Hot topics in Modern Cosmology
SW17, 23-29 March2025, IESC, Cargese

SO(10)-inspired Leptogenesis

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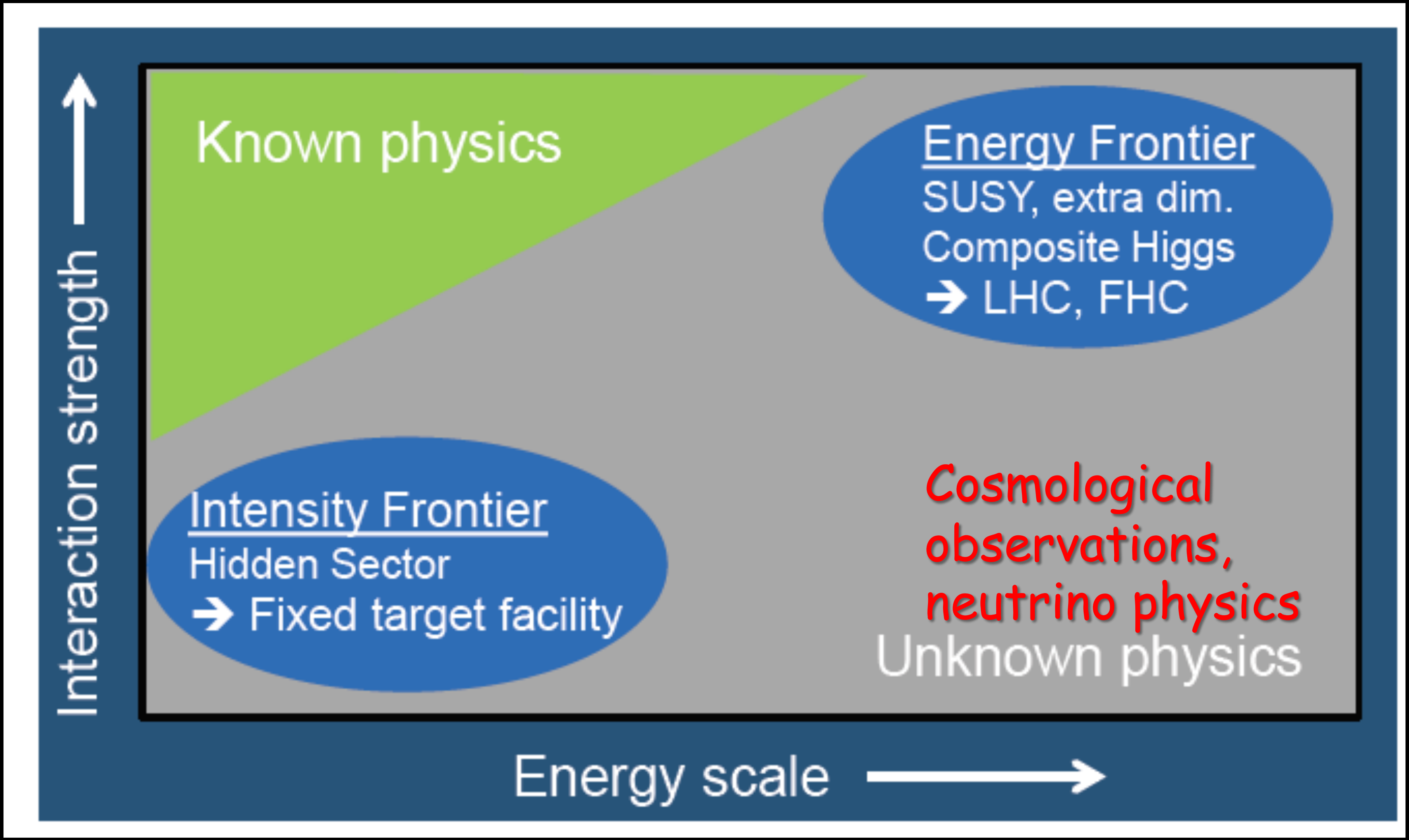
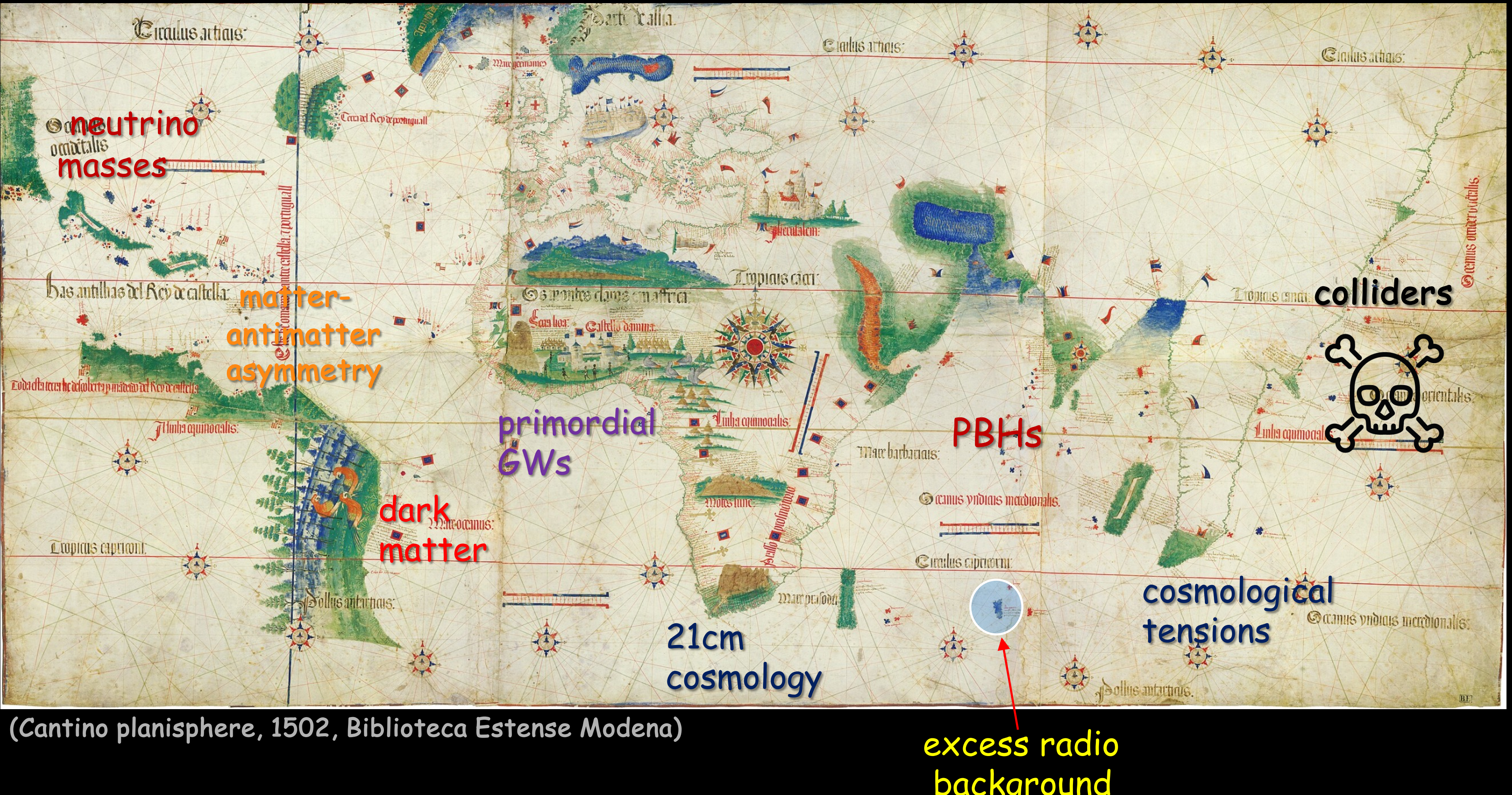
New frontiers

(SHIP proposal, 1504.04855)

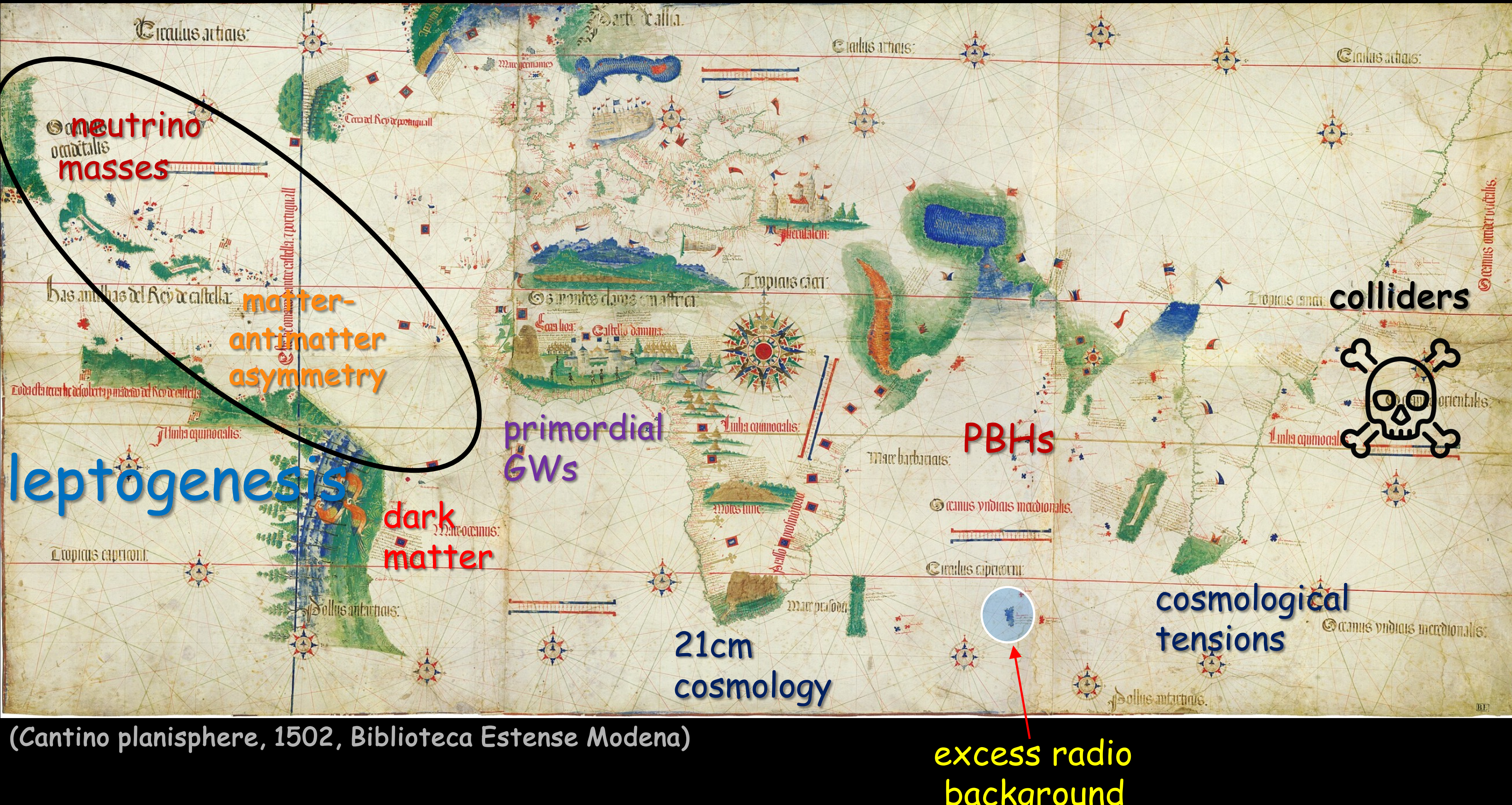


A map to new physics?



(Cantino planisphere, 1502, Biblioteca Estense Modena)

A map to new physics?

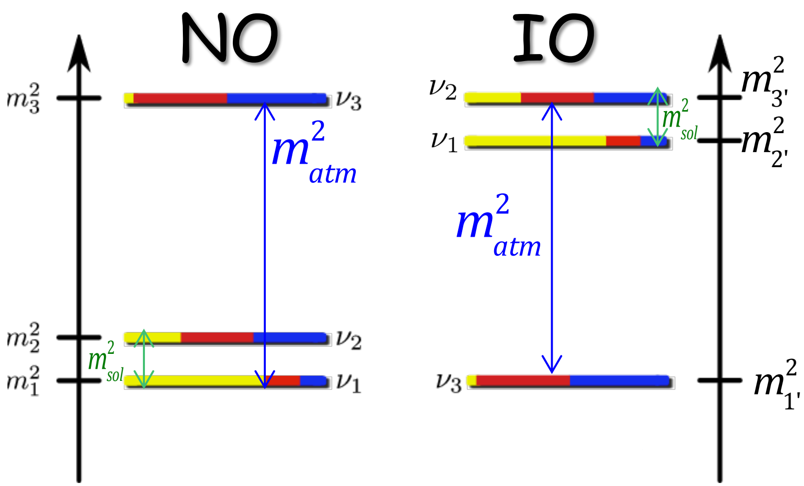


(Cantino planisphere, 1502, Biblioteca Estense Modena)

Preamble

- A common statement is that high scale leptogenesis is untestable.
- SO(10)-inspired leptogenesis provides a counter-example that clearly shows that, though challenging, it is possible, even just with standard low energy neutrino experiments, to have a high-scale leptogenesis scenario that is highly predictive, it is already getting tested now and has the potential for a high statistical significance support (or to be relatively quickly ruled out).
- Moreover new phenomenological avenues toward tests of high scale scenarios are now available and intensively explored, mainly thanks to GW discovery.

