Fermion Masses, Unification and Beyond

Lecture I Fermion Masses and Mixings
Lecture II Unification
Lecture III Beyond the Standard Model

Steve King
University of Southampton,
May 2011

References

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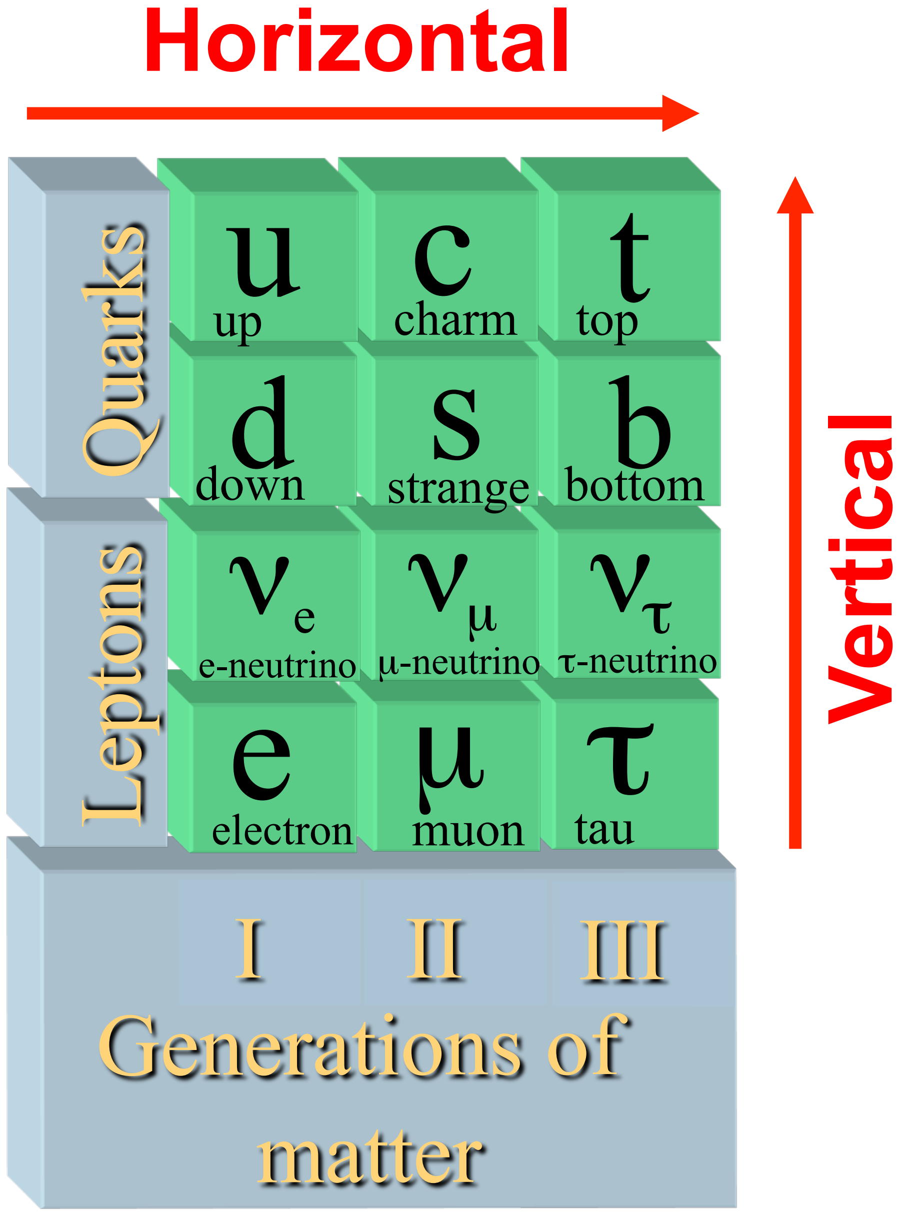
Lecture I

The Flavour Problem and See-Saw

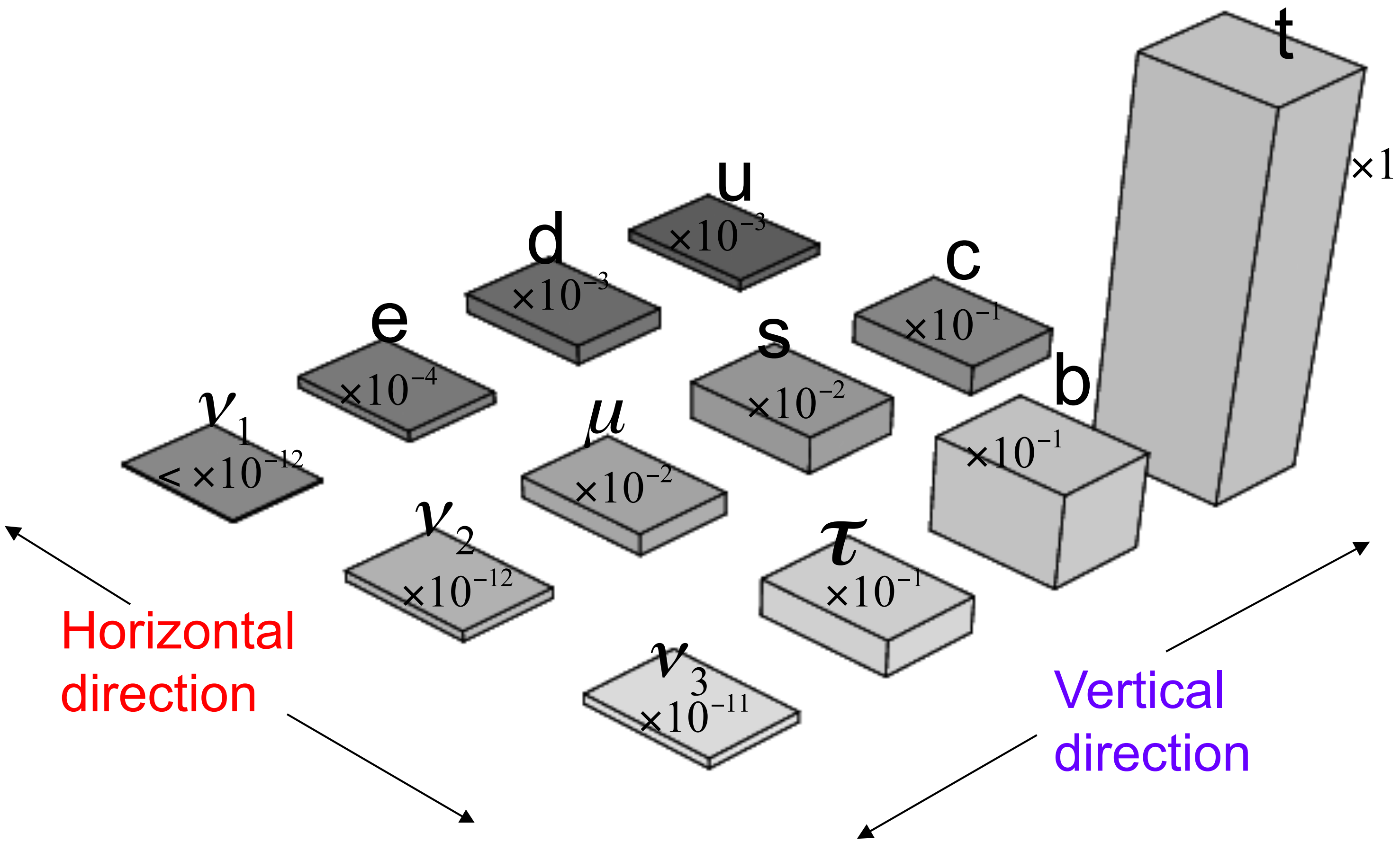
The Flavour Problem

1. Why are there three families of quarks and leptons?



The Flavour Problem

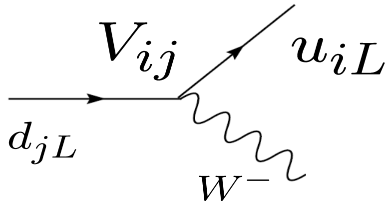
2. What is the origin of quark and lepton masses?



The Flavour Problem

3. Why is quark mixing so small?

Cabibbo
KobayashiM
askawa



$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\theta_{12} = 13^\circ \pm 0.1^\circ$$
$$\theta_{23} = 2.4^\circ \pm 0.1^\circ$$
$$\theta_{13} = 0.20^\circ \pm 0.05^\circ$$

All these angles are pretty small – why?

While the CP phase is quite large

$\delta_{CP} \approx 70^o \pm 5^o$

The Flavour Problem

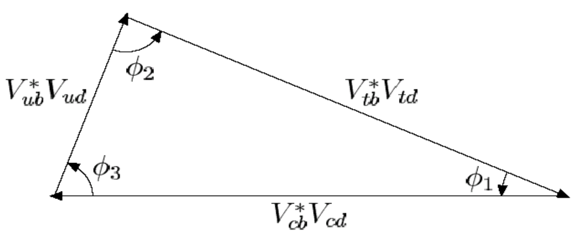
4. What is origin of quark CP violation?

Wolfenstein

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$

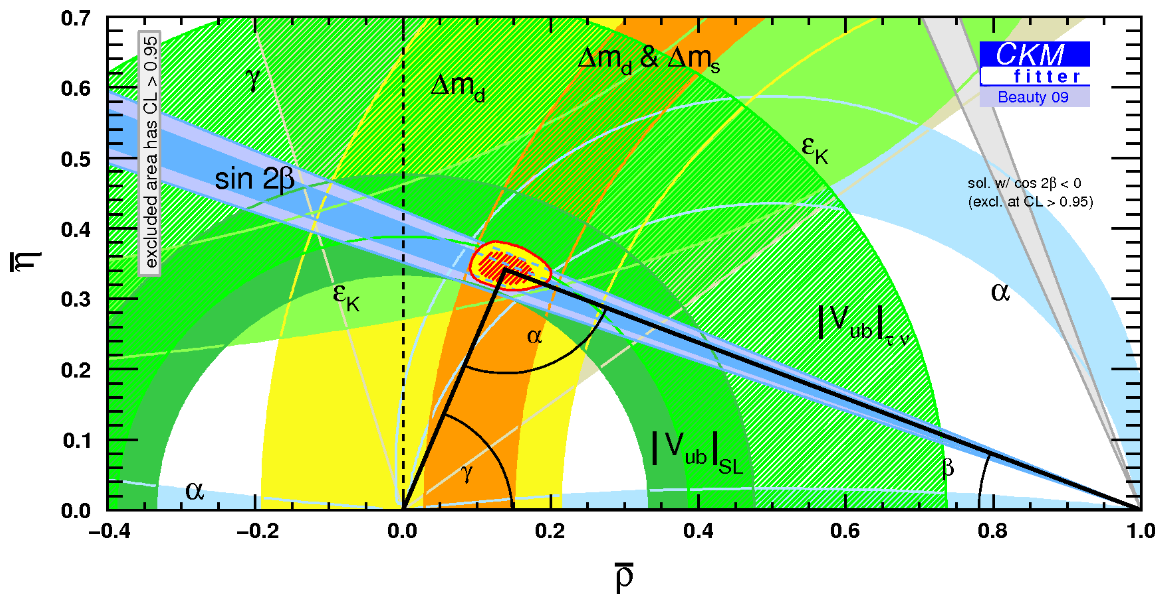
$\lambda \approx 0.226 \quad A \approx 0.81 \quad \rho \approx 0.13 \quad \eta \approx 0.35$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$\alpha \approx 90^o \pm 4^o$

