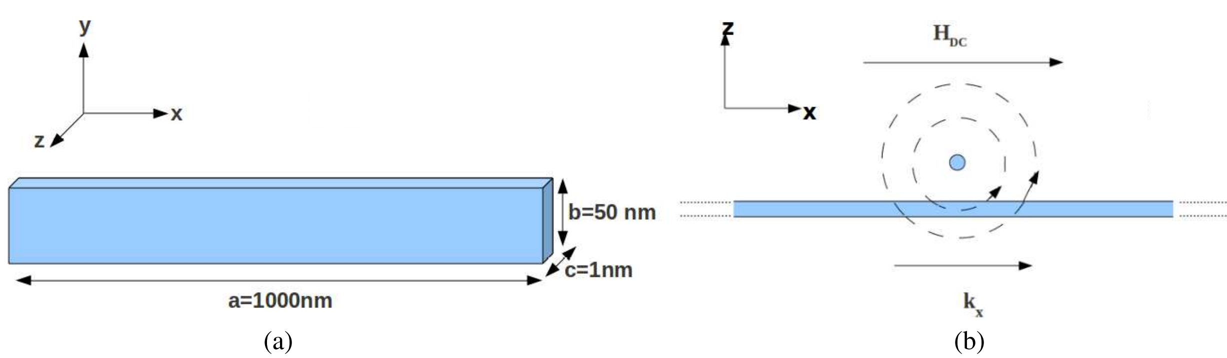


Figure 1: (a) The geometry of the nano stripe and (b) the applied excitation along with the DC bias field and the direction of wave propagation. Figures are not to scale.

The excitation, which was used for inducing spin waves, had the following features:

1. A cylindrical current carrying conductor, of radius 5 nm was assumed to be placed at $(500, y, 6)$ nm, in the x-z plane (shown in Fig. 1(b)).
2. A sinc excitation pulse was applied to the current conductor. The maximum amplitude of the excitation pulse ($I = 160\,\mu\text{A}$) was chosen so that a field of about 5 kOe was obtained at the circumference of the wire.
3. The Oersted fields were calculated for each finite element cell in the geometry based on its distance from the axis of the wire.

Thus the excitation, $h_{\text{exc}}(r, t)$, at a point can be expressed as:

$$h_{\text{exc}}(r, t) = \frac{\mu_0 I}{2\pi r} \frac{\sin(2\pi f t)}{2\pi f t}, \tag{1}$$

where r is the radial distance of the point from the centre of the conductor, and f is the cutoff frequency and was taken to be 500 GHz. The $\omega(k)$ curve is obtained by viewing a surface plot of $m_y(k_x, \omega)$, which is obtained by taking the two dimensional Fourier transform of $m_y(x, t)$, after allowing the simulation to run for 5 ns.

3. SIMULATION RESULTS

The spatial and temporal variation in the excitation field excites only the dominant lowest order SW mode, which is easily visible in Fig. 2. To facilitate a comparison with analytic models, we fit the points on the dispersion curve to a polynomial (in rad/s)

$$\omega(k_x) = (-1.76\times10^{-24})k_x^4 + (5.54\times10^{6})k_x^2 + 2.36\times10^{11}. \tag{2}$$

Table 1: Fields and constants used in the simulation.

Sr. No.	Parameter	Value
1	Saturation Magnetization (M_s)	8.6×10^5 A/m
2	Exchange Coupling Constant (A)	1.3×10^{-11} J/m
3	Anisotropy Constant (K)	0
4	Gyromagnetic ratio (γ)	1.7×10^9 m/A-s
5	Damping coefficient (α)	0
6	DC bias field (H_{DC})	10.1 kOe

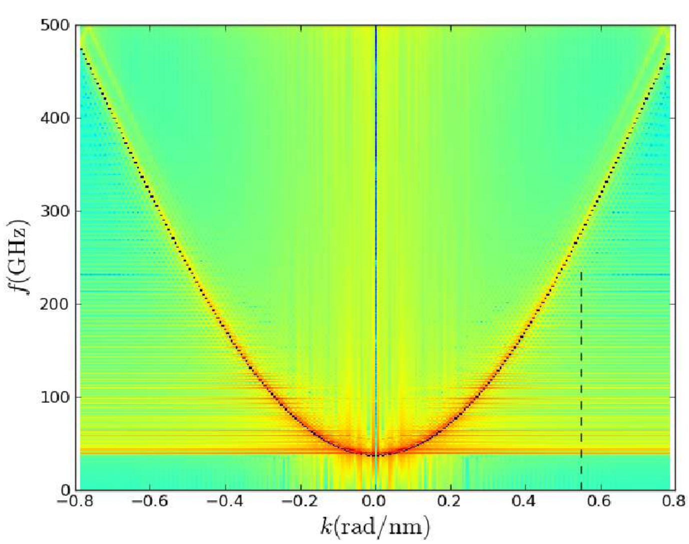


Figure 2: Dispersion curve for a BVSW configuration in a permalloy nano-stripe excited by Oersted fields from a current carrying filament along with the polynomial fit (2) (dotted line). The vertical dashed line corresponds to $k = 0.55$ rad/nm.