Neutrino masses ($m_{1'}<m_{2'}<m_{3'}$)



$$NO: m_2=\sqrt{m_1^2+m_{sol}^2}\,,\quad m_3=\sqrt{m_1^2+m_{atm}^2}$$

$$IO:\ m_{2'}=\sqrt{m_{1'}^2+m_{atm}^2-m_{sol}^2}\,,\quad m_{3'}=\sqrt{m_{1'}^2+m_{atm}^2}$$

m_{sol} = $(8.6 \pm 0.1)\ meV$

m_{atm} = $(50.0 \pm 0.3)\ meV$

$$\sum_i m_i < 0.23\, eV\ \ (95\%C.L.)$$

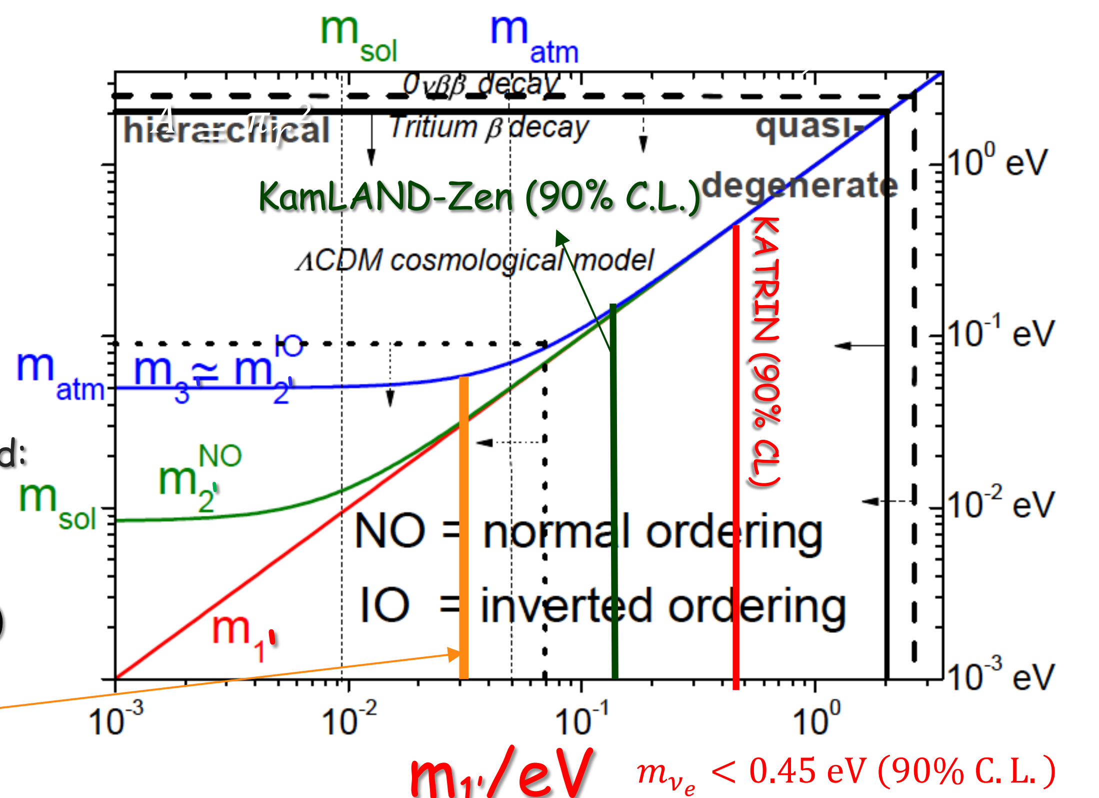
$$\Rightarrow m_{1'} \leq 0.07\ eV$$ (Planck 2015)

With DESI results +updated lensing likelihood:

$$\sum_i m_i < 120\,\mathrm{meV}\ \ (95\%\,\mathrm{C.L.})\,.$$

(Allali,Notari 2406.14556)

for NO: $m_1 < 30\,\mathrm{meV}\ \ (95\%\,\mathrm{C.L.})$



$m_{\nu_e} < 0.45\ \mathrm{eV}\ (90\%\ \mathrm{C.L.})$

Neutrino mixing parameters:

$$U_{\alpha i}=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}\begin{pmatrix} e^{i\rho} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & e^{i\sigma}\end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13} & 0 & c_{13}\end{pmatrix}\begin{pmatrix} c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix} e^{i\rho} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & e^{i\sigma}\end{pmatrix}$$

$\theta_{12}=[31.27^\circ,35.86^\circ]$
$\theta_{13}=[8.20^\circ,8.97^\circ]$
$\theta_{23}=[39.5^\circ,52.0^\circ]$
$\delta=[105^\circ,405^\circ]$
$\rho,\sigma=[0,360^\circ]$

Atmospheric, LB

Reactors, LB (CP violation)

Solar, Reactors

[illegible]ν decay

PDG :
$\alpha_{31}=2(\sigma-\rho)$
$\alpha_{21}=-2\rho$

$c_{ij}\equiv\cos\theta_{ij}\;,\; s_{ij}\equiv\sin\theta_{ij}$

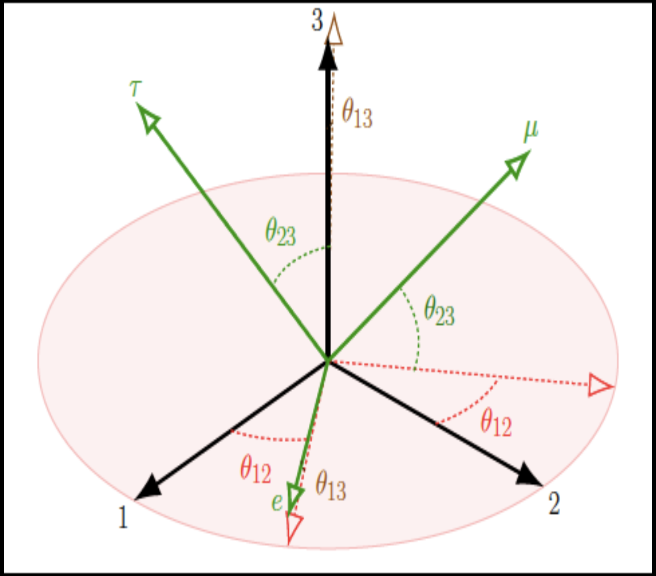
3σ ranges (NO)

$\theta_{12}=[31.63°, 35.95°]$
$\theta_{13}=[8.19°, 8.89°]$
$\theta_{23}=[41.3°, 49.9°]$
$\delta=[-236°, 4°]$
$\rho,\sigma=[0°, 360°]$

(νfit September 2024, with SK atm. data)

NO favoured over IO:

$\Delta\chi^2$ (IO-NO)=6.1



Minimally extended SM

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_Y^\nu$$

$$-\mathcal{L}_Y^\nu = \overline{\nu_L}\, h^\nu\, \nu_R\, \phi \Rightarrow -\mathcal{L}_{\rm mass}^\nu = \overline{\nu_L}\, m_D\, \nu_R$$

Dirac Mass

(in a basis where charged lepton mass matrix is diagonal)

diagonalising m_D : $m_D = V_L^\dagger D_{m_D} U_R$ $\qquad D_{m_D} \equiv \begin{pmatrix} m_{D1} & 0 & 0 \\ 0 & m_{D2} & 0 \\ 0 & 0 & m_{D3} \end{pmatrix}$

$\Rightarrow$ neutrino masses: $m_i = m_{Di}$

leptonic mixing matrix: $U = V_L^\dagger$

<u>But many unanswered questions:</u>

- Why neutrinos are much lighter than all other fermions?
- Why large mixing angles (differently from CKM angles)?
- Cosmological puzzles?
- Why not a Majorana mass term as well?

Minimal seesaw mechanism (type I)

• Dirac + (right-right) Majorana mass terms

(Minkowski '77; Gell-mann,Ramond,Slansky; Yanagida; Mohapatra,Senjanovic '79)

$$-\mathcal{L}^{\nu}_{\rm mass} = \overline{\nu_L}\, m_D\, \nu_R + \frac{1}{2}\, \overline{\nu^c_R}\, M\, \nu_R + {\rm h.c.}$$

violates lepton number

In the see-saw limit ($M \gg m_D$) the mass spectrum splits into 2 sets:

- 3 light **Majorana neutrinos** with masses (seesaw formula):

$$\boxed{{\rm diag}(m_1, m_2, m_3) = -U^\dagger\, m_D\, \frac{1}{M}\, m_D^T\, U^\star}$$

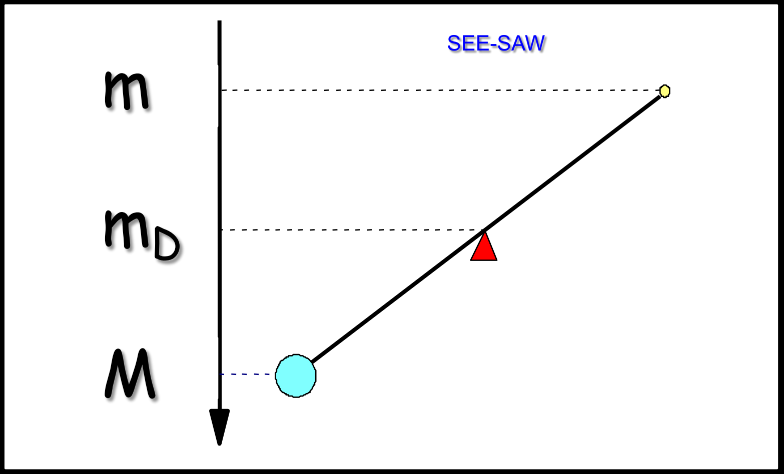
- 3(?) very heavy Majorana neutrinos N_I, N_{II}, N_{III} with $M_{III} > M_{II} > M_I \gg m_D$

1 generation toy model :

$m_D \sim m_{top}$,

$m \sim m_{atm} \sim 50$ meV

$\Rightarrow M \sim M_{GUT} \sim 10^{16}$GeV



3 generation seesaw models: two limits

In the flavour basis (both charged lepton mass and Majorana mass matrices are diagonal):

$$-\mathcal{L}^{\nu+\ell}_{\rm mass} = \overline{\alpha_L}\, m_\alpha\, \alpha_R + \overline{\nu_{L\alpha}}\, m_{D\alpha I}\, \nu_{RI} + \frac{1}{2}\,\overline{\nu^c_{RI}}\, M_I\, \nu_{RI} + \text{h.c.}$$

$\alpha = e, \mu, \tau$

$I = 1,2,3$

bi-unitary parameterisation: $m_D = V_L^\dagger D_{m_D} U_R \qquad D_{m_D} \equiv diag(m_{D1}, m_{D2}, m_{D3})$

FIRST (EASY) LIMIT: ALL MIXING FROM THE LEFT-HANDED SECTOR

- $U_R = I \Rightarrow$ again $U = V_L^\dagger$ and neutrino masses: $m_i = \dfrac{m^2_{Di}}{M_I}$

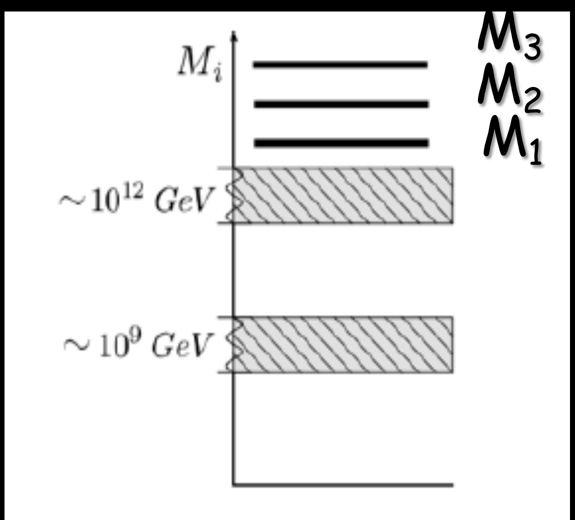
If also $m_{D1} = m_{D2} = m_{D3} = \lambda$ then simply: $M_I = \dfrac{\lambda^2}{m_i}$