$\delta_{CP} \approx \gamma \approx 70^o \pm 5^o$



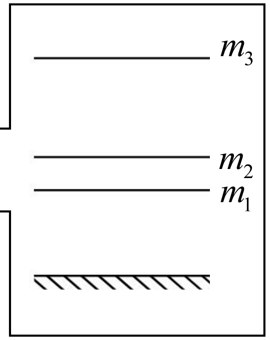
The Flavour Problem

5. Why is lepton mixing so large?

$$\begin{pmatrix}\nu_e\\ e^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\mu\\ \mu^-\end{pmatrix}_L \quad \begin{pmatrix}\nu_\tau\\ \tau^-\end{pmatrix}_L$$

Standard Model states

Neutrino mass states



Pontecorvo
Maki
Nakagawa
Sakata

$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}e^{-i\delta}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\alpha_1/2} & 0 & 0\\ 0 & e^{i\alpha_2/2} & 0\\ 0 & 0 & 1\end{pmatrix}$$

Atmospheric | Reactor | Solar | Majorana

$s_{ij} = \sin\theta_{ij}$

$c_{ij} = \cos\theta_{ij}$

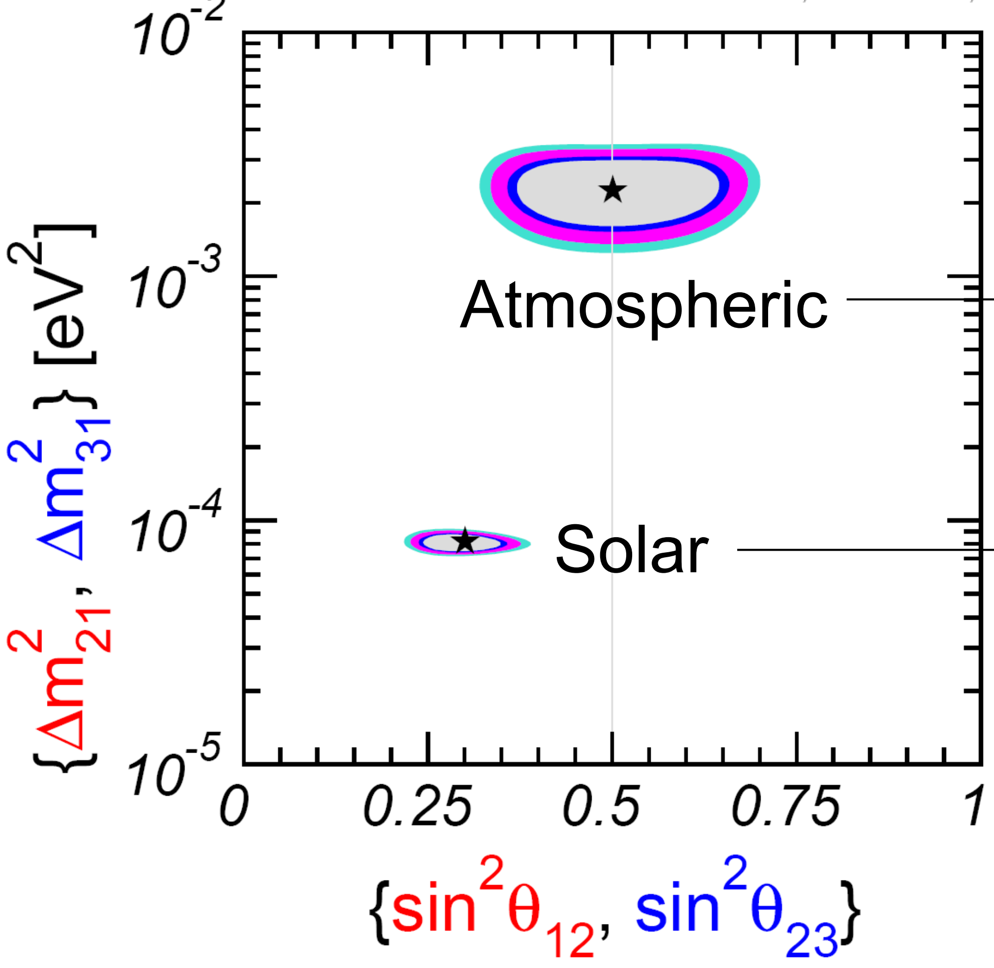
Oscillation phase δ

Majorana phases α_1, α_2

3 masses + 3 angles + 1(3) phase(s)
= 7(9) new parameters for SM

Global Fit to Atmospheric and Solar Data

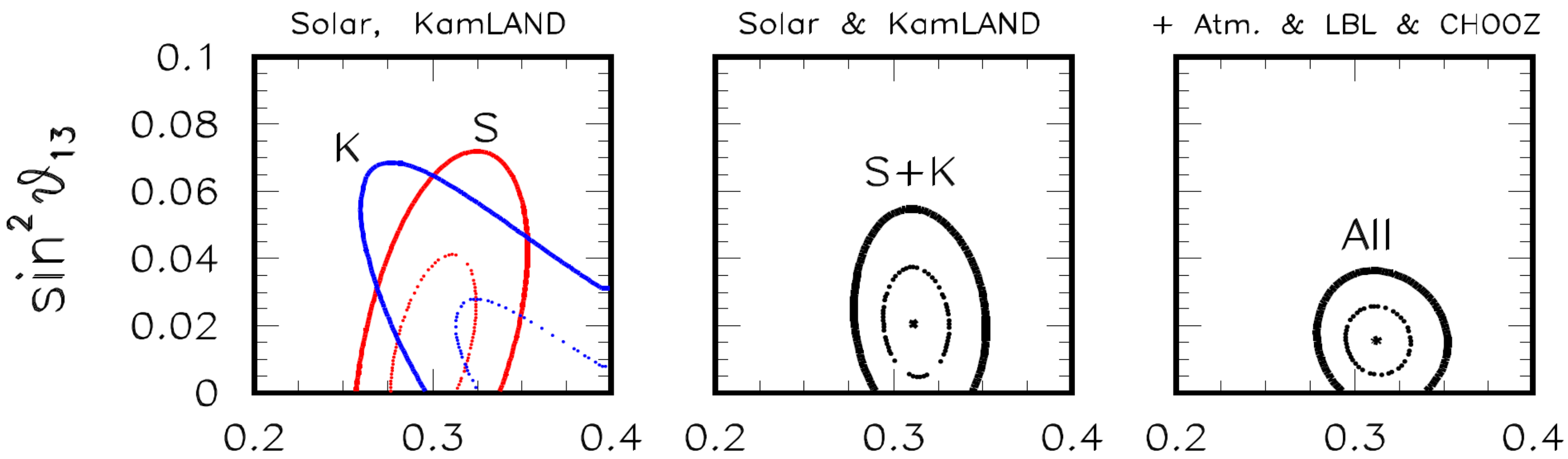
M. Maltoni, T. Schwetz, M. A. Tortola and J. W. F. Valle,



$\theta_{23} = 43.1^{o}{}^{+4.4^o}_{-3.5^o}$

$\theta_{12} = 34.5^o \pm 1.4^o$

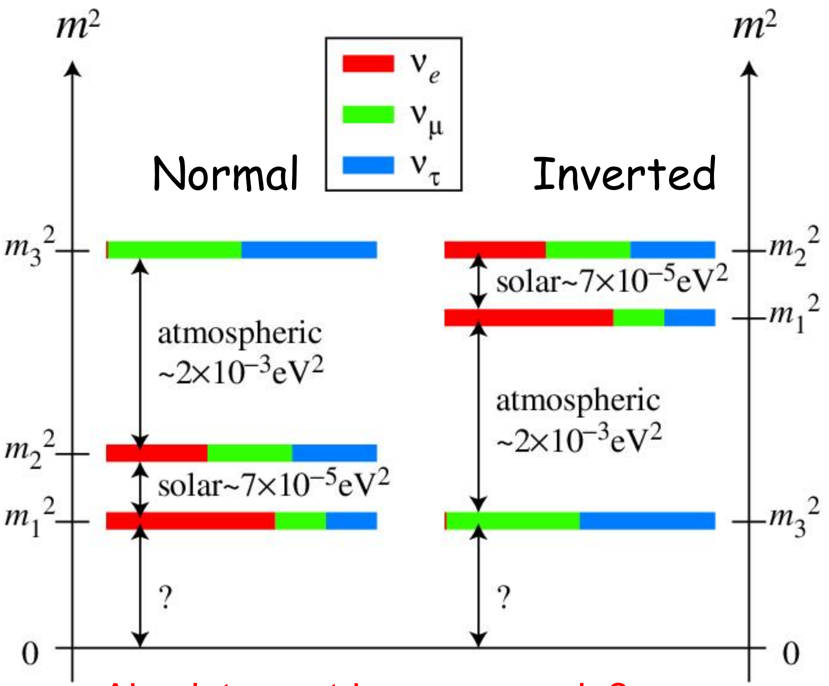
There is a hint for θ_{13} being non-zero



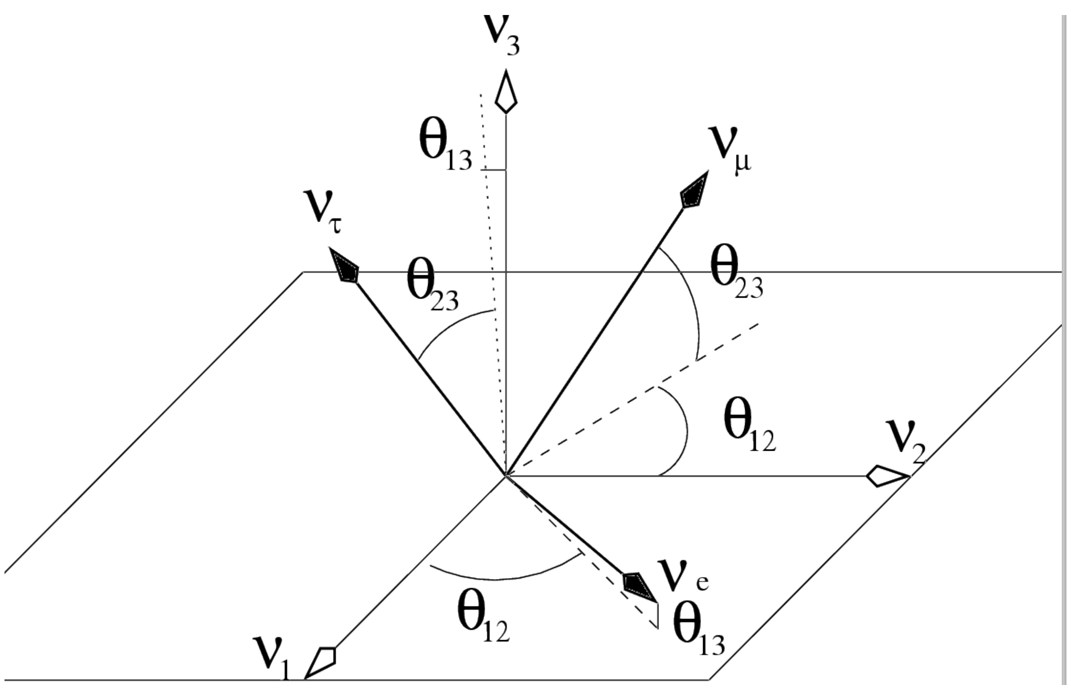
$$\sin^2\theta_{13} = 0.016 \pm 0.010$$

Fogli et al '08

Neutrino mass squared splittings and angles



Absolute neutrino mass scale?