Figure 2 shows (2) superimposed, as black dots, on a surface color plot of $m_y(k_x, \omega)$. A number of horizontal lines are also seen in Fig. 2. These are a manifestation of the exchange splitting of the dispersion relation due to interactions between the lowest and higher order longitudinal modes [12]. One such mode is shown schematically in Fig. 3.

The resonances due to standing wave modes will follow a relation of the form [13]

$$\omega_n = \omega_0 + \omega_M \lambda_{\rm ex} \left(\frac{n\pi}{d}\right)^2, \tag{3}$$

where d is the length along the dimension of interest. The exchange constant ($\lambda_{\rm ex}$) is given by

$$\lambda_{\rm ex} = \frac{2A}{\mu_0 M_s^2}. \tag{4}$$

In our case, we are exciting longitudinal modes that have a maxima at the centre of the film and a particular width mode. Hence, (3) gets modified to

$$\omega_n = \omega_0 + \omega_M \lambda_{\rm ex} \left(\frac{(2n+1)\,\pi}{a}\right)^2, \tag{5}$$

$$\text{or} \quad \Delta f = \frac{1}{2\pi}(\omega_{n+1} - \omega_n) = \frac{4\omega_M \lambda_{\rm ex}\pi}{a^2}(n+1). \tag{6}$$

Resonance peaks in the power spectrum, for $k_x = 0.55\,\mathrm{rad/nm}$ (which corresponds to the dashed vertical line in Fig. 2), are shown in Fig. 4(a). We fit the data to a line, shown in Fig. 4(b). The slope is found to be $57.53 \pm 0.76\,\mathrm{MHz}$, from which, using (6), we extract $\lambda_{\rm ex} = 2.59\times10^{-17}\,\mathrm{m}^2$. This is in reasonable agreement with the values given in Table 1 that yield $\lambda_{\rm ex} = 3.23\times10^{-17}\,\mathrm{m}^2$.

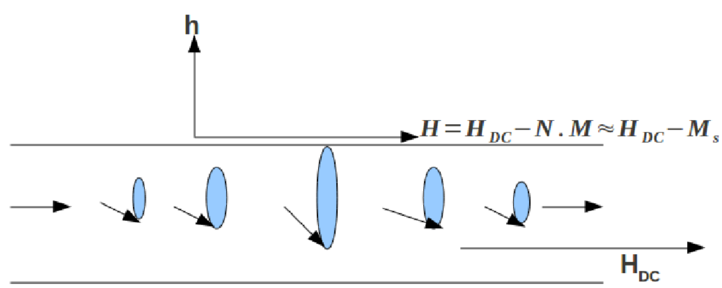
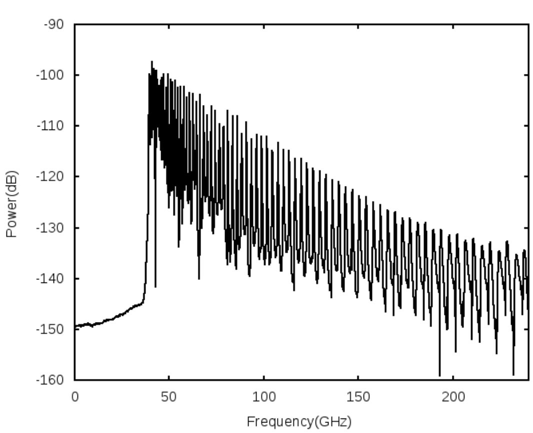
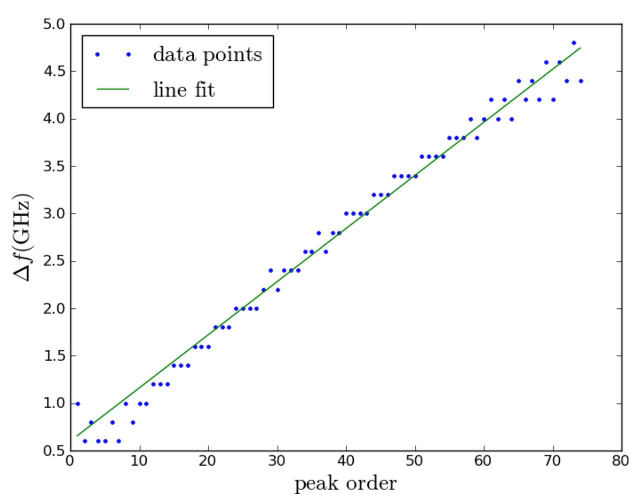