Exercise: $\lambda \sim 100\, GeV$

$m_1 \sim 10^{-4}\, eV \Rightarrow M_3 \sim 10^{17}\, GeV$

$m_2 = m_{sol} \sim 10\, meV \Rightarrow M_2 \sim 10^{15}\, GeV$

$m_3 = m_{atm} \sim 50\, meV \Rightarrow M_1 \sim 10^{14}\, GeV$



Typically RH neutrino mass spectrum emerging in simple discrete flavour symmetry models

A SECOND LIMIT: ALL MIXING FROM THE RH SECTOR

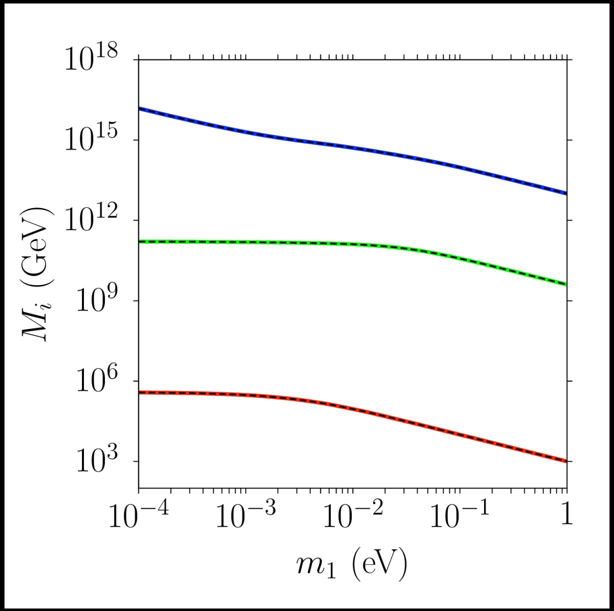
(Branco et al. '02; Nezri, Orloff '02; Akhmedov, Frigerio, Smirnov '03; PDB, Riotto '08; PDB, Re Fiorentin '12)

- $V_L = I \;\Rightarrow\; M_1 = \dfrac{m_{D1}^2}{m_{\beta\beta}};\quad M_2 = \dfrac{m_{D2}^2}{m_1 m_2 m_3}\,\dfrac{m_{\beta\beta}}{|(m_\nu^{-1})_{\tau\tau}|};\quad M_3 = m_{D3}^2\,|(m_\nu^{-1})_{\tau\tau}|$

If one also imposes (SO(10)-inspired models)

$$m_{D1} = \alpha_1\, m_{up};\quad m_{D2} = \alpha_2 m_{charm};\quad m_{D3} = \alpha_3 m_{top};\quad \alpha_i = O(1)$$

Barring very fine-tuned solutions, one obtains a very hierarchical RH neutrino mass spectrum



Combining discrete flavour + grand unified symmetries one can obtain all mass spectra between these two limits

How can we test the existence of these very heavy seesaw neutrinos and their mass spectrum?

Minimal scenario of leptogenesis

- Type I seesaw mechanism (Fukugita,Yanagida '86)

- Thermal production of RH neutrinos: $T_{RH} \gtrsim T_{lep} \simeq M_i / (2 \div 10)$

heavy neutrinos decay $N_I \xrightarrow{\Gamma_I} L_I + \phi^\dagger \qquad N_I \xrightarrow{\bar{\Gamma}} \bar{L}_I + \phi$

total CP asymmetries $\varepsilon_I \equiv -\dfrac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$

$$\Rightarrow N_{B-L}^{fin} = \sum_{I=1,2,3} \varepsilon_I \times \kappa_I^{fin}$$

efficiency factors

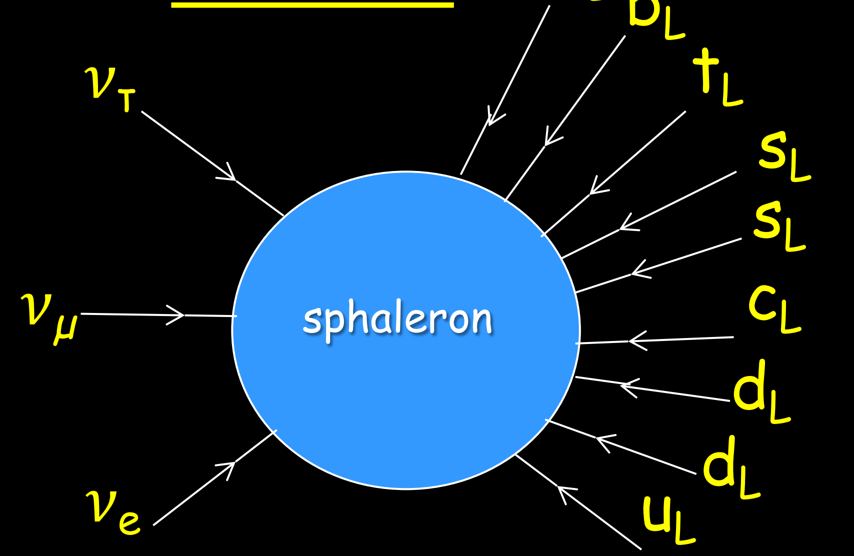
- Sphaleron processes in equilibrium

$\Rightarrow T_{lep} \gtrsim T_{sphalerons}^{off} \simeq 132$ GeV

(Kuzmin,Rubakov,Shaposhnikov '85
D'Onofrio, Rummukainen, Tranberg 1404.3565)

$$\Rightarrow \boxed{\eta_{B0}^{lep} = \frac{a_{sph} N_{B-L}^{fin}}{N_\gamma^{rec}} \simeq 0.01 N_{B-L}^{fin}}$$

ΔB=ΔL=3



Seesaw parameter space

Combining $\eta_{B0}^{lep} \simeq \eta_{B0}^{CMB} \simeq 6\times 10^{-10}$ with low energy neutrino data can we test seesaw and leptogenesis?

(Casas, Ibarra'01) $m_\nu = -m_D \frac{1}{M} m_D^T \Leftrightarrow \boxed{\Omega^T\Omega = I}$ Orthogonal parameterisation

$$\boxed{m_D} = \boxed{U \begin{pmatrix} \sqrt{m_1} & 0 & 0 \\ 0 & \sqrt{m_2} & 0 \\ 0 & 0 & \sqrt{m_3} \end{pmatrix} \Omega \begin{pmatrix} \sqrt{M_1} & 0 & 0 \\ 0 & \sqrt{M_2} & 0 \\ 0 & 0 & \sqrt{M_3} \end{pmatrix}}$$

(in a basis where charged lepton and Majorana mass matrices are diagonal)

light neutrino parameters

heavy neutrino parameters escaping experimental information

- Popular solution: *low-scale* leptogenesis, potential direct discovery of RH neutrinos in lab neutrino experiments (no signs so far).

- *High-scale* leptogenesis is challenging to test but there are a few strategies able to reduce the number of parameters in order to obtain testable predictions on low energy neutrino parameters

Vanilla leptogenesis ⇒ upper bound on ν masses

(Buchmüller,PDB,Plümacher '04; Blanchet, PDB '07,Garbrecht et al 2025)

1) Lepton flavor composition is neglected

2) Hierarchical spectrum ($M_2 \gtrsim 2M_1$)

3) Strong lightest RH neutrino wash-out

$$\eta_{B0} \simeq 0.01 N_{B-L}^{final} \simeq 0.01 \varepsilon_1 \kappa_1^{fin}(K_1, m_1)$$

decay parameter: $K_1 \equiv \frac{\Gamma_{N_1}(T=0)}{H(T=M_1)}$

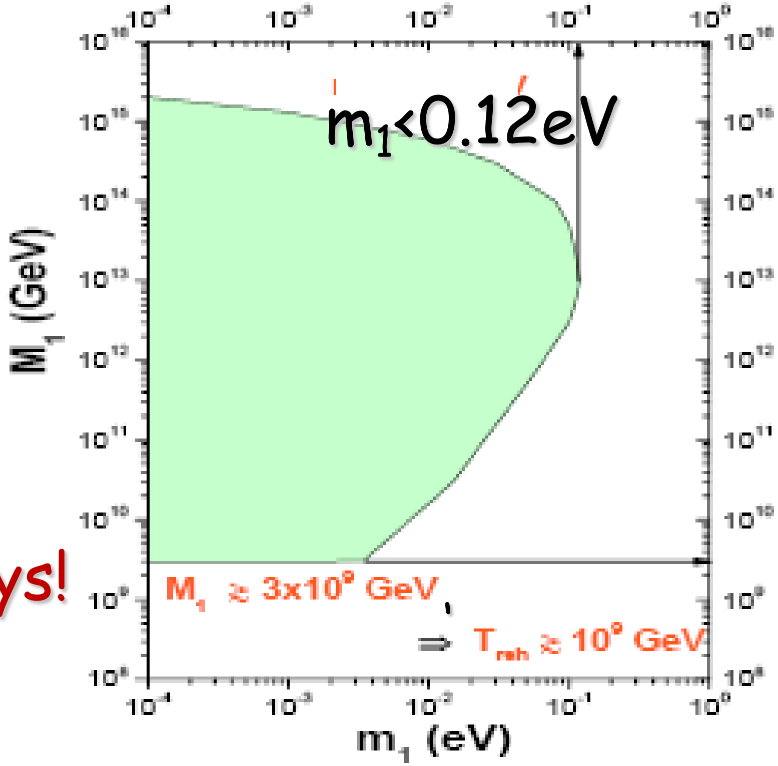
All the asymmetry is generated by the lightest RH neutrino decays!

4) Barring fine-tuned cancellations

(Davidson, Ibarra '02)

$$\varepsilon_1 \leq \varepsilon_1^{\max} \simeq 10^{-6} \left(\frac{M_1}{10^{10}\,\mathrm{GeV}}\right) \frac{m_{\rm atm}}{m_1 + m_3}$$

$$\eta_B^{\max}(m_1, M_1) \geq \eta_B^{CMB}$$



No dependence on the leptonic mixing matrix U: it cancels out!

IS SO(10)-INSPIRED LEPTOGENESIS RULED OUT ?

Independence of the initial conditions (strong thermal leptogenesis)

(Buchmüller,PDB,Plümacher '04)

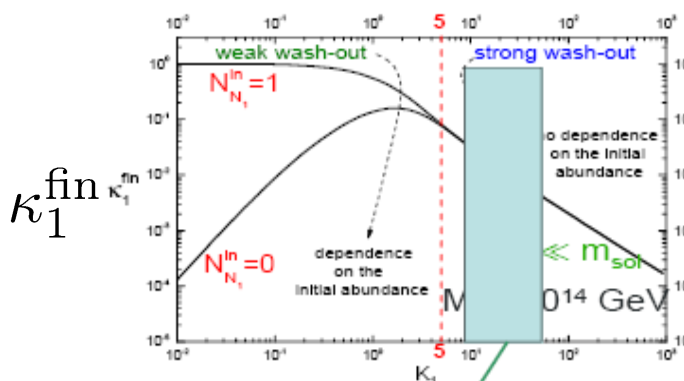
wash-out of a pre-existing asymmetry N^{p}_{B-L}

$$\boxed{N_{B-L}^{\rm p,final} = N_{B-L}^{\rm p,initial}\, e^{-\frac{3\pi}{8}\,K_1} \ll N_{B-L}^{\rm f,N_1}}$$

decay parameter: $$K_1 \equiv \frac{\Gamma_{N_1}}{H(T=M_1)} \sim \frac{m_{\rm sol,atm}}{m_\star \sim 10^{-3}\,{\rm eV}} \sim 10 \div 50$$ Just a coincidence?

equilibrium neutrino mass: $$m_* = \frac{16\pi^{5/2}\sqrt{g_*}}{3\sqrt{5}}\,\frac{v^2}{M_{\rm Pl}} \simeq 1.08\times 10^{-3}\ {\rm eV}.$$

Independence of the initial N_1 abundance



$$\boxed{K_{\rm sol} \simeq 9 \lesssim K_1 \lesssim 50 \simeq K_{\rm atm}}$$

Charged lepton flavour effects

(Barbieri et al '98; Abada et al '06; Nardi et al. '06; Blanchet, PDB, Raffelt '06; Riotto, De Simone '06)

Flavor composition of lepton quantum states matters!

$$|l_1\rangle = \sum_\alpha \langle l_\alpha | l_1 \rangle \, |l_\alpha\rangle \qquad (\alpha = e, \mu, \tau)$$

$$|\bar{l}'_1\rangle = \sum_\alpha \langle l_\alpha | \bar{l}'_1 \rangle \, |\bar{l}_\alpha\rangle$$