Why are neutrino masses so small ?

$$\theta_{12} = 34.5^\circ \pm 1.4^\circ$$
$$\theta_{23} = 43^\circ \pm 4^\circ$$
$$\theta_{13} \leq 10^\circ$$

Two of these angles are pretty large – why?

Tri-bimaximal mixing matrix U_{TB}

Harrison, Perkins, Scott

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

TB angles $\theta_{12} = 35^\circ,\qquad \theta_{23} = 45^\circ,\qquad \theta_{13} = 0^\circ.$

c.f. data $\theta_{12} = 34.5^\circ \pm 1.4^\circ, \theta_{23} = 43.1^\circ \pm 4^o,\ \theta_{13} < 10^o$

Discrete neutrino flavour symmetry

Consider the TB Neutrino Mass Matrix

$$M_{TB}^{\nu} = U_{TB}\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} U_{TB}^{T}$$

$$M_{TB}^{\nu} = m_1\Phi_1\Phi_1^T + m_2\Phi_2\Phi_2^T + m_3\Phi_3\Phi_3^T$$

$$\Phi_1\Phi_1^T = \frac{1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}, \quad \Phi_2\Phi_2^T = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \Phi_3\Phi_3^T = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

TB Neutrino Mass Matrix is invariant under a discrete $Z_2 \times Z_2$ group generated by S,U

$$M_{TB}^{\nu} = S\, M_{TB}^{\nu} S^T \qquad M_{TB}^{\nu} = U\, M_{TB}^{\nu} U^T$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \qquad U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

S_4 family symmetry

Lam

In this basis the charged lepton matrix is invariant under a diagonal phase symmetry T

$$M^E = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix} = T M^E T^\dagger \qquad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} \qquad \omega = e^{2\pi i/3}$$

S,T,U → generate the discrete group S_4

Suggests using a discrete family symmetry S_4 broken by three types of flavons ϕ_S , ϕ_T, ϕ_U which each preserve a particular generator

$$S\langle\phi_S\rangle = +1\langle\phi_S\rangle, \quad U\langle\phi_U\rangle = +1\langle\phi_U\rangle, \quad T\langle\phi_T\rangle = +1\langle\phi_T\rangle$$

$$\mathcal{L}^{Yuk} \sim \psi(\phi_T + \phi_I)\psi^c H \ ,$$

Charged leptons preserve T

$$\mathcal{L}^{Maj} \sim \psi(\phi_S + \phi_U + \phi_I)\psi H H$$

Neutrinos preserve S,U

Indirect models

SFK, Luhn

Alternatively it is possible to realise the neutrino flavour symmetry indirectly as an accidental symmetry

Introduce three triplet flavons ϕ_1 , ϕ_2 , ϕ_3 with VEVs along columns of U_{TB}

$$\langle \phi_1 \rangle = v_1 \Phi_1, \quad \langle \phi_2 \rangle = v_2 \Phi_2, \quad \langle \phi_3 \rangle = v_3 \Phi_3$$

$$\Phi_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$$

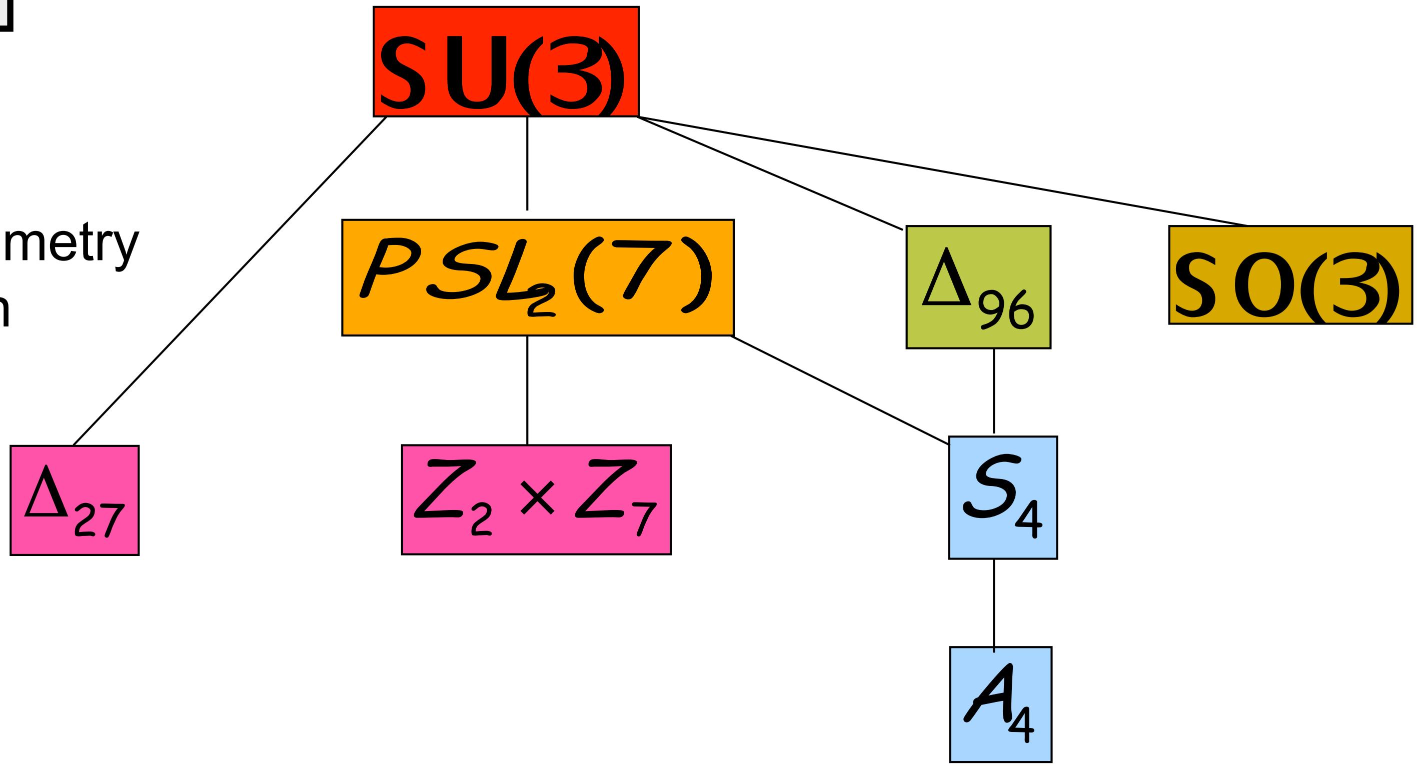
These flavons break S,U

The following Majorana Lagrangian preserves S,U accidentally

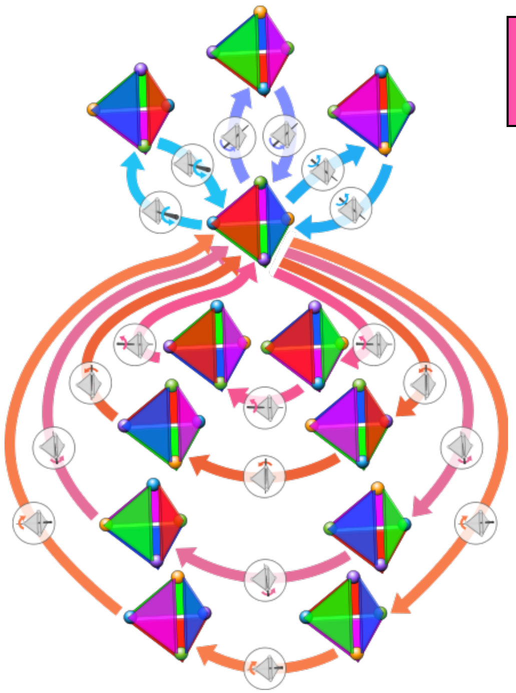
$$\mathcal{L}^{Maj} \sim \psi(\phi_1 \phi_1^T + \phi_2 \phi_2^T + \phi_3 \phi_3^T) \psi H H$$

$$\rightarrow M_{TB}^{\nu} = m_1 \Phi_1 \Phi_1^T + m_2 \Phi_2 \Phi_2^T + m_3 \Phi_3 \Phi_3^T$$

GFamily



e.g. A_4 is the symmetry of the tetrahedron



Discrete family symmetry suggested by TB mixing

Cooper, SFK, Luhn

Aside on Family Symmetry

- Assume the 3 families of quarks and leptons ψ are triplets of some family symmetry e.g. SU(3)
- Higgs fields H are typically singlets of family symmetry
- The family symmetry must be spontaneously broken by new Higgs fields called “flavons” denoted ϕ
- The flavons are typically antitriplets under the family symmetry group and different flavons develop VEVs in different components, E.g.:
- $< \phi_3 > = (0,0,v)$
- $< \phi_{23} > = (0,v,v)$
- $< \phi_{123} > = (v,v,v)$

Neutrinos can have Dirac and/or Majorana Mass

Majorana masses

CP conjugate

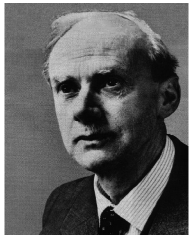
$$m_{LL}\bar{\nu}_L \nu_L^c$$

$$M_{RR}\bar{\nu}_R \nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino



$$m_{LR}\bar{\nu}_L \nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

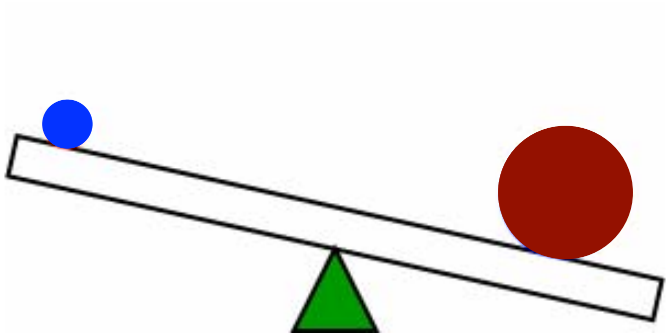
Neutrino $\neq$ antineutrino

Dirac mass

The See-Saw Mechanism

Basic idea $$m_\nu = \frac{m_t^2}{M_{GUT}} \sim eV$$

Light neutrinos



Heavy particles

See-Saw Standard Model (type I)

Yukawa couplings to 2 Higgs doublets (or one with $H_d \equiv H_u^c$)

$$\mathcal{L}_{mass} = -\epsilon_{ab}\left[\tilde{Y}^e_{ij} H^a_d L^b_i e^c_j - \tilde{Y}^\nu_{ij} H^a_u L^b_i \nu^c_j + \frac{1}{2}\nu^c_i \tilde{M}^{ij}_{RR}\nu^c_j\right] + H.c.$$

$$< H^2_u >= v_2, < H^1_d >= v_1 \quad \tan\beta \equiv v_2/v_1 \quad \epsilon_{ab} = -\epsilon_{ba},\ \epsilon_{12} = 1$$

