

Third problem sheet for Simulation Modelling for Computer Science (COMP1216)

January 19, 2022

1 The towers of Hanoi

In the popular children's game the "Towers of Hanoi" towers consist of a stack of discs of different sizes always placed in such a way that a larger disc has to be under a smaller disc. One typically considers three piles of discs (labelled A, B, and C) and starts with a tower of discs of size n placed at location A, while there are no towers at location B and location C. All discs have different sizes and are arranged such that the largest disc is at the bottom of the pile, followed by the next largest disc, and so on, with the smallest disc being placed on top of the tower. Moving discs one by one, always obeying the rule that a larger disc cannot be under a smaller disc, the tower has to be moved from A to C. As a function of the size n of the tower, how many moves are required to relocate the tower from A to C? Write down a recursion relation and solve using the method discussed in the lecture.

2 Recursion relations – 1

Consider the following recursion relations. Classify them and describe the temporal dynamics (in case they can be solved using the methods discussed in the lecture):

- $X_n = X_{n-1} + X_{n-2} + X_{n-4}$
- $X_n = nX_{n-1}$
- $X_n = X_{n-1}/2 + 1$
- $X_n = 6X_{n-1} - 18X_{n-2}$

3 Recursion relations – 2

Give an example of a 2nd order recursion relation X_t whose solution

- Has an exponentially growing umbrella and oscillates between positive and negative values with period one.
- Has an umbrella that exponentially declines to zero and displays sinusoidal oscillations.
- Is exponentially increasing.

4 Recursion relations – 3

Consider the recursion relation $X_n + X_{n-2} = 0$ with $X_0 = 0$ and $X_1 = 1$. Find the characteristic equation and solve by making an appropriate exponential ansatz.

5 Recursion relations – 4

Consider the recursion relations $X_n = X_{n-1}/2 + 1$ and $X_n = 6X_{n-1}^2 - 2$. Find all stationary points and discuss their stability.

6 Recursion relations – 5*

Consider the system of recursion relations given by $X_n = 2Y_{n-1}^2 - 1$ and $Y_n = X_{n-1}/2 - 2$. Find all stationary points and discuss their stability.