- $T \ll 10^{12}$ GeV $\Rightarrow$ τ-Yukawa interactions are fast enough break the coherent evolution of $|l_1\rangle$ and $|\bar{l}'_1\rangle$

$\Rightarrow$ incoherent mixture of a τ and of a ∝+e components $\Rightarrow$ 2-flavour regime

- $T \ll 10^{9}$ GeV then also ∝-Yukawas in equilibrium $\Rightarrow$ 3-flavour regime

M_1

UNFLAVOURED $\quad N_{B-L}^{final} = \varepsilon_1 \kappa_1^{fin}$

~ 10^{12} GeV — TRANSITION REGIME: DENSITY MATRIX APPROACH NEEDED

2 Flavour regime (τ, e+μ) $\quad \varepsilon_{1\tau}\kappa_1^{fin}(K_{1\tau}) + \varepsilon_{1e+\mu}\kappa_1^{fin}(K_{1e+\mu})$

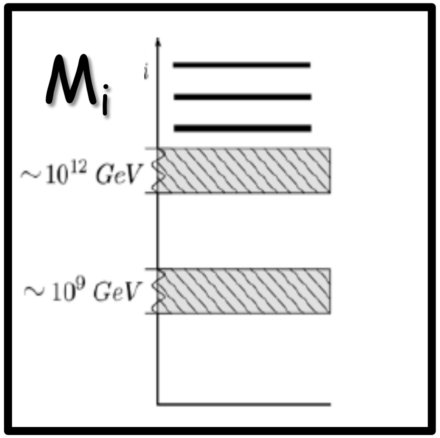
~ 10^{9} GeV — TRANSITION REGIME: DENSITY MATRIX APPROACH NEEDED

3 Flavour regime (e, μ, τ) $\quad \varepsilon_{1\tau}\kappa_1^{fin}(K_{1\tau}) + \varepsilon_{1\mu}\kappa_1^{fin}(K_{1\mu}) + \varepsilon_{1e}\kappa_1^{fin}(K_{1e})$

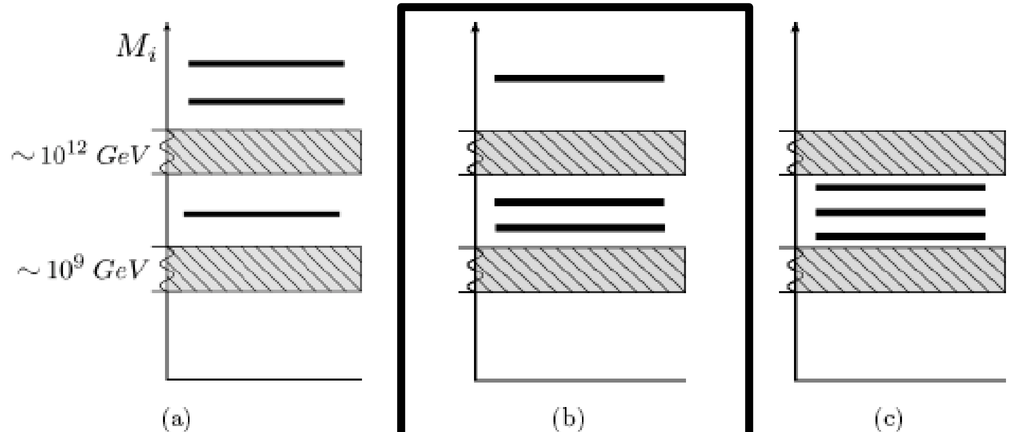
Heavy neutrino lepton flavour effects: 10 scenarios

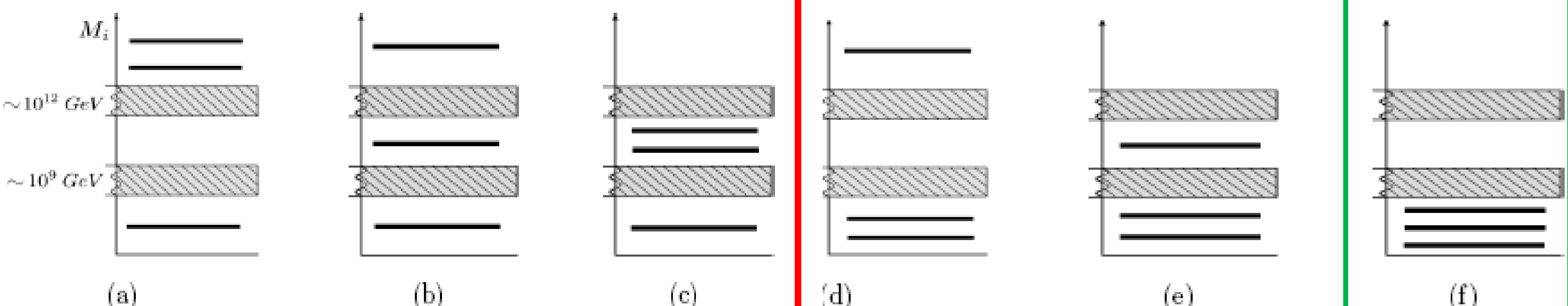
Typically rising in discrete flavour symmetry models

Heavy neutrino flavored scenario



2 RH neutrino scenario





N_2-dominated scenario:

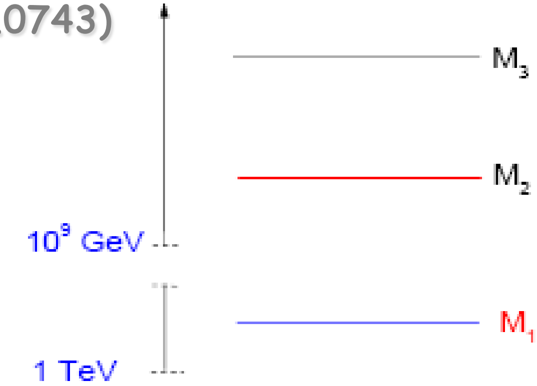
☛N_1 produces negligible asymmetry;

Low scale leptogenesis

Examples: Resonant+ ARS leptogenesis

N_2-leptogenesis

(PDB hep-ph/0502082, Vives hep-ph/0512160; Blanchet,PDB 0807.0743)



- **Unflavoured case:** asymmetry produced from N_2 - RH neutrinos is typically washed-out

$$\eta_{B0}^{lep(N_2)} \simeq 0.01 \cdot \varepsilon_2 \cdot \kappa^{fin}(K_2) \cdot e^{-\frac{3\pi}{8}K_1} << \eta_{B0}^{CMB}$$

- **Adding flavour effects:** lighest RH neutrino wash-out acts on individual flavour $\Rightarrow$ much weaker

$$\boxed{N_{B-L}^{\mathrm{f}}(N_2) = P_{2e}^0\, \varepsilon_2\, \kappa(K_2)\, e^{-\frac{3\pi}{8}\, K_{1e}} + P_{2\mu}^0\, \varepsilon_2\, \kappa(K_2)\, e^{-\frac{3\pi}{8}\, K_{1\mu}} + P_{2\tau}^0\, \varepsilon_2\, \kappa(K_2)\, e^{-\frac{3\pi}{8}\, K_{1\tau}}}$$

- With flavor effects the domain of successful N_2 dominated leptogenesis greatly enlarges: the probability that $K_1 < 1$ is less than 0.1% but the probability that either K_{1e} or $K_{1\mu}$ or $K_{1\tau}$ is less than 1 is ~23%

  (PDB, Michele Re Fiorentin, Rome Samanta)

- Existence of the heaviest RH neutrino N_3 is necessary for the $\varepsilon_{2\alpha}$'s not to be negligible

- It is the only hierarchical scenario that can realise strong thermal leptogenesis (independence of the initial conditions) if the asymmetry is tauon-dominated and if $m_1 \gtrsim 10$ meV (corresponding to $\Sigma_i m_i \gtrsim 80$meV)

  (PDB, Michele Re Fiorentin, Sophie King arXiv 1401.6185)

- N_2-leptogenesis rescues SO(10)-inspired models!

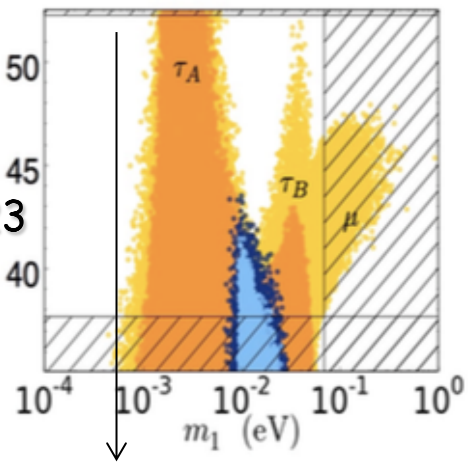
N_2-leptogenesis rescues SO(10)-inspired leptogenesi

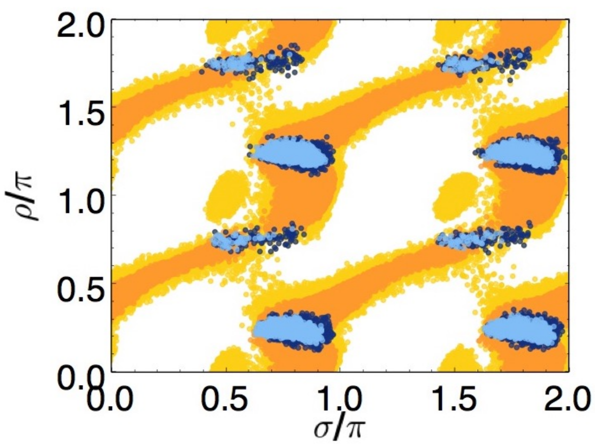
(PDB, Riotto 0809.2285;1012.2343;He,Lew,Volkas 0810.1104)

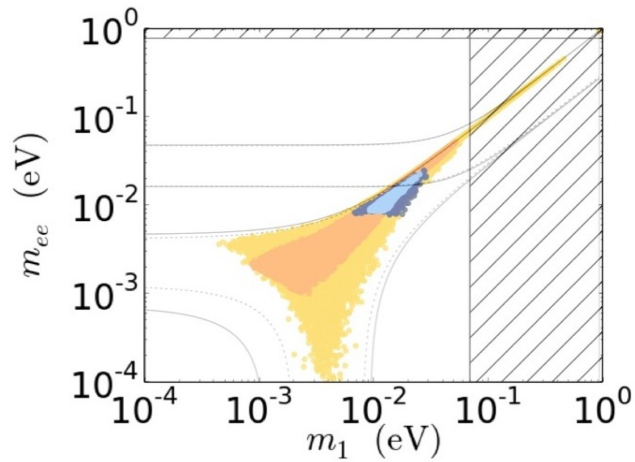
- dependence on α_1 and α_3 cancels out $\Rightarrow$
  the asymmetry depends only on $\alpha_2 \equiv m_{D2}/m_{charm}$: $\eta_B \propto \alpha_2^2$

$\alpha_2=5$ <u>NORMAL ORDERING</u> $I \leq V_L \leq V_{CKM}$ $V_L = I$

Θ_{23}







- ➢ Lower bound $m_1 \gtrsim 10^{-3}$ eV
- ➢ θ_{23} upper bound

- ➢ Majorana phases constrained about specific regions

- ➢ Effective $0\nu\beta\beta$ mass can still vanish but bulk of points above meV

- ➢ INVERTED ORDERING IS NOW EXCLUDED (it requires too large sum o neutrino masses + too large θ_{23})
- ➢ Tauon + muon-dominated solutions
- ➢ Strong thermal leptogenesis is realised for a subset of tauon solutions (blue points)

Imposing SO(10)-inspired conditions

Seesaw formula

$$m_\nu = -m_D \, \frac{1}{D_M} \, m_D^T \, .$$

Leptonic mixing matrix

$$U^\dagger \, m_\nu \, U^\star = -D_m$$

Bi-unitary parameterisation

$$m_D = V_L^\dagger \, D_{m_D} \, U_R$$

SO(10)-inspired conditions

$$m_{D1} = \alpha_1 \, m_u \, , \; m_{D2} = \alpha_2 \, m_c \, , \; m_{D3} = \alpha_3 \, m_t \, , \quad (\alpha_i = \mathcal{O}(1))$$

Majorana mass matrix (in the Yukawa basis)

$$U_R^\star \, D_M \, U_R^\dagger = M = D_{m_D} \, V_L^\star \, U^\star \, D_m^{-1} \, U^\dagger \, V_L^\dagger \, D_{m_D} \simeq -D_{m_D} \, m_\nu^{-1} \, D_{m_D}$$

RH neutrino mass spectrum (V_L=I)