Insert the vevs

$$\mathcal{L}_{mass} = -v_1 \tilde{Y}^e_{ij} e_i e^c_j - v_2 \tilde{Y}^\nu_{ij} \nu_i \nu^c_j - \frac{1}{2}\tilde{M}^{ij}_{RR}\nu^c_i \nu^c_j + H.c.$$

Rewrite in terms of L and R chiral fields, in matrix notation

$$\mathcal{L}_{mass} = -\bar{e}_L v_1 \tilde{Y^e}^* e_R - \bar{\nu}_L v_2 \tilde{Y^\nu}^* \nu_R - \frac{1}{2}\nu^T_R \tilde{M}^*_{RR}\nu_R + H.c.$$

$$= -\overline{e}_L m^E_{LR} e_R - \overline{\nu}_L m^\nu_{LR}\nu_R - \tfrac{1}{2}\overline{\nu}^c_R M_{RR}\nu_R + H.c.$$

The See-Saw Matrix

Type II contribution (ignored here) → m_{LL}^{II}

Dirac matrix → m_{LR}

$$\begin{pmatrix} \overline{\nu_L} & \overline{\nu_R^c} \end{pmatrix} \begin{pmatrix} m_{LL}^{II} & m_{LR} \\ m_{LR}^T & M_{RR} \end{pmatrix} \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}$$

Heavy Majorana matrix → M_{RR}

Diagonalise to give effective mass $\longrightarrow m_{LL}\overline{\nu}_L \nu_L^c$

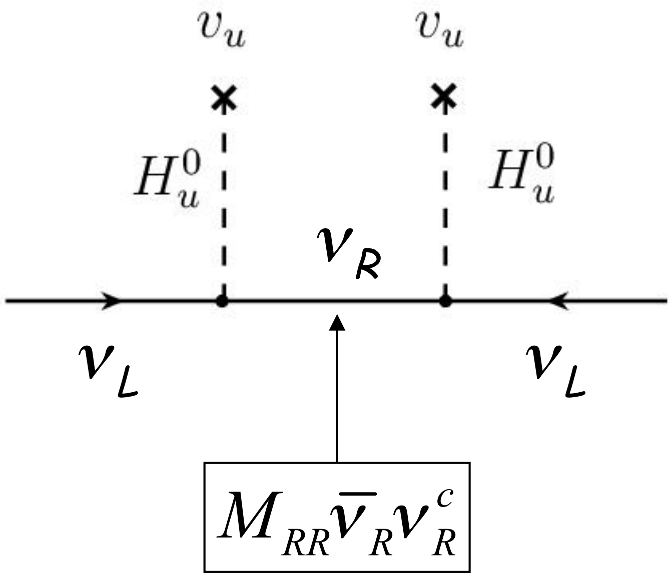
Light Majorana matrix ⟶

$$m_{LL}^{\nu} = m_{LL}^{II} - m_{LR} M_{RR}^{-1} m_{LR}^T$$

The see-saw mechanism

Type I see-saw mechanism

P. Minkowski (1977), Gell-Mann, Glashow, Mohapatra, Ramond, Senjanovic, Slanski, Yanagida (1979/1980)

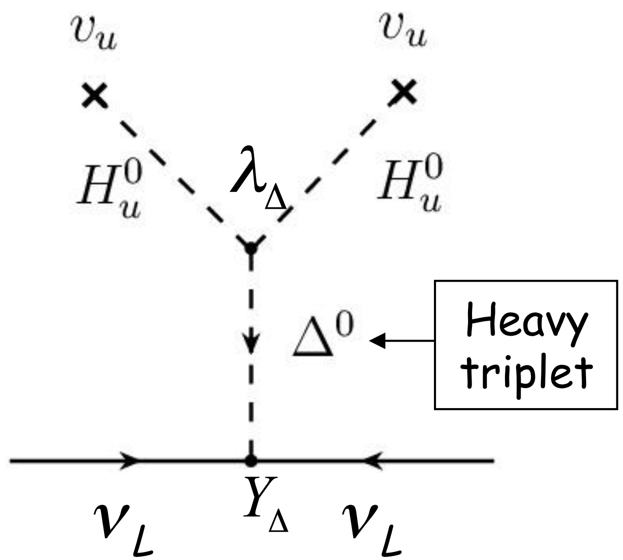


$$m_{LL}^I \approx -m_{LR} M_{RR}^{-1} m_{LR}^T$$

Type I

Type II see-saw mechanism (SUSY)

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



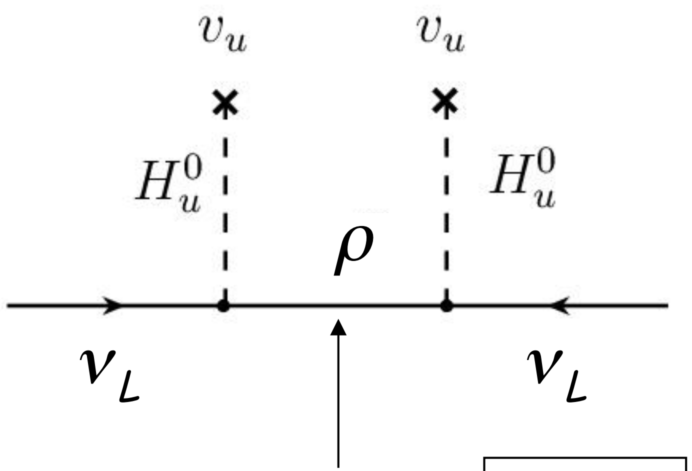
$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Type II

The see-saw mechanism cont'd

Type III see-saw mechanism

Foot, Lew, He, Joshi



SU(2)$_L$ fermion triplet

$$M_\rho \rho\rho$$

$$m_{LL}^{III} \approx -m_{LR} M_\rho^{-1} m_{LR}^T$$

Type III

Supersymmetric adjoint SU(5)

$$\psi_{24} \to \left(\rho_3^0 + \frac{\sqrt{15}}{10} \rho_0 \right)$$

SU(2)$_L$ triplet

SU(2)$_L$ singlet

Supersymmetric Adjoint A_4 x SU(5)

Cooper, SFK, Luhn

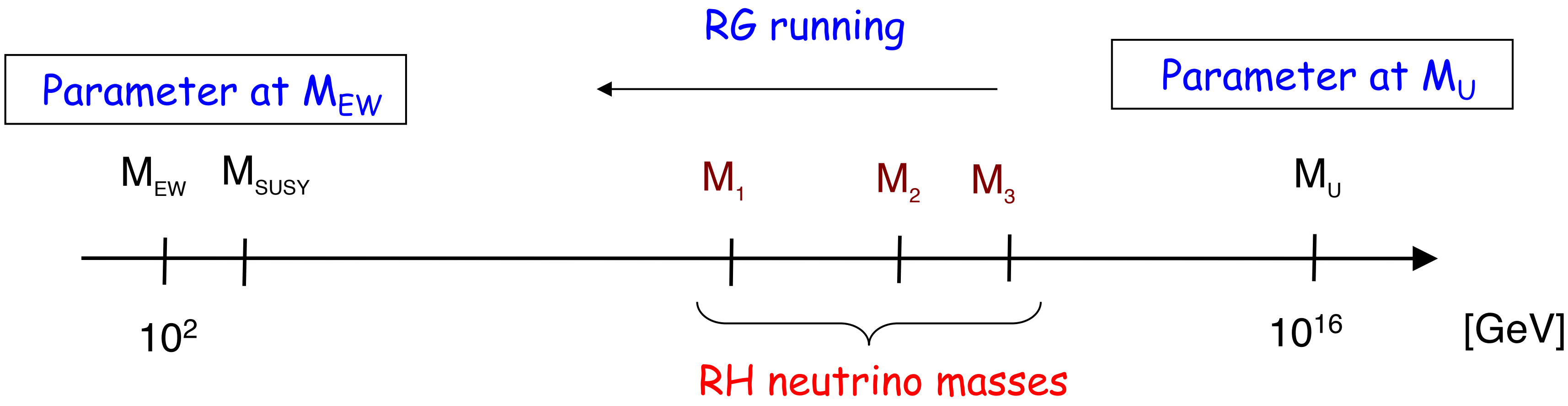
Lecture 2

Grand Unified Theories

1. SU(5)
2. SO(10)
3. Proton decay with triplets

Appendix on group theory

Renormalisation Group running



RGEs for gauge couplings
(to one loop accuracy)

$$\frac{dg_a}{dt} = \frac{g_a^3}{16\pi^2} b_a$$

where $t = \ln(\mu/M_X)$ (μ is the $\overline{MS}$ scale and M_X is the high energy scale)

SM beta functions

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_{Fam} \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$$

$$b_a = (\tfrac{41}{10}, -\tfrac{19}{6}, -7)$$

MSSM beta functions

$$b_i = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_{Fam} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_{Higgs} \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$$

