Second problem sheet for Simulation Modelling for Computer Science (COMP1216)

January 26, 2022

1 Taylor expansions – 1

Consider the functions $f(x) = \exp(x)$ and $f(x) = \sin(x)$ and develop into a Taylor series around $x = 0$. Find an approximation for the function $f(x) = \frac{1}{5+x}$ for $x \gg 1$ up to 3rd order.

2 Taylor expansions – 2

Develop the functions $f(x, y) = x^2 + y^2$ and $g(x, y) = \exp(x + y)$ into a Taylor series around $(x, y) = (0, 0)$ up to 2nd order in x and y. Verify the result for the (multivariate) expansion of g in x and y by comparison to the univariate expansion of $\exp(z)$ in z with $z = x + y$.

3 Numerical differentiation – 1

Estimate the derivative df/dx of the function $f(x) = \exp(x)$ for $x = 0$ using the forward, backward, and central difference schemes discussed in the lecture, using a step width of $h = 0.1$ and compare to the true value of the derivative.

4 Numerical differentiation – 2*

In the lecture, finite difference schemes accurate to first and second order in the step size were discussed. Can you find a finite difference scheme which allows to estimate derivatives accurate up to third order in the step size?

5 Numerical integration

Calculate $\int_0^1 f(x)dx$ for the function $f(x) = x$ using the square method and calculate the dependence of numerical errors on the step length h.

- How do numerical errors scale with h?
- How would this change if you were to approximate the function by trapezoids instead of by rectangles?
- For which class of functions is the square integration scheme exact?
- For which class of functions is integration by trapezoid approximation exact?