(Akhmedov,Frigerio,Smirnov, 2005; PDB, Re Fiorentin, Marzola,1411.5478)

$$U_R \simeq \begin{pmatrix} 1 & -\frac{m_{D1}}{m_{D2}}\frac{m^\star_{\nu e\mu}}{m^\star_{\nu ee}} & \frac{m_{D1}}{m_{D3}}\frac{(m_\nu^{-1})^\star_{e\tau}}{(m_\nu^{-1})^\star_{\tau\tau}} \\ \frac{m_{D1}}{m_{D2}}\frac{m_{\nu e\mu}}{m_{\nu ee}} & 1 & \frac{m_{D2}}{m_{D3}}\frac{(m_\nu^{-1})^\star_{\mu\tau}}{(m_\nu^{-1})^\star_{\tau\tau}} \\ \frac{m_{D1}}{m_{D3}}\frac{m_{\nu e\tau}}{m_{\nu ee}} & -\frac{m_{D2}}{m_{D3}}\frac{(m_\nu^{-1})_{\mu\tau}}{(m_\nu^{-1})_{\tau\tau}} & 1 \end{pmatrix} D_\Phi \qquad D_\phi \equiv (e^{-i\frac{\Phi_1}{2}}, e^{-i\frac{\Phi_2}{2}}, e^{-i\frac{\Phi_3}{2}})$$

$$\boxed{M_1 \simeq \frac{m_{D1}^2}{|m_{\nu ee}|} \simeq \frac{\alpha_1^2\, m_u^2}{|m_{\nu ee}|} \simeq \alpha_1^2\, 10^5\,\mathrm{GeV}\left(\frac{m_u}{1\mathrm{MeV}}\right)^2\left(\frac{10\,\mathrm{meV}}{|m_{\nu ee}|}\right)}$$

$$\Phi_1 = \mathrm{Arg}[-m^\star_{\nu ee}]\,.$$

0νββ neutrino mass

$$\boxed{M_2 \simeq \frac{\alpha_2^2\, m_c^2}{m_1\, m_2\, m_3}\frac{|m_{\nu ee}|}{|(m_\nu^{-1})_{\tau\tau}|} \simeq \alpha_2^2\, 10^{11}\,\mathrm{GeV}\left(\frac{m_c}{400\mathrm{MeV}}\right)^2\left(\frac{|m_{\nu ee}|}{10\,\mathrm{meV}}\right)}$$

$$\Phi_2 = \mathrm{Arg}\left[\frac{m_{\nu ee}}{(m_\nu^{-1})_{\tau\tau}}\right] - 2\,(\rho+\sigma)$$

$$\boxed{M_3 \simeq \alpha_3^2\, m_t^2\, |(m_\nu^{-1})_{\tau\tau}| \simeq \alpha_3^2\, 10^{15}\,\mathrm{GeV}\left(\frac{m_t}{100\mathrm{GeV}}\right)^2\left(\frac{\mathrm{meV}}{m_1}\right)}\,.$$

$$\Phi_3 = \mathrm{Arg}[-(m_\nu^{-1})_{\tau\tau}]\,.$$

Decrypting SO(10)-inspired leptogenesis (V_L=I

(PDB, Re Fiorentin, Marzola,1411.5478)

Finally, putting all together, one arrives to an expression for the final asymmetry:

$$\boxed{\begin{aligned} N_{B-L}^{\rm lep,f} \simeq & \frac{3}{16\,\pi}\,\frac{\alpha_2^2\,m_c^2}{v^2}\,\frac{|m_{\nu ee}|\,\left(|m_{\nu\tau\tau}^{-1}|^2+|m_{\nu\mu\tau}^{-1}|^2\right)^{-1}}{m_1\,m_2\,m_3}\,\frac{|m_{\nu\tau\tau}^{-1}|^2}{|m_{\nu\mu\tau}^{-1}|^2}\,\sin\alpha_L \\ & \times\ \kappa\left(\frac{m_1\,m_2\,m_3}{m_\star}\,\frac{|(m_\nu^{-1})_{\mu\tau}|^2}{|m_{\nu ee}|\,|(m_\nu^{-1})_{\tau\tau}|}\right) \\ & \times\ e^{-\frac{3\pi}{8}\frac{|m_{\nu e\tau}|^2}{m_\star\,|m_{\nu ee}|}}\,. \end{aligned}}$$

$K_{1\tau}$

SO(10)-inspired leptogenesis phase

$$\boxed{\alpha_L = {\rm Arg}\,[m_{\nu ee}] - 2\,{\rm Arg}[(m_\nu^{-1})_{\mu\tau}] + \pi - 2\,(\rho+\sigma)\,.}$$

successful leptogenesis condition

$$\eta_B^{SO10\,lep}(m_1, m_{sol}, m_{atm}, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \rho, \sigma; \alpha_2) = \eta_B^{\rm obs}$$

This condition identifies an hypersurface in the space of low energy neutrino parameters

All numerical results are accurately reproduced for V_L=I

In particular, one has a strong tau-dominance:

$$\boxed{\varepsilon_{2\tau} : \varepsilon_{2\mu} : \varepsilon_{2e} = \alpha_3^2\,m_t^2 : \alpha_2^2\,m_c^2 : \alpha_1^2\,m_u^2\,\frac{\alpha_3 m_t}{a_2\,m_c}\,\frac{\alpha_1^2\,m_u^2}{\alpha_2^2\,m_c^2}\,.}$$

Some insight into τ solutions

They split into two (bordering) regions. Both of course realise the crucial condition $K_{1\tau} \lesssim 1$ but in a different way:

τ_A solutions:

- 1 meV $\lesssim m_1 \lesssim$ 30 meV
- $K_{2\tau} \gtrsim 20$ (strong washout at the production)
- $2\sigma - \delta \simeq 2n\pi$ (n integer) for $m_1 \ll m_{sol}$
- They can realise strong thermal leptogenesis for $m_1 \gtrsim 10$ meV

τ_B solutions:

- 30 meV $\lesssim m_1 \lesssim$ 70 meV
- $1 \leq K_{2\tau} \lesssim 10$ (mild washout at the production)
- $\rho \simeq 2n\pi$ (n integer)
- They cannot realise strong thermal leptogenesis since $K_{1\mu} \lesssim 4$ (they cannot wash-out efficiently a large pre-existing muonic asymmetry)

Turning on a mismatch between neutrino Yukawa and weak basis ($V_L \neq 1$)

$$V_L = \begin{pmatrix} c_{12}^L c_{13}^L & s_{12}^L c_{13}^L & s_{13}^L e^{-i\delta_L} \\ -s_{12}^L c_{23}^L - c_{12}^L s_{23}^L s_{13}^L e^{i\delta_L} & c_{12}^L c_{23}^L - s_{12}^L s_{23}^L s_{13}^L e^{i\delta_L} & s_{23}^L c_{13}^L \\ s_{12}^L s_{23}^L - c_{12}^L c_{23}^L s_{13}^L e^{i\delta_L} & -c_{12}^L s_{23}^L - s_{12}^L c_{23}^L s_{13}^L e^{i\delta_L} & c_{23}^L c_{13}^L \end{pmatrix} \begin{pmatrix} e^{i\rho_L} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\sigma_L} \end{pmatrix}$$

$$s_{ij}^L \equiv \sin\theta_{ij}^L\,,\;\; c_{ij}^L \equiv \cos\theta_{ij}^L$$

By definition in SO(10)-inspired leptogenesis: $0 \leq \theta^L_{ij} \lesssim \theta^{CKM}_{ij}$ ($\Leftrightarrow I \leq V_L \lesssim V_{CKM}$)

The upper bounds are not strictly determined, as far as the RH neutrino mass spectrum is such that one can assume N_2-dominated leptogenesis.

Full analytical solution (relaxing $V_L=I$): RH neutrino mass spectrum and mixing matrix

light neutrino mass matrix in the Yukawa basis

$$m_\nu \to \tilde{m}_\nu = V_L m_\nu V_L^T$$

RH neutrino masses

$$M_1 \simeq \frac{\alpha_1^2 m_u^2}{|(\tilde{m}_\nu)_{11}|}, \;\; M_2 \simeq \frac{\alpha_2^2 m_c^2}{m_1 m_2 m_3} \frac{|(\tilde{m}_\nu)_{11}|}{|(\tilde{m}_\nu^{-1})_{33}|}, \;\; M_3 \simeq \alpha_3^2 m_t^2 \, |(\tilde{m}_\nu^{-1})_{33}|$$

RH neutrino phases

$$\Phi_1 \simeq -\mathrm{Arg}[-(\tilde{m}_\nu)^*_{11}], \;\; \Phi_2 \simeq \mathrm{Arg}\left[\frac{(\tilde{m}_\nu)_{11}}{(\tilde{m}_\nu^{-1})_{33}}\right] - 2(\rho+\sigma) - 2(\rho_L+\sigma_L), \;\; \Phi_3 \simeq \mathrm{Arg}[(\tilde{m}_\nu^{-1})_{33}]$$

RH neutrino mixing matrix

$$U_R \simeq \begin{pmatrix} 1 & -\frac{m_{D1}}{m_{D2}} \frac{(\tilde{m}_\nu)^*_{12}}{(\tilde{m}_\nu)^*_{11}} & \frac{m_{D1}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})^*_{13}}{(\tilde{m}_\nu^{-1})^*_{33}} \\ \frac{m_{D1}}{m_{D2}} \frac{(\tilde{m}_\nu)_{12}}{(\tilde{m}_\nu)_{11}} & 1 & \frac{m_{D2}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})^*_{23}}{(\tilde{m}_\nu^{-1})^*_{33}} \\ \frac{m_{D1}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{13}}{(\tilde{m}_\nu^{-1})_{33}} & -\frac{m_{D2}}{m_{D3}} \frac{(\tilde{m}_\nu^{-1})_{23}}{(\tilde{m}_\nu^{-1})_{33}} & 1 \end{pmatrix} D_\Phi \; , \quad D_\Phi \equiv \left(e^{-i\frac{\Phi_1}{2}}, e^{-i\frac{\Phi_2}{2}}, e^{-i\frac{\Phi_3}{2}}\right)$$

Full analytical solution for the asymmetry ($I \leq V_L \lesssim V_{CKM}$)

Flavoured decay parameters

$$K_{I\alpha} = \frac{\sum_{k,l} m_{Dk} m_{Dl} V_{Lk\alpha} V^{*}_{Ll\alpha} U^{*}_{RkI} U_{Rli}}{M_I m_*}$$

Flavoured CP asymmetries

$$\varepsilon_{2\alpha} = \frac{3}{16\pi v^2} \frac{|(\tilde{m}_\nu)_{11}|}{m_1 m_2 m_3} \frac{\sum_{k,l} m_{Dk} m_{Dl} \, \mathrm{Im}[V_{Lk\alpha} V^{*}_{Ll\alpha} U^{*}_{Rk2} U_{Rl3} U^{*}_{R32} U_{R33}]}{|(\tilde{m}^{-1}_\nu)_{33}|^2 + |(\tilde{m}^{-1}_\nu)_{23}|^2}$$

Final B-L asymmetry

$$N^{\mathrm{lep,f}}_{B-L} = \varepsilon_{2e} \kappa(K_{2e} + K_{2\mu})\, e^{-\frac{3\pi}{8} K_{1e}} + \varepsilon_{2\mu} \kappa(K_{2e} + K_{2\mu})\, e^{-\frac{3\pi}{8} K_{1\mu}} + \varepsilon_{2\tau} \kappa(K_{2\tau})\, e^{-\frac{3\pi}{8} K_{1\tau}}$$

This time one has: $\eta_B^{SO10\,lep}(m_1, m_{sol}, m_{atm}, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \rho, \sigma; \alpha_2, V_L) = \eta_B^{\mathrm{obs}}$