$$b_a = (\tfrac{33}{5}, 1, -3)$$

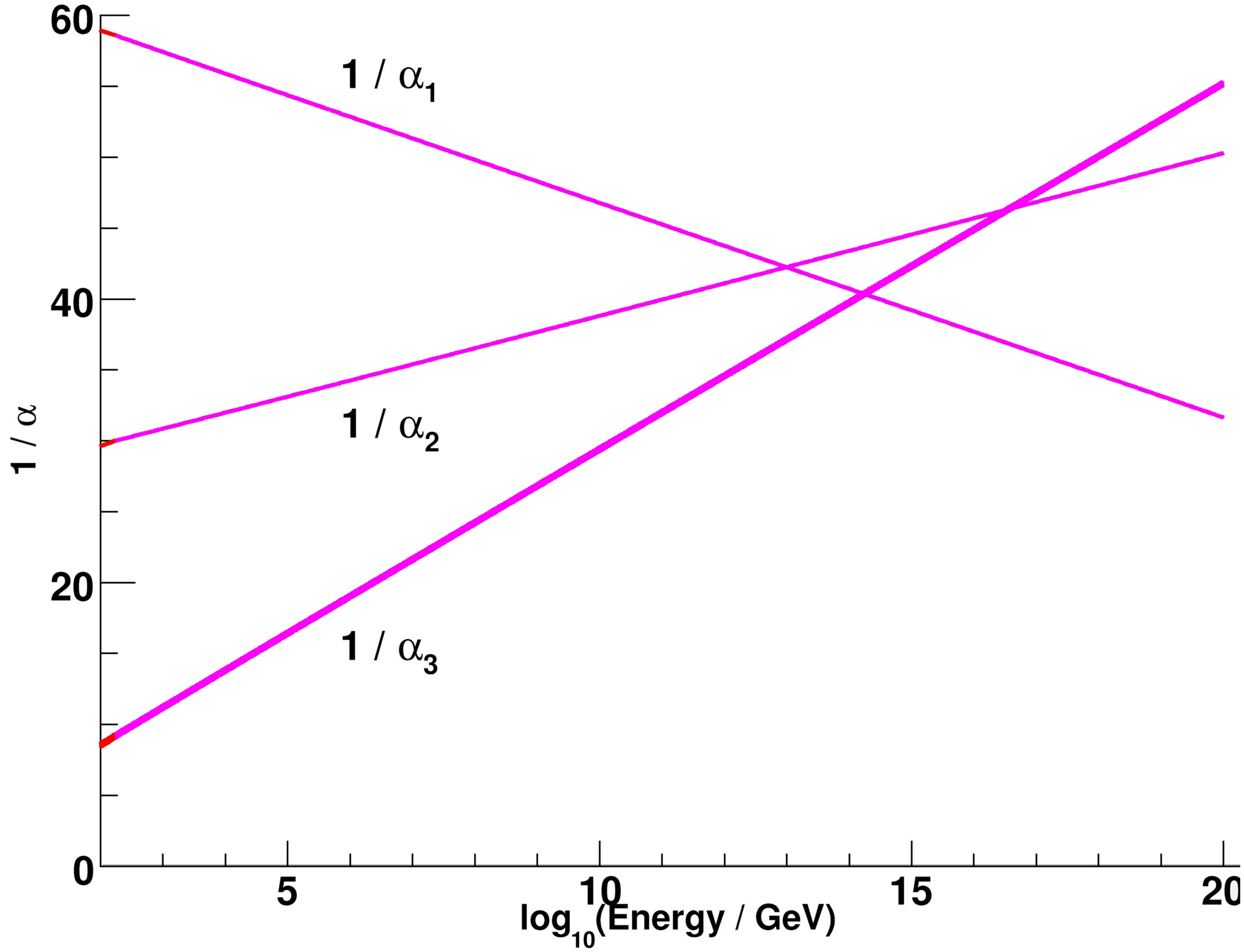
SM couplings at low energy

Latest coupling constant measurements at M_Z **energy scale:**

- $$\alpha_1\ (M_Z)\ (\overline{MS}) = 0.016947(6)\ (RPP\ 2006)$$
- $$\alpha_2\ (M_Z)\ (\overline{MS}) = 0.033813(27)\ (RPP\ 2006)$$
- $$\alpha_3\ (M_Z)\ (\overline{MS}) = 0.1187(20)\ (RPP\ 2006)$$

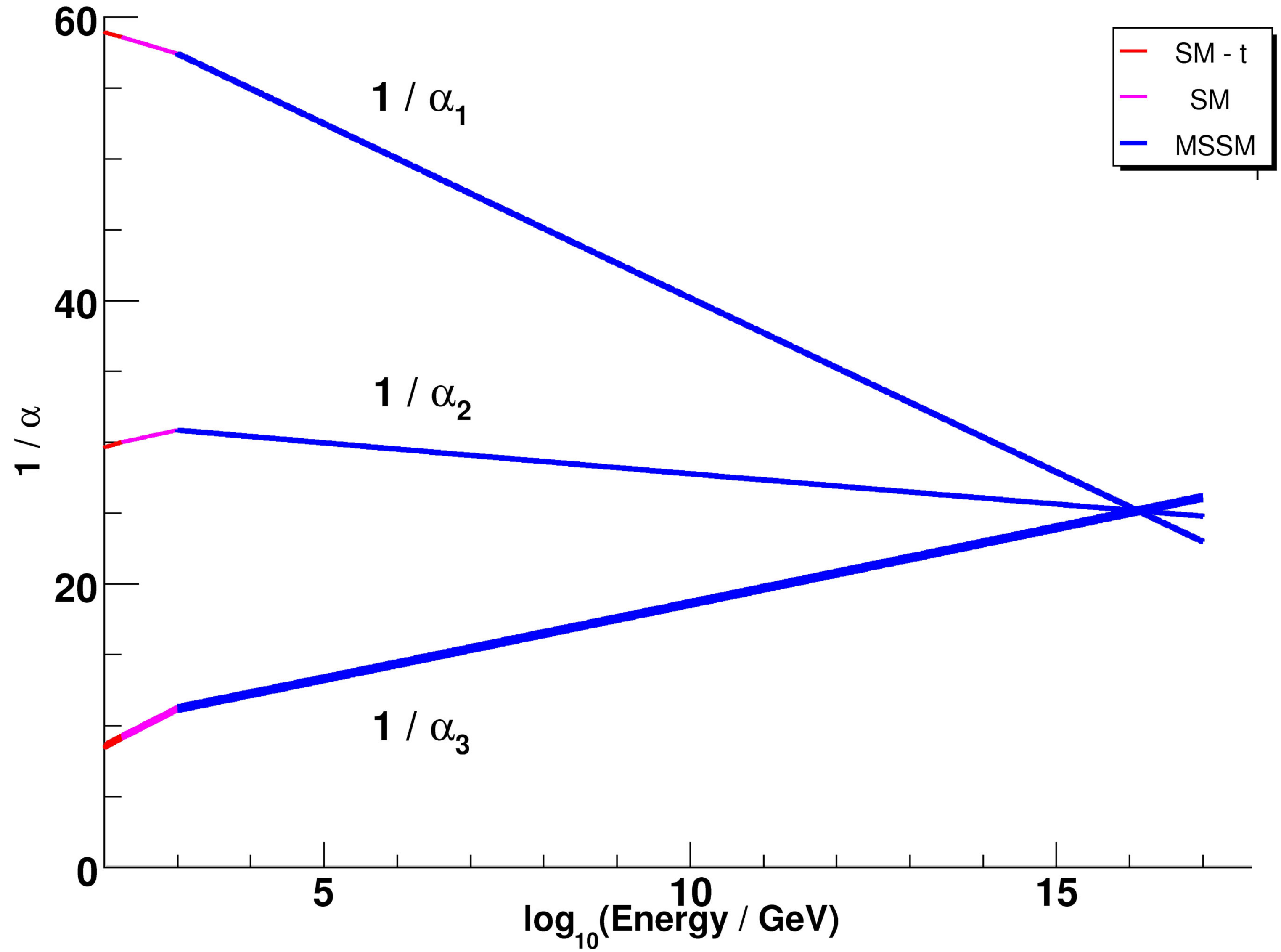
Evolution of SM couplings

Two-loop RGEs for the SM:



MSSM

Two-loop RGEs for the MSSM with 1 TeV effective SUSY threshold:



RGEs for t,b,τ in the MSSM

$$Y_u \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & Y_t \end{pmatrix}, \quad Y_d \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & Y_b \end{pmatrix}, \quad Y_e \approx \begin{pmatrix} 0 & & \\ & 0 & \\ & & Y_\tau \end{pmatrix}$$

$$\begin{aligned}
\frac{dY_t}{dt} &= \frac{1}{16\pi^2} Y_t [6|Y_t|^2 + |Y_b|^2 - (\frac{16}{3} g_3^2 + 3 g_2^2 + \frac{13}{15} g_1^2)] \\
\frac{dY_b}{dt} &= \frac{1}{16\pi^2} Y_b [6|Y_b|^2 + |Y_t|^2 + |Y_\tau|^2 - (\frac{16}{3} g_3^2 + 3 g_2^2 + \frac{7}{15} g_1^2)] \\
\frac{dY_\tau}{dt} &= \frac{1}{16\pi^2} Y_\tau [4|Y_\tau|^2 + 3|Y_b|^2 - (3 g_2^2 + \frac{9}{5} g_1^2)],
\end{aligned}$$

Grand Unified Theories (GUTs)

Basic idea is to embed the SM gauge group into a simple gauge group G with a single coupling constant, broken at a high energy scale

$$G \longrightarrow SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

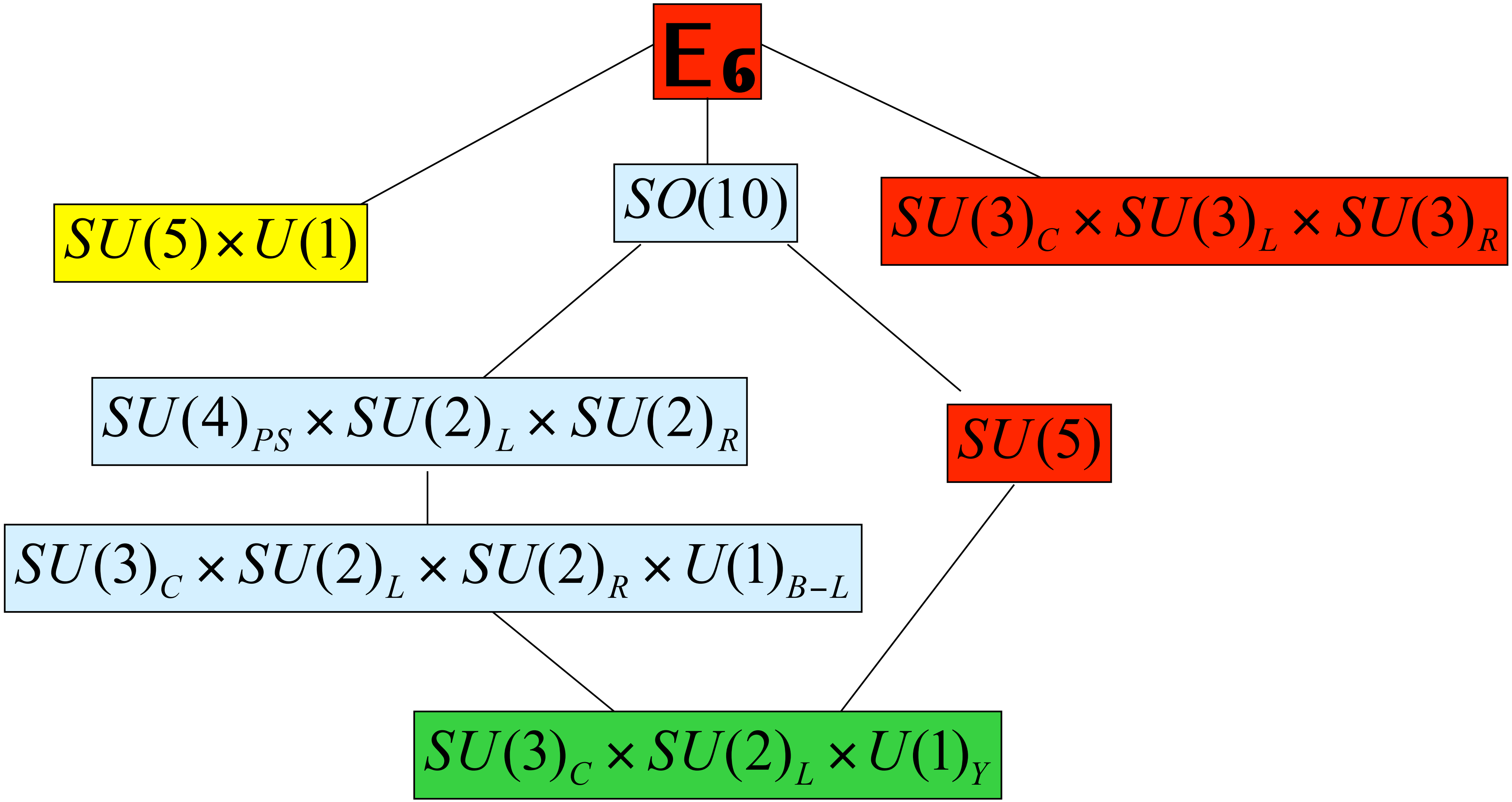
$$R \longrightarrow u_i^c = (\bar{3},\, 1,\, -\tfrac{2}{3}), \quad d_i^c = (\bar{3},\, 1,\, \tfrac{1}{3}), \quad e^c = (1,\, 1,\, 1),$$

$$Q^{\alpha\, i} = (u^i,\, d^i) = (3,\, 2,\, \tfrac{1}{6}), \qquad L^\alpha = (\nu,\, e) = (1,\, 2,\, -\tfrac{1}{2}),$$