Figure 3: Standing spin wave resonance in a magnetic thin film. The precession cone angle of the magnetization has been exaggerated for clarity. N is the demagnetizing tensor. It was assumed that the applied field, $H_{\rm DC}$, is strong enough to saturate the stripe so that M_s is parallel to $H_{\rm DC}$, everywhere.



(a)



(b)

Figure 4: (a) The power spectrum at $k = 0.55\,\mathrm{rad/nm}$ and (b) frequency separation (Δf) dependence on mode order (n_x) with a linear fit to the equation $\Delta f = 57.53n + 523.88\,\mathrm{MHz}$, with $R^2 = 0.98$.

4. RADIATION RESISTANCE

Radiation resistance determines the amount of power transferred between the current in a microstrip transducer and the excited SWs. For the lowest order BVSW mode [13],

$$r_{\mathrm{rad}}(\omega) = \frac{\omega\mu_0}{k_x c}\sin^2\left[\frac{k_x c}{2\sqrt{-(1+\chi)}}\right]\left|\frac{F}{I}\right|^2. \tag{7}$$

The array factor

$$F = Ie^{-k_x s}J_0\left(\frac{k_x w'}{2}\right), \tag{8}$$

where J_0 is the zeroth Bessel function, $s = 6\,\mathrm{nm}$ is the spacing of the of the current filament from the film, I is the current carried by the conductor, $w' = 10\,\mathrm{nm}$ is the width of the current filament and the susceptibility χ is given by

$$\chi = \frac{\omega_{\mathrm{ex}}\omega_M}{\omega_{\mathrm{ex}}^2 - \omega^2}. \tag{9}$$

To calculate $r_{\mathrm{rad}}(\omega)$, we must first estimate ω_0 and λ_{ex}.

The dispersion curve for the lowest order BVSW, in an infinite film, is approximated as [14]

$$\omega = \sqrt{\omega_{\mathrm{ex}}.\left(\omega_{\mathrm{ex}} + \omega_M\frac{1-e^{-k_x c}}{k_x c}\right)}, \tag{10}$$

where

$$\omega_{\mathrm{ex}} = \gamma\mu_0 H_{\mathrm{DC}} + \frac{2\gamma A}{M_s}k_x^2 = \omega_0 + \lambda_{\mathrm{ex}}\omega_M k^2. \tag{11}$$

When can fit the (ω, k_x) dispersion curve in Fig. 2, to (10), we obtain the values $\omega_0 = 2.21\times10^{11}$ rad/s and $\lambda_{\mathrm{ex}} = 2.61\times10^{-17}\,\mathrm{m}^2$.

Figure 5 shows a minima in $r_{\mathrm{rad}}(f)$ at 217 GHz. This corresponds to the zero of the Bessel function in (8). The position of this zero can be shifted by considering current filaments of different widths.

The return loss, for a microstrip transducer (in our case the current carrying conductor), is defined as [13]

$$RL = -10\log\left[\frac{(R_c + R_{\mathrm{rad}} - R_g)^2 + (X_l + X_{\mathrm{rad}})^2}{(R_c + R_{\mathrm{rad}} + R_g)^2 + (X_l + X_{\mathrm{rad}})^2}\right], \tag{12}$$

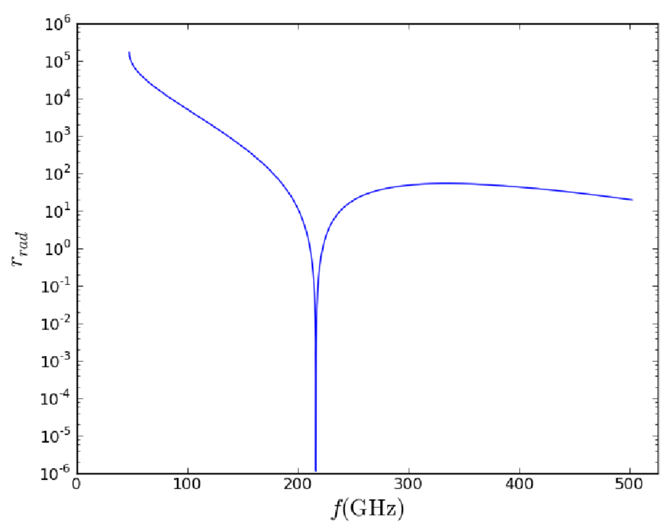


Figure 5: Frequency dependence of radiation resistance per unit length for exchange dominated BVSWs.