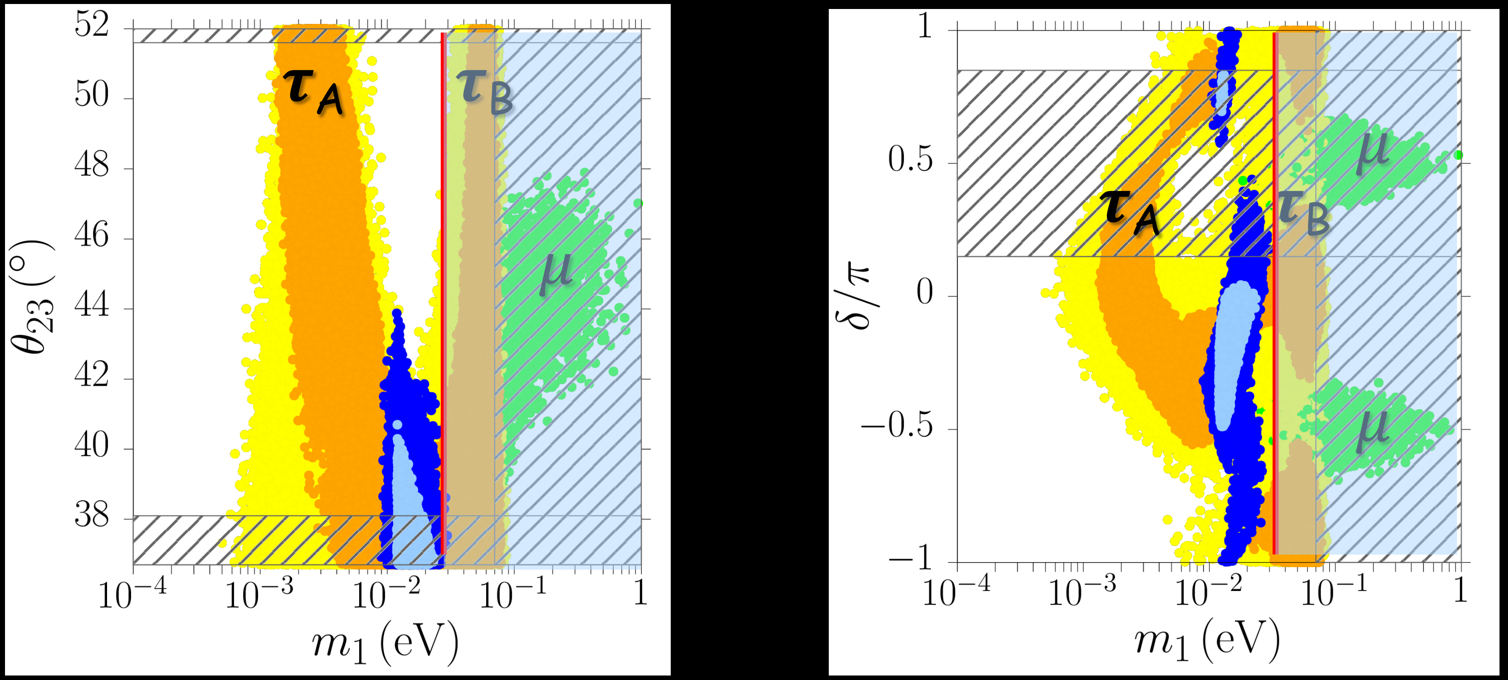
The dependence on the 6 parameters in V_L give some thickness to the hypersurface that becomes a layer but the smallness of the θ^L_{ij} however still make in a way that constraints do relax but in general do not evaporate.

Also notice that now: $\varepsilon^{\max}_{2e} : \varepsilon^{\max}_{2\mu} : \varepsilon^{\max}_{2\tau} \simeq 1 : |V_{L23}| : |V_{L21} V_{L31}|$

This explains why tauon solutions are still favoured but this time also muon solutions appear and in the supersymmetric case even very marginal electron solutions

SO(10)-inspired leptogenesis confronting long baseline and absolute neutrino mass experiments

α_2=5

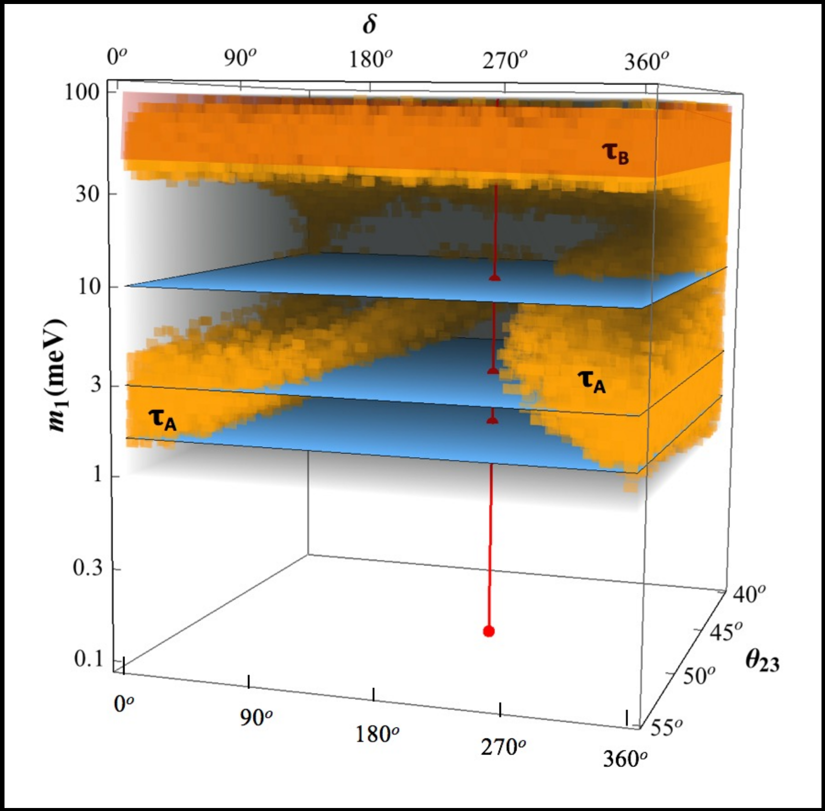


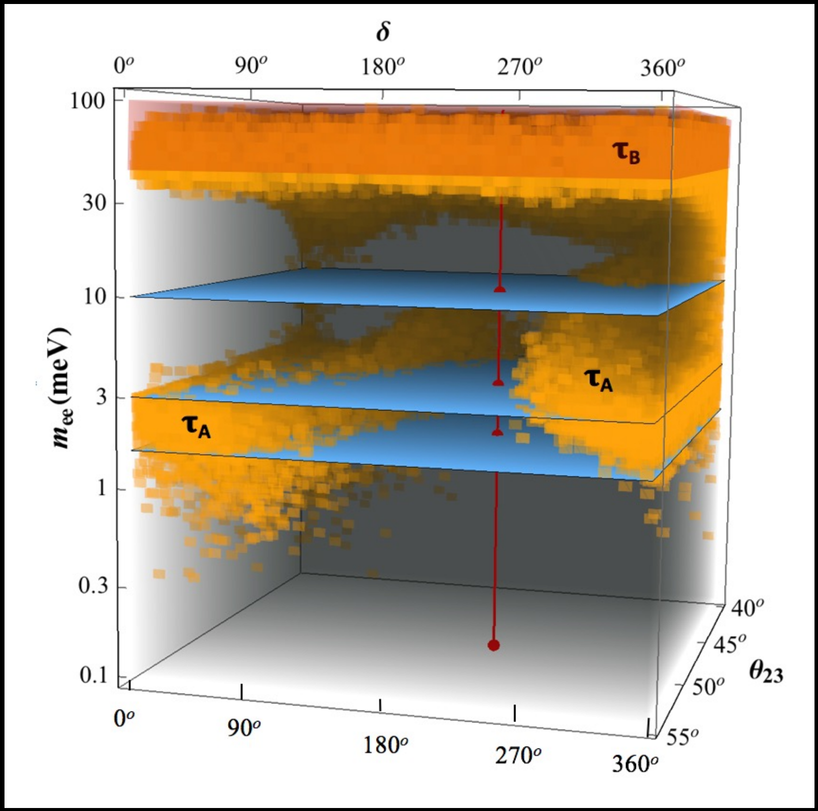
Projecting the allowed region (an hypersurface in the space of neutrino parameters) on planes can hide a more complex structure corresponding potentially to stronger predictions.

SO(10)-inspired leptogenesis confronting long baseline and absolute neutrino mass experiments…..in 3D

(PDB, R. Samanta 2005.03057)

α_2=5



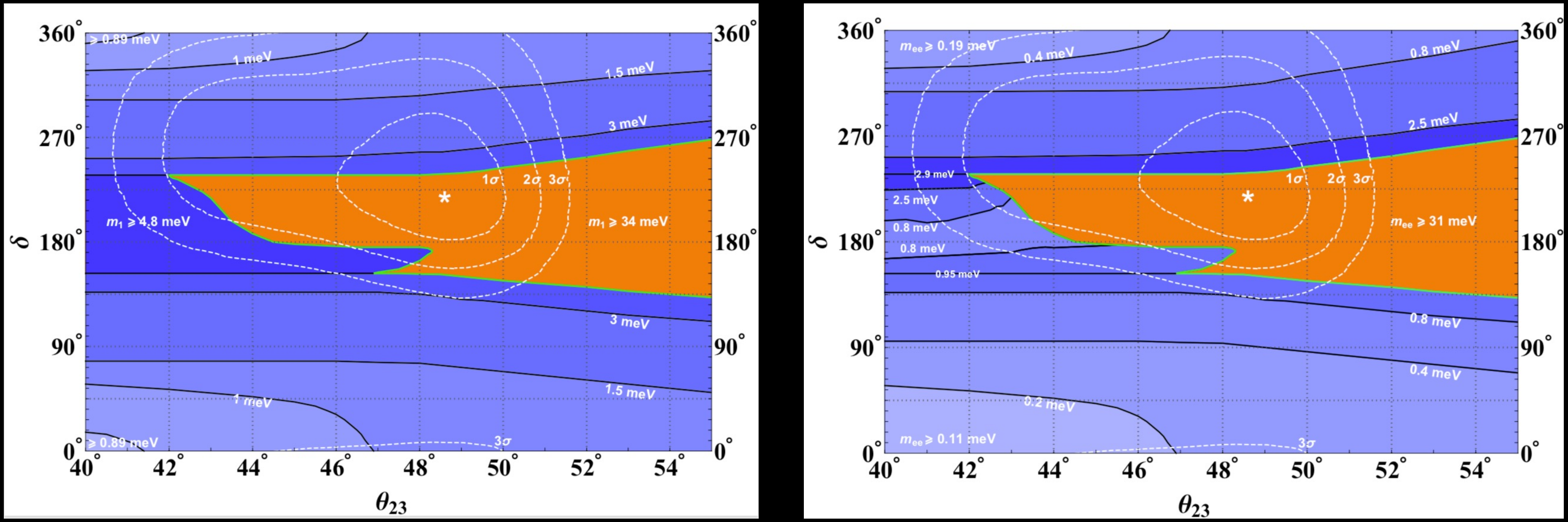


For certain values of δ and θ_{23} the lower bound on the absolute neutrino mass scale is much more stringent: $m_1, m_{ee} \gtrsim 30$ meV

SO(10)-inspired leptogenesis: lower bound on the absolute neutrino mass scale as a function of δ and θ_{23}

(PDB, R. Samanta 2005.03057)

α_2=5



Future precise measurements of δ and θ_{23} will have an important impact on SO(10)-inspired leptogenesis, in particular a precise determination of δ might be crucial. Ultimately if measured neutrino mixing parameters will lie on the hypersurface (implying $0\nu\beta\beta$ discovery) a strong case for discovery can be made (this has to take into account also θ_{13}, θ_{12}, m_{sol}, m_{atm})

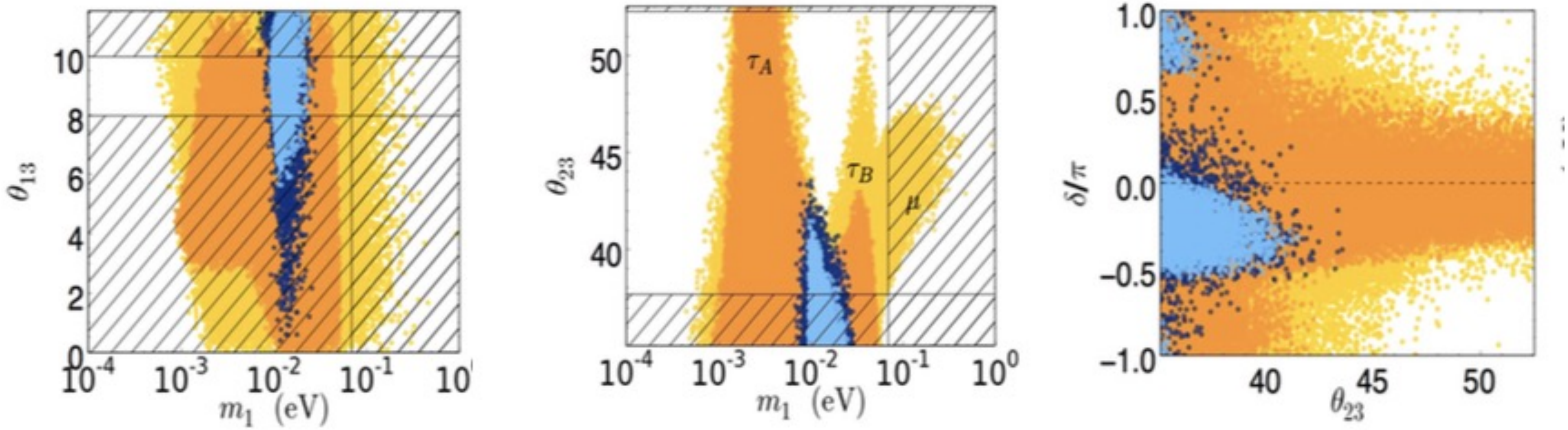
Notice that CP conserving values of δ are possible since CP violation comes from high energy phases (they can be identified with those in the orthogonal matrix)