Motivations

1. Continuation of process of unification of physics starting with Maxwell
2. Remarkable fit of SM multiplets into Pati-Salam, SU(5), SO(10), E_6…
3. Unification of gauge couplings at high energy scale M_{GUT}
4. Charge quantization: equality of electron and proton charges
5. High energy fermion mass relations e.g. $m_b = m_\tau$

Candidate GUTs



SU(5) GUT

Georgi and Glashow

With the hypercharge embedding

$$Y = -2\sqrt{\frac{5}{3}}Y_5 \text{ and } Y_5 \equiv \frac{1}{\sqrt{60}}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \qquad TrY_5^2 = 1/2$$

Each family fits nicely into the SU(5) multiplets

$$\overline{5}_i \equiv \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu \end{pmatrix}_L \qquad 10^{[ij]} \equiv \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ . & 0 & u_1^c & u^2 & d^2 \\ . & . & 0 & u^3 & d^3 \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}_L$$

$$\overline{5} = (\overline{3}, 1, +1/3) \oplus (1, \overline{2}, -1/2) \quad \text{and} \quad 10 = (\overline{3}, 1, -2/3) \oplus (3, 2, +1/6) \oplus (1, 1, +1)$$

N.B in minimal SU(5) neutrino masses are zero.

Right-handed neutrinos may be added to give neutrino masses but they are not predicted.

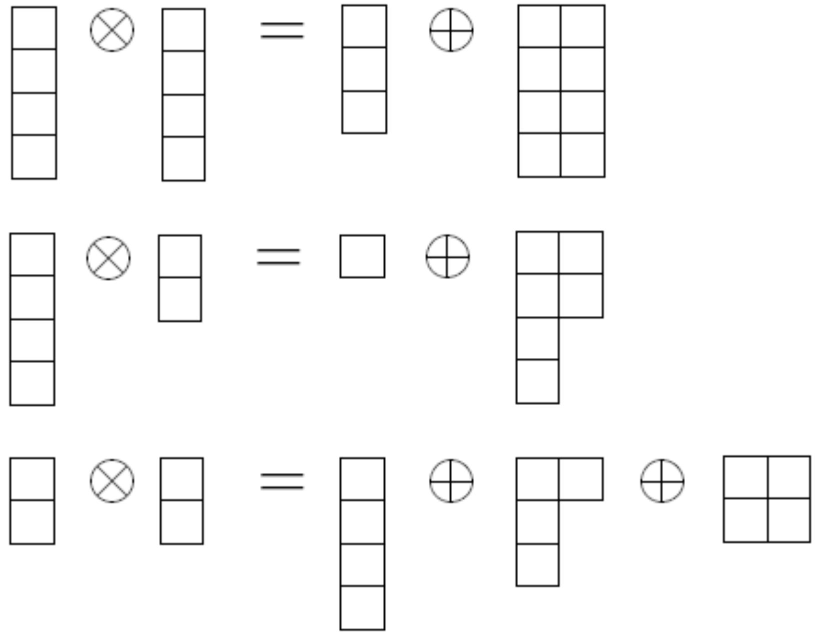
Higgs Sector of SU(5)

Candidate Higgs reps of SU(5) are contained in matter bilinears constructed from 5^* and 10

$$\overline{5} \otimes \overline{5} = \overline{10} \oplus \overline{15}$$

$$\overline{5} \otimes 10 = 5 \oplus \overline{45}$$

$$10 \otimes 10 = \overline{5} \oplus 45 \oplus 50$$



Minimal suitable Higgs reps for fermion masses consist of $5_H + 5^*_H$

Fermion Masses in SU(5)

The Yukawa superpotential for one family with Higgs H=5, $H^*=5^*$

$$\lambda_u H_i 10_{jk} 10_{lm} \epsilon^{ijklm} + \lambda_d \overline{H}^i 10_{ij} \overline{5}^j$$

$$\longrightarrow \quad \lambda_u H_u Q u^c + \lambda_d (H_d Q d^c + H_d L e^c)$$

$$\lambda_d = \lambda_e \text{ at the GUT scale}$$

Assuming this relation holds for all 3 families

$$\longrightarrow \quad \underbrace{\lambda_b = \lambda_\tau,}_{\text{good}} \quad \underbrace{\lambda_s = \lambda_\mu, \quad \lambda_d = \lambda_e}_{\text{almost good}} \text{ at } M_{GUT}$$

good — almost good

c.f. Georgi-Jarlskog relations at M_{GUT}: $\lambda_b = \lambda_\tau, \quad \lambda_s = \dfrac{\lambda_\mu}{3}, \quad \lambda_d = 3\lambda_e$

Breaking SU(5)

The smallest Higgs rep which contains a singlet under the SM subgroup is the 24 Higgs rep and is a candidate to break SU(5)

$$24_H = (1,1,0) \oplus (8,1,0) \oplus (1,3,0) \oplus (3,2,-5/6) \oplus (\overline{3},2,+5/6)$$

The Higgs superpotential involving the minimal Higgs sector of SU(5) consisting of the 24_H plus $H=5_H$ plus $H^*=5_H^*$

$$\mathcal{W}_H = m_5 \overline{5}_{Hi} 5^i_H + m_{24} 24^j_{Hi} 24^i_{Hj} + \lambda 24^j_{Hi} 24^k_{Hj} 24^i_{Hk} + \eta \overline{5}_{Hi} 24^i_{Hj} 5^j_H$$

$$\langle 24_H \rangle = v_{24} \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ \hline & & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{array} \right) \qquad \langle 5_H \rangle = v_5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

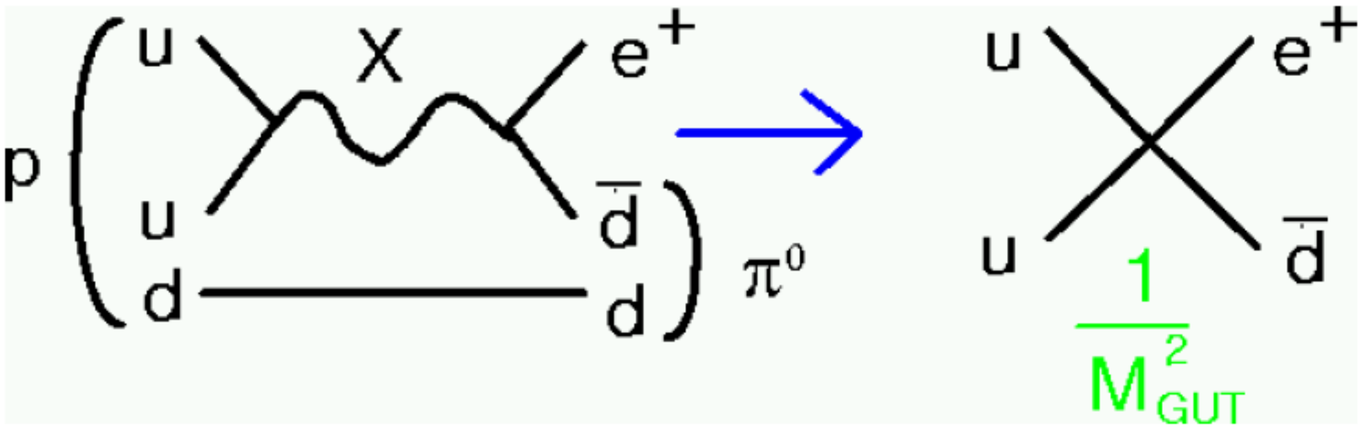
With some tuning (see later) one can achieve light Higgs doublets which can develop weak scale vevs $v_5 \ll v_{24}$

Proton Decay in Non-SUSY SU(5)

Gauge bosons in adjoint of SU(5) contain SM gauge bosons G,W,B plus new gauge bosons X,Y

$$24 = \left(\begin{array}{ccc|cc} G_{11}-\frac{2B}{\sqrt{30}} & G_{12} & G_{13} & X_1^C & Y_1^C \\ G_{21} & G_{22}-\frac{2B}{\sqrt{30}} & G_{23} & X_2^C & Y_2^C \\ G_{31} & G_{32} & G_{33}-\frac{2B}{\sqrt{30}} & X_3^C & Y_3^C \\ \hline X_1 & X_2 & X_3 & \frac{W^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W^3}{\sqrt{2}}+\frac{3B}{\sqrt{30}} \end{array}\right)$$

– **Proton decay mediated by X gauge bosons at $M_{\rm GUT} \sim 10^{15}$ GeV**



$$\Gamma_{\rm SM} \sim \frac{\alpha_{\rm GUT}^2}{M_{\rm GUT}^4} m_p^5 \quad \Rightarrow \quad \tau_p^{\rm SM} \sim 10^{31} \text{ years}$$

Decay modes

$p \to e^+\pi^0$ $\quad n \to e^+\pi^-$
$p \to e^+\rho^0$ $\quad n \to e^+\rho^-$
$p \to e^+\omega^0$ $\quad n \to \nu\omega^0$
$p \to e^+\eta$ $\quad n \to \overline{\nu}\pi^0$
$p \to \overline{\nu}\pi^+$ $\quad n \to \overline{\nu}_\mu K^0$
$p \to \overline{\nu}\rho^+$
$p \to \overline{\nu}_\mu K^+$

– **Experimentally ruled out** $\tau\left(p \to \pi^0 e^+\right) > 5.0 \times 10^{33}\, y\ (SK)$

– **Also, SM couplings DO NOT unify.**

Proton Decay in SUSY SU(5)

- **Supersymmetric GUTs**
  - Couplings **DO** unify
  - ... and at a higher scale: $M_{\mathrm{GUT}} \sim 10^{16}$ GeV

$$\Gamma_{\mathrm{SUSY}} \sim \frac{\alpha_{\mathrm{GUT}}^2}{M_{\mathrm{GUT}}^4} m_p^5 \quad \Rightarrow \quad \tau_p^{\mathrm{SUSY}} \sim 10^{35} \text{ years}$$

  - Such a lifetime is not yet probed experimentally.
  - But these are dimension-6 operators...

There are also in addition dimension 5 proton decay operators arising from colour triplet exchange (see later)

However the main drawback of SU(5) is that it does not predict right-handed neutrinos….

SO(10) GUT

Georgi; Fritzsch and Minkowski

The 16 of SO(10) contains a single quark and lepton family and also predicts a single right-handed neutrino per family.