where R_c is the resistance from the finite conductivity of the transducer, R_{rad} and X_{rad} are the radiation resistance and reactance resulting from coupling to spin waves, X_l is the inductive reactance of the microstrip transducer in the absence of dipolar spin wave coupling and R_g is the input impedance if the transducer is modelled as a transmission line. Here $R_{\text{rad}} = r_{\text{rad}} \times L_{\text{fil}}$ where L_{fil} is the length of the microstrip transducer. With $L_{\text{fil}} = b = 50\,\text{nm}$, $R_{\text{rad}} \ll \{R_c, R_g\}$. Therefore, RL shows no significant frequency dependence and the return loss will be largely determined by the impedance characteristics of the external circuitry connected to the microstrip transducer.

5. CONCLUSION

We have shown how micromagnetic simulations can be used to obtain valuable information about the exchange splitting of the dispersion curve and the radiation resistance of the structure. The effects of exchange can be extracted by looking at SW resonances in the transmission characteristics, and the value of λ_{ex} thus extracted seems to agree with the values used as input to the simulation. This provides a validation of the method used to obtain the SW dispersion relation.

Excitation of SWs using an Oersted field would be a possible experimental geometry. The radiation resistance of SWs helps design transducers with proper impedance matching characteristics. r_{rad} was estimated from $\omega(k)$, and was found to have a minimum at 217 GHz. However, we would need to know the impedance characteristics of the microstrip transducer fairly accurately before we can estimate return and insertion losses in an experiment.

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REFERENCES

1. Damon, R. and J. Eshbach, "Magnetostatic modes of a ferromagnet slab," *J. Phys. Chem. Solids*, Vol. 19, 308–320, 1961.
2. Damon, R. W. and H. Van De Vaart, "Propagation of magnetostatic spin waves at microwave frequencies in a normally-magnetized disk," *J. Appl. Phys.*, Vol. 36, 3453–3459, 1965.
3. Khitun, A., M. Bao, and K. Wang, "Spin wave magnetic nanofabric: A new approach to spin-based logic circuitry," *IEEE Trans. Magn.*, Vol. 44, 2141–2152, 2008.
4. Khitun, A., D. E. Nikonov, M. Bao, K. Galatsis, and K. L. Wang, "Feasibility study of logic circuits with a spin wave bus," *Nanotechnology*, Vol. 18, 465202, 2007.
5. Kim, S.-K., "Micromagnetic computer simulations of spin waves in nanometre-scale patterned magnetic elements," *J. Phys. D: App. Phys.*, Vol. 43, 264004, 2010.
6. Kruglyak, V. V. and R. J. Hicken, "Magnonics: Experiment to prove the concept," *J. Magn. Magn. Mater.*, Vol. 306, 191–194, 2006.
7. Dvornik, M. and V. V. Kruglyak, "Dispersion of collective magnonic modes in stacks of nanoscale magnetic elements," *Phys. Rev. B*, Vol. 84, 140405, Oct. 2011.
8. Gilbert, T., "A phenomenological theory of damping in ferromagnetic materials," *IEEE Trans. Magn.*, Vol. 40, 3443–3449, 2004.
9. Fischbacher, T., et al., "A systematic approach to multiphysics extensions of finite-element-based micromagnetic simulations: N_{mag}," *IEEE Trans. Magn.*, Vol. 43, 2896–2898, 2007.
10. Kajiwara, Y., et al., "Transmission of electrical signals by spin-wave interconversion in a magnetic insulator," *Nature*, Vol. 464, 262–266, 2010.
11. Venkat, G., et al., "Proposal for a standard micromagnetic problem: Spin wave dispersion in a magnonic waveguide," *IEEE Trans. Magn.*, submitted for publication.
12. Kalinikos, B. and A. N. Slavin, "Theory of dipole-exchange spin wave spectrum for ferromagnetic films with mixed exchange boundary conditions," *J. Phys. C: Solid Sate Phys.*, 19, 1986.
13. Stancil, D. D. and A. Prabhakar, *Spin Waves Theory and Applications*, 1st Edition, Springer, New York, 2008.
14. Kalinikos, B. A., "Excitation of propagating spin waves in ferromagnetic films," *IEE Proc.*, Vol. 127, 4, 1980.