Strong thermal SO(10)-inspired leptogenesis

(PDB, Marzola 09/2011, DESY workshop and 1308.1107; PDB, Re Fiorentin, Marzola 1411.5478

- Strong thermal leptogenesis condition can be satisfied for a subset of the solutions only for NORMAL ORDERING

α_2=5 ❑ blue regions: $N_{B-L}^{pre-ex} = 10^{-3}$ ($I \leq V_L \leq V_{CKM}$; $V_L = I$)



- Absolute neutrino mass scale: $8 \lesssim m_1/\text{meV} \lesssim 30 \Leftrightarrow 70 \lesssim \sum_i m_i/\text{meV} \lesssim 120$
- Non-vanishing Θ_{13} (first results presented before Daya Bay discovery)
- Θ_{23} preferably in the first octant;

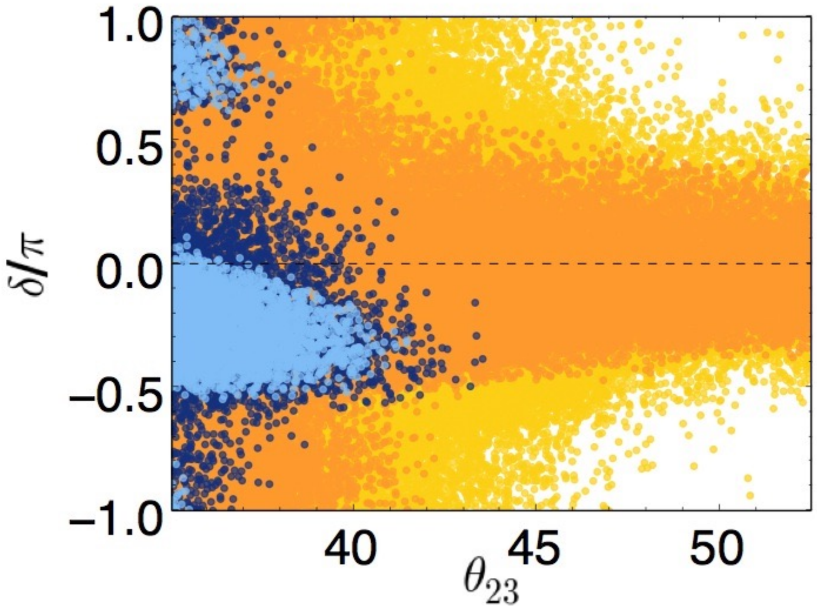
Why do we live in a matter (and not antimatter) dominated universe?

(PDB, Marzola, Re Fiorentin, 1411.5478)

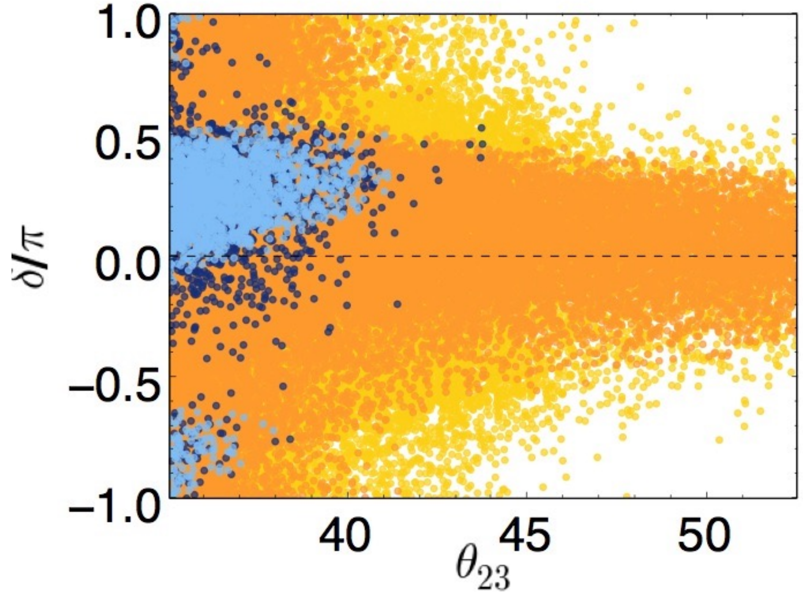
$\alpha_2=5$

- blue regions: $N_{B-L}^{pre-ex} = 10^{-3}$ ($I \le V_L \le V_{CKM}$; $V_L = I$)

Matter dominated universe ($\eta_B \sim + 6\times10^{-10}$)



Antimatter dominated universe ($\eta_B \sim - 6\times10^{-10}$)



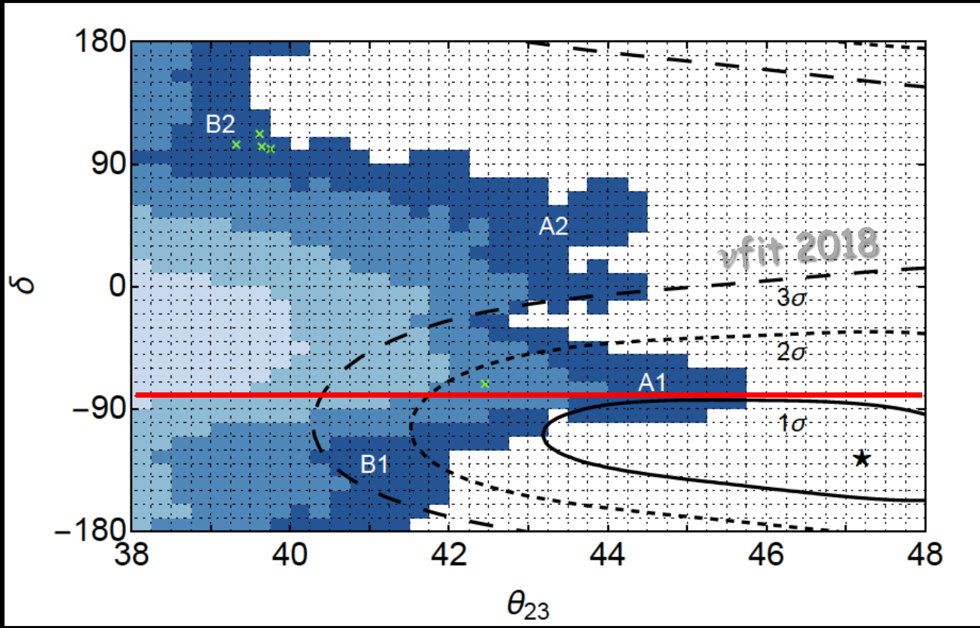
For sufficiently large θ_{23} one has sign(η_B)=-sign(sin δ)

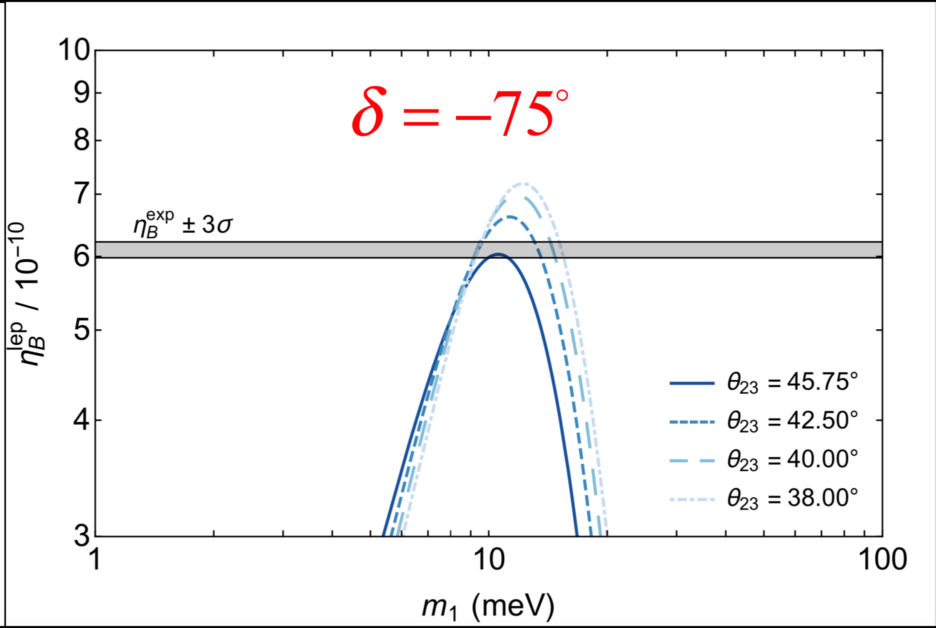
$\Rightarrow$ We would live in a matter dominated universe because sin δ < 0

Strong SO(10)-inspired leptogenesis confronting long baseline experiments
(PDB, Marco Chianese 1802.07690)

Pre-existing initial asymmetry: $N^{p,i}_{B-L} = 10^{-3}$

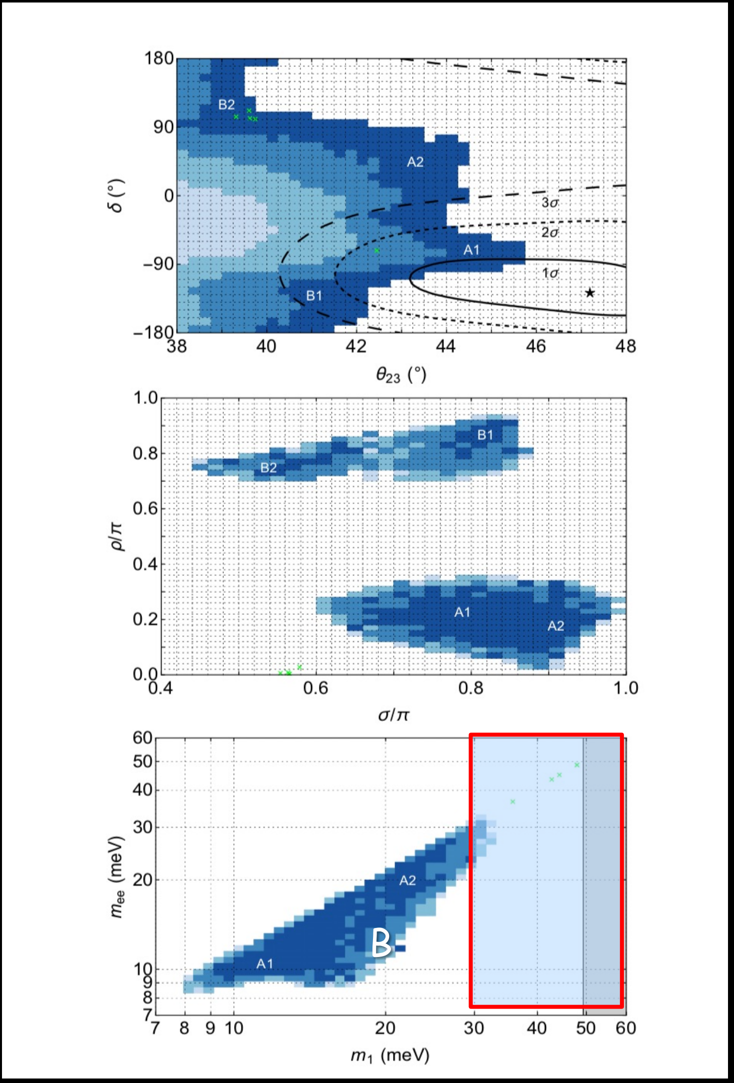
$$\alpha_2 = m_{D2} / m_{charm} = 5$$





"The more stringent experimental lower bound on atmospheric mixing angle starts to corner STSO10-leptogenesis"

Strong SO(10)-inspired leptogenesis, Majorana phases and $0\nu\beta\beta$ decay
(PDB, Marco Chianese 1802.07690)



A determination of δ and θ_{23} would correspondingly determine the Majorana phases.