The SU(5) reps are unified into SO(10):

$$\mathbf{10} + \bar{\mathbf{5}} + \bar{\nu} \quad \subset \quad \mathbf{16}$$

The two Higgs doublets are contained in a 10 of SO(10)

$$\mathbf{5_H}, \quad \bar{\mathbf{5}}_{\mathbf{H}} \quad \subset \quad \mathbf{10_H}$$

Fermion masses arise from the coupling

$$\lambda 16.16.10_H \rightarrow \lambda \left(Qh_2u^c + Qh_1d^c + Lh_1e^c + Lh_2\nu^c \right)$$

c.f. Pati-Salam

Neutrino masses in SO(10)

$$16.16.10_H \rightarrow (\bar{\nu} \quad \bar{e})_L \begin{pmatrix} H^0 \\ H^- \end{pmatrix} \nu_R \rightarrow m_{LR} \bar{\nu}_L \nu_R$$

Dirac mass

$$16.16.126_H \rightarrow < 126_H > \nu_R \nu_R$$

$$\frac{16.16.\overline{16}_H \overline{16}_H}{M} \rightarrow \frac{< \overline{16}_H >^2}{M} \nu_R \nu_R$$

Heavy Majorana mass

SO(10) contains all the ingredients for the see-saw mechanism and tends to predict a hierarchical pattern of neutrino masses

Troublesome Colour Triplet Higgs

Low energy MSSM Higgs doublets must be embedded into representations of the GUT group

This Leads to new (colour triplet) particles D e.g. $5_H = \begin{pmatrix} H_u \\ D \end{pmatrix}$

SU(5)	SO(10)	E_6
$H_u + H_d \rightarrow \mathbf{5_H} \oplus \overline{\mathbf{5_H}}$	$H_u + H_d \rightarrow \mathbf{10_H}$	$H_u + H_d \rightarrow \mathbf{27_H}$

All give new colour triplet particles: $D \sim (3,1)_{-\frac{1}{3}}$; $D \sim (3,1)_{\frac{1}{3}}$

($\times 3$ in E_6)

Problems:

1. **Spoil Unification of MSSM gauge couplings**
2. **Cause rapid proton decay**

Proton Decay with Triplet Higgs

Say $H_u, H_d, D, \overline{D} \rightarrow$ $\mathbf{H}$ representation of G

e.g. **10** for SO(10)

And quarks and leptons ! **F** representation of G

e.g. **16** for SO(10)

To produce SM Yukawa terms one generally uses **F F H** terms

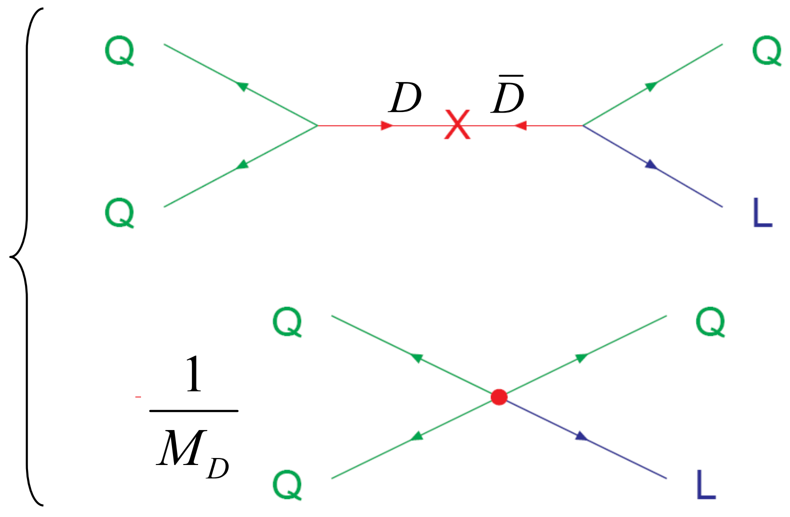
Gives following SM interactions: $H_u Q u^c, \;\; H_d Q d^c, \;\; H_d L e^c$

But also gives 'dangerous' terms involving D ; D with SM particles:

$D\,QQ; \;\; D\,d^c u^c; \;\; e^c D\,u^c; QLD$ $\longrightarrow$ Proton decay

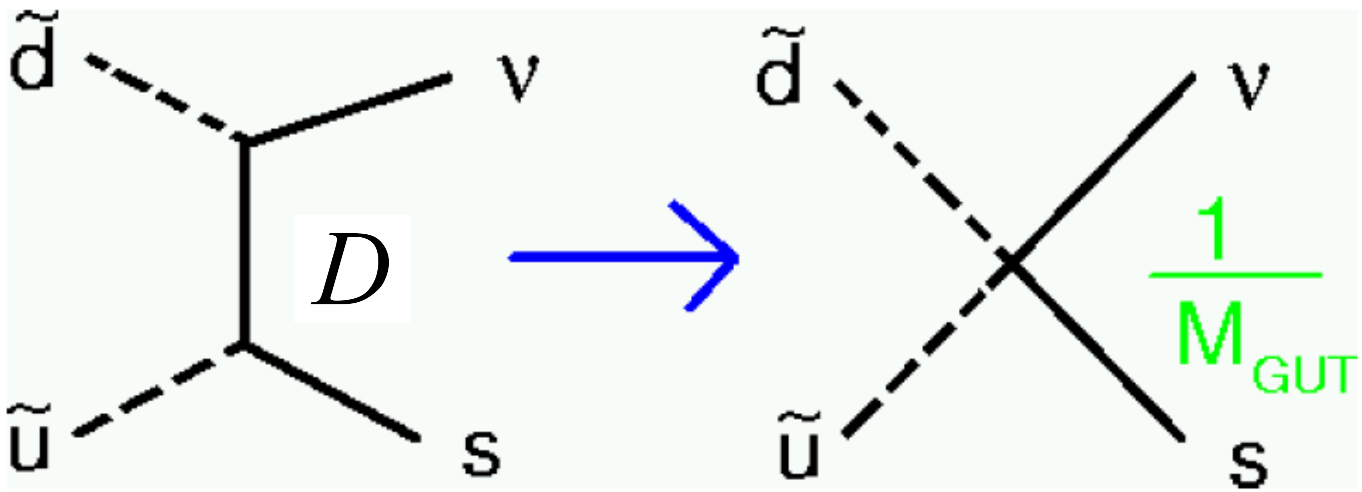
$D\,QQ;\;\; D\,d^c u^c;\;\; e^c D\,u^c; QLD$

D-exchange generates superfield operators→

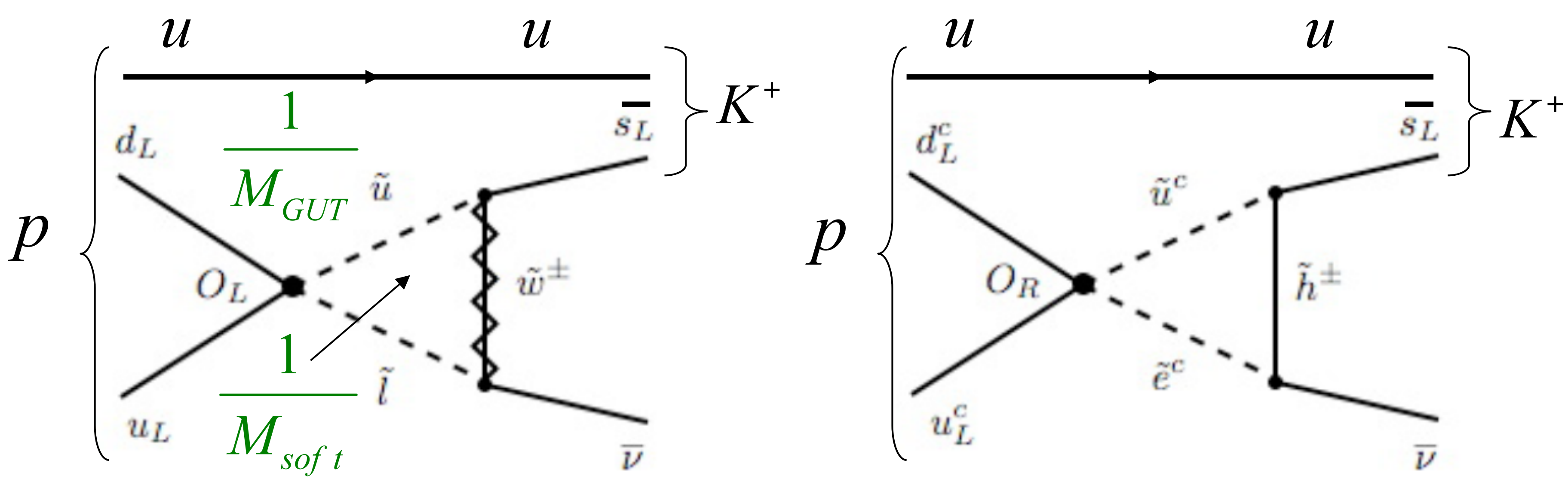


In terms of scalar and fermion components some examples of dangerous operators are shown below

- **Colored Higgs exchange generates dimension-5 operators.**



Proton Decay with Dim 5 Operators



Thus $\tau_p \sim M_{GUT}^2 \, M_{soft}^2$ instead of M_{GUT}^4

$$\tau(p \to K^+\bar{\nu}) \sim c^2 \times loop \times RG \times matrix\, element$$

$$c_L^{ijkl} \sim c_R^{ijkl} \sim \frac{1}{M_D} \cdot Y_u^{ij} Y_d^{kl}$$

$$\tau\left(p \to K^+\bar{\nu}\right) > 1.6 \times 10^{33}\, y\, (SK)$$

Minimal SU(5) turns out to be ruled out by proton decay -- but it gives unacceptable fermion masses anyway

Appendix 1 Group Theory of SU(5)

SU(5) and SU(N) groups

- A set of $N \times N$ unitary matrices form fundamental representation of $U(N)$ group.
  - Generators of $U(N)$ group are N^2 hermitian $N \times N$ matrices.
  - As before one generator T^0 may be taken to be $\sim I$ while other $N^2 - 1$ generators can be chosen so that their traces are equal to zero.
- The set of $N^2 - 1$ traceless generators form invariant subalgebra of $U(N)$ that correspond to $SU(N)$ group, i.e.

$$\left[T^a, T^b\right] = i f_{abc} T^c \,, \qquad \left[T^a, T^0\right] = 0 \,.$$

- Therefore $U(N)$ is not a simple group.

- The Cartan subalgebra of $SU(N)$ group involves $N-1$ traceless diagonal matrices

$$H_1 = \frac{1}{2}\begin{pmatrix} 1 & & & & \\ & -1 & & & \\ & & 0 & & \\ & & & \dots & \\ & & & & 0 \end{pmatrix}, \quad H_2 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & -2 & & \\ & & & \dots & \\ & & & & 0 \end{pmatrix}$$

$$\dots H_{N-1} = \frac{1}{\sqrt{2N(N-1)}}\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \dots & \\ & & & & -(N-1) \end{pmatrix}.$$

- As a result $SU(N)$ is a group of rank $N-1$.
- Among $SU(N)$ groups $SU(5)$ plays a very special role because this is a minimal special unitary group that contains $SU(3)_C \times SU(2)_W \times U(1)_Y$ subgroup which is a group of the standard model (SM).
  - SM incorporates electromagnetic, weak and strong interactions.
  - It describes all available experimental data with high accuracy.
  - In the SM one family of quarks and leptons includes

$$u_i^c = (\bar{3},\, 1,\, -\tfrac{2}{3}), \quad d_i^c = (\bar{3},\, 1,\, \tfrac{1}{3}), \quad e^c = (1,\, 1,\, 1),$$

$$Q^{\alpha\, i} = (u^i,\, d^i) = (3,\, 2,\, \tfrac{1}{6}), \qquad L^{\alpha} = (\nu,\, e) = (1,\, 2,\, -\tfrac{1}{2}),$$

where the quantities in brackets are $SU(3)$ and $SU(2)$ representations and $U(1)_Y$ charges.