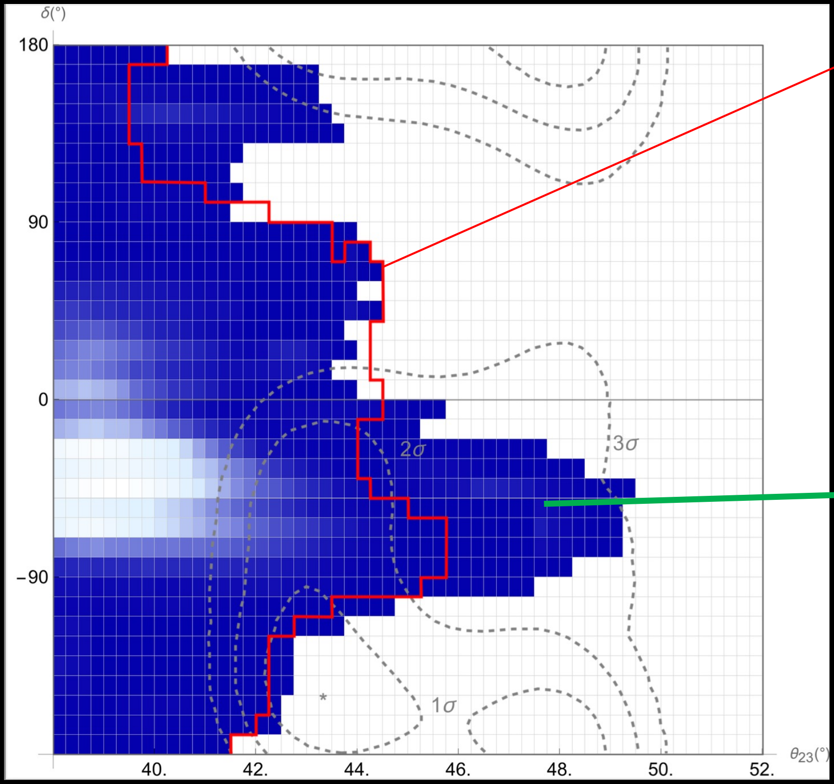
New atmospheric neutrino data seem to remove the tension

(PDB, Xubin Hu, in preparation)

The new SK atmospheric data seem to favour first octant when combined in global analysis (νfit September 2024) and moreover **$\Delta\chi^2$ (IO-NO)=6.1:** there is a potential interesting overlap now

Pre-existing initial asymmetry: $N_{B-L}^{p,i} = 10^{-3}$

$$\alpha_2 = m_{D2} / m_{charm} = 5$$



"old" solutions confirmed (no flavour coupling effects)

new solutions found including flavour coupling effects

Is the asymmetry correctly calculated?

The are 4 main effects that are neglected in the calculation of the asymmetry:

- Flavour coupling effects from spectator processes
- Radiative corrections and running of the parameters
- Full density matrix calculation
- Momentum dependence

Each of this effect is expected to give corrections without changing the main features. At the same time they slow down the calculation and scatter plots with millions of points are hard to obtain including all of them.

Including flavour coupling effects

(Antusch, PDB, Jones and King 2010; PDB, Xubin Hu, in preparation)

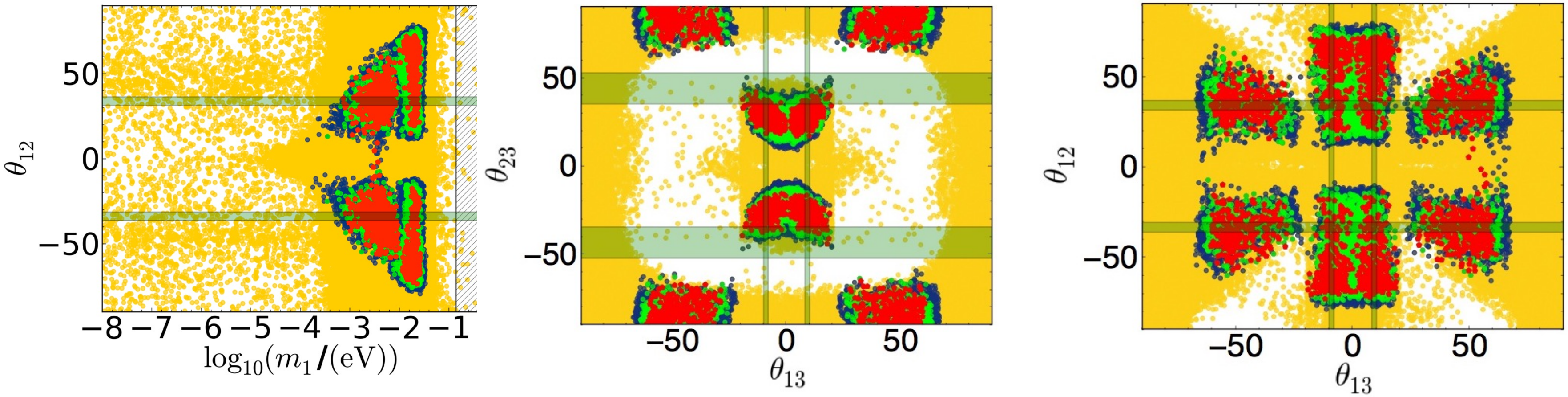
$$\frac{dN_{\Delta_\alpha}}{dz_1} = -P^0_{1\alpha} \sum_\beta C^{(3)}_{\alpha\beta} \, W^{\rm ID}_1 \, N_{\Delta_\beta} \, ,$$

$$\begin{aligned}
N^{\rm f}_{\Delta_\alpha} \;&=\; V^{-1}_{\alpha e''} \left[\sum_\beta V_{e''\beta} \, N^{T\sim T_L}_{\Delta_\beta}\right] e^{-\frac{3\pi}{8}\, K_{1e''}} \\
&+\; V^{-1}_{\alpha\mu''} \left[\sum_\beta V_{\mu''\beta} \, N^{T\sim T_L}_{\Delta_\beta}\right] e^{-\frac{3\pi}{8}\, K_{1\mu''}} \\
&+\; V^{-1}_{\alpha\tau''} \left[\sum_\beta V_{\tau''\beta} \, N^{T\sim T_L}_{\Delta_\beta}\right] e^{-\frac{3\pi}{8}\, K_{1\tau''}} \, .
\end{aligned}$$

How significantly can the STSO10 solution be supported by data?

(PDB, Marzola '13)

(N^{p}_{B-L}= 0, 0.001, 0.01, 0.1)



If θ_{23} is found in the first octant then p$\lesssim$10%
If NO is confirmed then p$\lesssim$5%
If δ is measured in the fourth quadrant p$\lesssim$1%

This would sum up to the coincidence $m_{sol}, m_{atm} \sim 10\, m_*$
If also absolute neutrino mass scales (m_1 and m_{ee}) will fall within the expected range (implying $0\nu\beta\beta$ signal) then strong case for discovery
(notice also that Majorana phases impose non arbitrary m_{ee}/m_1)
What about if one gives up strong thermal leptogenesis?

A popular class of SO(10) models

(Fritzsch, Minkowski, Annals Phys. 93 (1975) 193-266; R.Slansky, Phys.Rept. 79 (1981) 1-128; G.G. Ross, GUTs, 1985; Dutta, Mimura, Mohapatra, hep-ph/0507319; G. Senjanovic hep-ph/0612312)

In SO(10) models each SM particles generation + 1 RH neutrino are assigned to a single 16-dim representation. Masses of fermions arise from Yukawa interactions of two 16s with vevs of suitable Higgs fields. Since:

$$16 \otimes 16 = 10_S \oplus \overline{126}_S \oplus 120_A ;$$

The Higgs fields of renormalizable SO(10) models can belong to 10-, 126-,120-dim representations yielding Yukawa part of the Lagrangian

$$\mathcal{L}_Y = 16\,(Y_{10} 10_H + Y_{126} \overline{126}_H + Y_{120} 120_H)\, 16 \,.$$

After SSB of the fermions at $M_{GUT}=2\times10^{16}$ GeV one obtains the masses:

up-quark mass matrix $\quad M_u = \boxed{v_{10}^u Y_{10}} + v_{126}^u Y_{126} + v_{120}^u Y_{120} ,$

down-quark mass matrix $\quad M_d = \boxed{v_{10}^d Y_{10}} + v_{126}^d Y_{126} + v_{120}^d Y_{120} ,$

neutrino mass matrix $\quad M_D = \boxed{v_{10}^u Y_{10}} - 3 v_{126}^u Y_{126} + v_{120}^D Y_{120} ,$

charged lepton mass matrix $\quad M_l = \boxed{v_{10}^d Y_{10}} - 3 v_{126}^d Y_{126} + v_{120}^l Y_{120} ,$

RH neutrino mass matrix $\quad M_R = v_{126}^R Y_{126} ,$

LH neutrino mass matrix $\quad M_L = v_{126}^L Y_{126} ,$

Simplest case but clearly non-realistic: it predicts no mixing at all (both in quark and lepton Sectors). For realistic models one has to add at least the 126 contribution

NOTE: these models do respect SO(10)-inspired conditions

A recent realistic fit

(K Babu, PDB, C.S. Fong, S. Saad 2409.03840)

Observables (Δm^2_{ij} in eV^2)	Values at M_Z scale		
	Input	Benchmark Fit: NO	Benchmark Fit: IO
$y_u/10^{-6}$	6.65±2.25	7.30	10.0
$y_c/10^{-3}$	3.60±0.11	3.59	3.57
y_t	0.986±0.0086	0.986	0.986
$y_d/10^{-5}$	1.645±0.165	1.636	1.635
$y_s/10^{-4}$	3.125±0.165	3.122	3.148
$y_b/10^{-2}$	1.639±0.015	1.639	1.637
$y_e/10^{-6}$	2.7947±0.02794	2.7945	2.7906
$y_\mu/10^{-4}$	5.8998±0.05899	5.9011	5.9080
$y_\tau/10^{-2}$	1.0029±0.01002	1.0022	1.0023
$\theta_{12}^{\rm CKM}/10^{-2}$	22.735±0.072	22.729 ($\theta_{12}^{\rm CKM}$ =13.023°)	22.730 ($\theta_{12}^{\rm CKM}$ =13.023°)
$\theta_{23}^{\rm CKM}/10^{-2}$	4.208±0.064	4.206 ($\theta_{23}^{\rm CKM}$ =2.401°)	4.204 ($\theta_{23}^{\rm CKM}$ =2.408°)
$\theta_{13}^{\rm CKM}/10^{-3}$	3.64±0.13	3.64 ($\theta_{13}^{\rm CKM}$ =0.208°)	3.64 ($\theta_{13}^{\rm CKM}$ =0.208°)
$\delta_{\rm CKM}$	1.208±0.054	1.209 ($\delta_{\rm CKM}$ =69.322°)	1.212 ($\delta_{\rm CKM}$ =69.457°)
$\Delta m^2_{21}/10^{-5}$	7.425±0.205	7.413	7.506
$\Delta m^2_{31}/10^{-3}$ (NO)	2.515±0.028	2.514	-
$\Delta m^2_{32}/10^{-3}$ (IO)	-2.498±0.028	-	-2.499
$\sin^2\theta_{12}$	0.3045±0.0125	0.3041 ($\theta_{12} = 33.46°$)	0.3067 ($\theta_{12} = 33.63°$)
$\sin^2\theta_{23}$ (NO)*	0.5705±0.0205	0.4473 ($\theta_{23} = 41.98°$)	-
$\sin^2\theta_{23}$ (IO)*	0.576±0.019	-	0.5784 ($\theta_{23} = 49.51°$)
$\sin^2\theta_{13}$ (NO)	0.02223±0.00065	0.02223 ($\theta_{13} = 8.57°$)	-
$\sin^2\theta_{13}$ (IO)	0.02239±0.00063	-	0.02238 ($\theta_{13} = 8.60°$)
$\delta^\circ_{\rm CP}$ (NO)	207.5±38.5	240.49	-
$\delta^\circ_{\rm CP}$ (IO)	284.5±29.5	-	263.49
$\eta_B/10^{-10}$	6.12±0.04‡	7.6 (7.6)	9.6 (51)
χ^2	-	1.45	5.76†

For NO:

light neutrino masses

m_1= 0.038 meV
m_2= 8.6 meV
m_3= 50.1 meV
m_{ee}=3.7 meV

heavy neutrino masses

M_1=6.6x10^4 GeV

M_2=2.1x10^{12} GeV

M_3=8.1x10^{14} GeV

Conclusions

- The matter-antimatter asymmetry puzzle might be related to an explanation of neutrino masses, this seems today the most attractive scenario. Discovery of $0\nu\beta\beta$ would provide a very strong support.

- SO(10)-inspired leptogenesis provide a well motivated class of scenarios relying on N_2-leptogenesis. They lead to interesting predictions, in particular there is a lower bound on the absolute neutrino mass scale and we are now starting to probe the bulk of the solutions with absolute neutrino mass scale experiments. NO should be confirmed.

- A subset of the solutions realizes strong thermal leptogenesis, this is highly non-trivial. In this case the atmospheric neutrino mixing angle should be strictly in the first octant and CP Dirac phase in the 4th quadrant. $0\nu\beta\beta$ signal is not arbitrarily low and a discovery should be within reach in next generation experiments.

- SO(10)-inspired leptogenesis can be realized within a realistic minimal SO(10) model. In this case the $\theta_{23,L} \sim 45^\circ$: this might signal the presence of an additional discrete symmetry.