- In the fundamental representation the eigenvectors of Cartan algebra of $SU(5)$ group can be written as

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

  - Each eigenvector can be associated with either quark or lepton.

- We can always choose the fundamental representation of $SU(5)$ so that

$$\bar{5}_i = \begin{pmatrix} d_i^c \\ L_\alpha \end{pmatrix} = (\bar{3},\, 1) \oplus (1,\, \bar{2}), \qquad 5^i = (3,\, 1) \oplus (1,\, 2)$$

- Let us consider tensor representation of rank 2

$$5^i \otimes 5^j = V^{ij} = \tfrac{1}{2}V^{\{ij\}} + \tfrac{1}{2}V^{[ij]} = 15 \oplus 10,$$

$$5 \otimes 5 = \left((3,\,1,\,-\tfrac{1}{3}) \oplus (1,\,2,\,\tfrac{1}{2})\right) \otimes \left((3,\,1,\,-\tfrac{1}{3}) \oplus (1,\,2,\,\tfrac{1}{2})\right),$$

$$(3,\,1,\,-\tfrac{1}{3}) \otimes (3,\,1,\,-\tfrac{1}{3}) = (\bar{3},\,1,\,-\tfrac{2}{3}) \oplus (6,\,1,\,-\tfrac{2}{3}),$$

$$(1,\,2,\,\tfrac{1}{2}) \otimes (3,\,1,\,-\tfrac{1}{3}) = (3,\,1,\,-\tfrac{1}{3}) \otimes (1,\,2,\,\tfrac{1}{2}) = (3,\,2,\,\tfrac{1}{6}),$$

$$(1,\,2,\,\tfrac{1}{2}) \otimes (1,\,2,\,\tfrac{1}{2}) = (1,\,3,\,1) \oplus (1,\,1,\,1),$$

$$10 = (\bar{3},\,1,\,-\tfrac{2}{3}) \oplus (3,\,2,\,\tfrac{1}{6}) \oplus (1,\,1,\,1) = u_i^c + Q^i + e^c,$$

$$15 = (6,\,1,\,-\tfrac{2}{3}) \oplus (3,\,2,\,\tfrac{1}{6}) \oplus (1,\,3,\,1).$$

- Thus each family of quarks and leptons fill in complete $SU(5)$ representations: 10 and $\bar{5}$.

Gauge Sector of SU(5)

- Adjoint representation of $SU(5)$ group, in which gauge fields lie, comes from the product of $5 \otimes \bar{5}$

$$5^i \otimes \bar{5}_j = V^i_j = (V^i_j - \tfrac{1}{5}\delta^i_j \delta^k_l V^l_k) + \tfrac{1}{5}\delta^i_j \delta^k_l V^l_k = 24 \oplus 1,$$

$$5 \otimes \bar{5} = \Big((3,\, 1,\, -\tfrac{1}{3}) \oplus (1,\, 2,\, \tfrac{1}{2}) \Big) \otimes \Big((\bar{3},\, 1,\, \tfrac{1}{3}) \oplus (1,\, \bar{2},\, -\tfrac{1}{2}) \Big),$$

$$(3,\, 1,\, -\tfrac{1}{3}) \otimes (\bar{3},\, 1,\, \tfrac{1}{3}) = (8,\, 1,\, 0) \oplus (1,\, 1,\, 0),$$

$$(1,\, 2,\, \tfrac{1}{2}) \otimes (\bar{3},\, 1,\, \tfrac{1}{3}) = (\bar{3},\, 2,\, 5/6),$$

$$(3,\, 1,\, -\tfrac{1}{3}) \otimes (1,\, \bar{2},\, -\tfrac{1}{2}) = (3,\, \bar{2},\, -5/6),$$

$$(1,\, 2,\, \tfrac{1}{2}) \otimes (1,\, \bar{2},\, -\tfrac{1}{2}) = (1,\, 3,\, 0) \oplus (1,\, 1,\, 0),$$

$$24 = (8,\, 1,\, 0) \oplus (\bar{3},\, 2,\, 5/6) \oplus (3,\, \bar{2},\, -5/6) \oplus (1,\, 3,\, 0) \oplus (1,\, 1,\, 0) =$$
$$= G^a + X + \bar{X} + W^b + B,$$

where G^a are gluons, W^b and B are $SU(2)_W$ and $U(1)_Y$ gauge bosons while X and $\bar{X}$ are leptoquarks.

Summary of Matter and Gauge Sector of SU(5)

- These representations can be written in the matrix form

$$\bar{F}\{\bar{5}\} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu \end{pmatrix}, \quad T\{10\} = \begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & e^c & 0 \end{pmatrix},$$

$$A\{24\} = \begin{pmatrix} G^a T^a + B/3 & \bar{X} \\ X & W^b \tau^b - B/2 \end{pmatrix},$$

where T^a and τ^b are generators of $SU(3)_C$ and $SU(2)_W$.

- $SU(5)$ predicts leptoquarks which give rise to proton decay ($p \to e^+ \pi^0$) and implies that the gauge couplings of $SU(3)_C$, $SU(2)_W$ and $U(1)_Y$ interactions are equal.

Lecture III

Beyond the Standard Model

Standard Model

$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

	Interactions			
	strong	electro-weak	gravitational	unified ?
Theory	QCD	GSW	quantum gravity ?	SUGRA ?
Symmetry	$SU(3)$	$SU(2) \times U(1)$	?	SU(5)?
Gauge bosons	$g_1 \cdots g_8$ gluons	photon $W^{\pm}$,Z^0 bosons	G graviton	X,Y ? GUT bosons?
charge	colour	weak isospin weak hypercharge	mass	?

$$Q = T_3 + \frac{1}{2} Y_W$$

Yukawa matrices

$$H \psi_L^i Y_{ij} \psi_R^j$$

$$Y_{ij} \rightarrow Y_{ij}^U, Y_{ij}^D, Y_{ij}^E, Y_{ij}^N$$

$$e.g. \quad Y_{ij}^E \left(\bar{\nu}_e \quad \bar{e}\right)_L^i \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} e_R^j$$

helicity	Generations 1.	2.	3.	Quantum Numbers Q	T_3	Y_W
ψ_L	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	0 -1	1/2 $-1/2$	-1 -1
	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	2/3 $-1/3$	1/2 $-1/2$	1/3 1/3
ψ_R	e_R	μ_R	τ_R	-1	0	-2
	u_R d_R	c_R s_R	t_R b_R	2/3 $-1/3$	0 0	4/3 $-2/3$

Standard Models Puzzles

The origin and fate of the Universe – and why is it so big and flat?

The dark side of the Universe – why is 95% of mass-energy in a form that is presently unknown, including 23% dark matter and 72% dark energy?

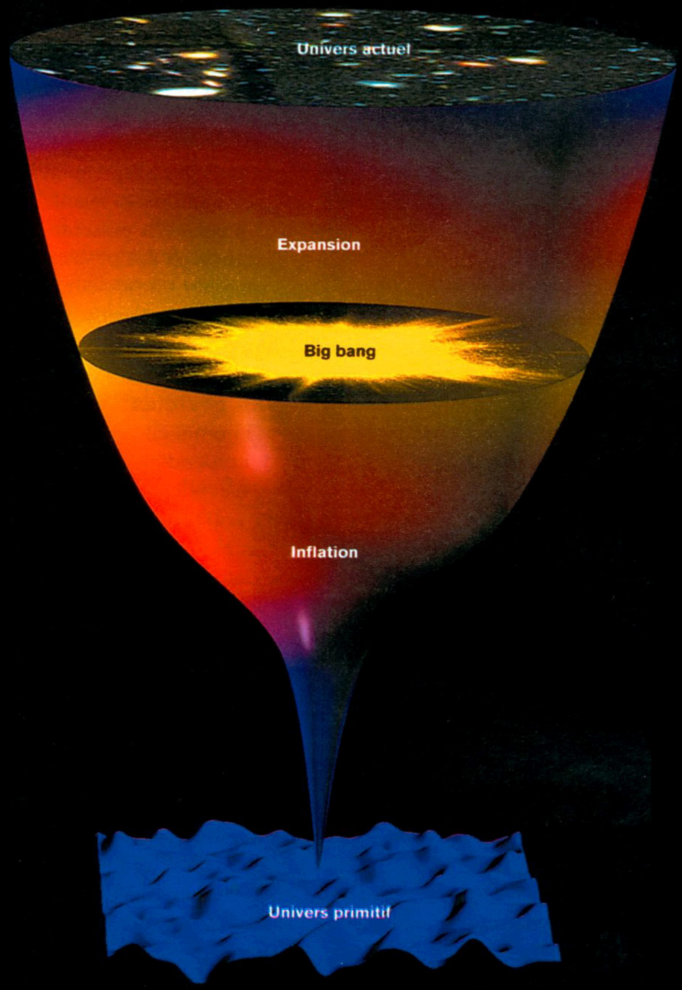
The origin of matter – the problem of why there is a tiny excess of matter over antimatter in the Universe, at a level of one part in a billion.

The origin of mass - the origin of the weak scale, its stability under radiative corrections, and the solution to the hierarchy problem.

The quest for unification - the question of whether the three known forces of the standard model (and gravity) may be unified.

The problem of flavour - the problem of the three generations with fermion (incl. neutrino) masses and mixing angles and CPV phases, giving small FCNCs and tiny strong CPV.

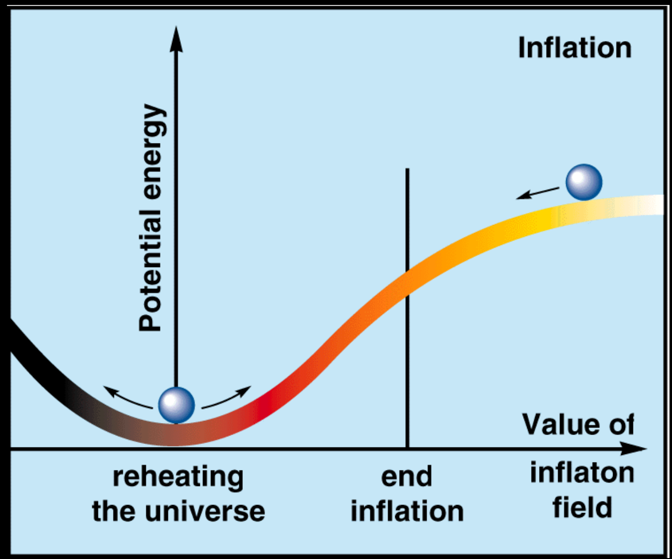
The Origin of the Universe



Why is the Universe so big and flat?

What seeds the density perturbations?

-- Inflation!

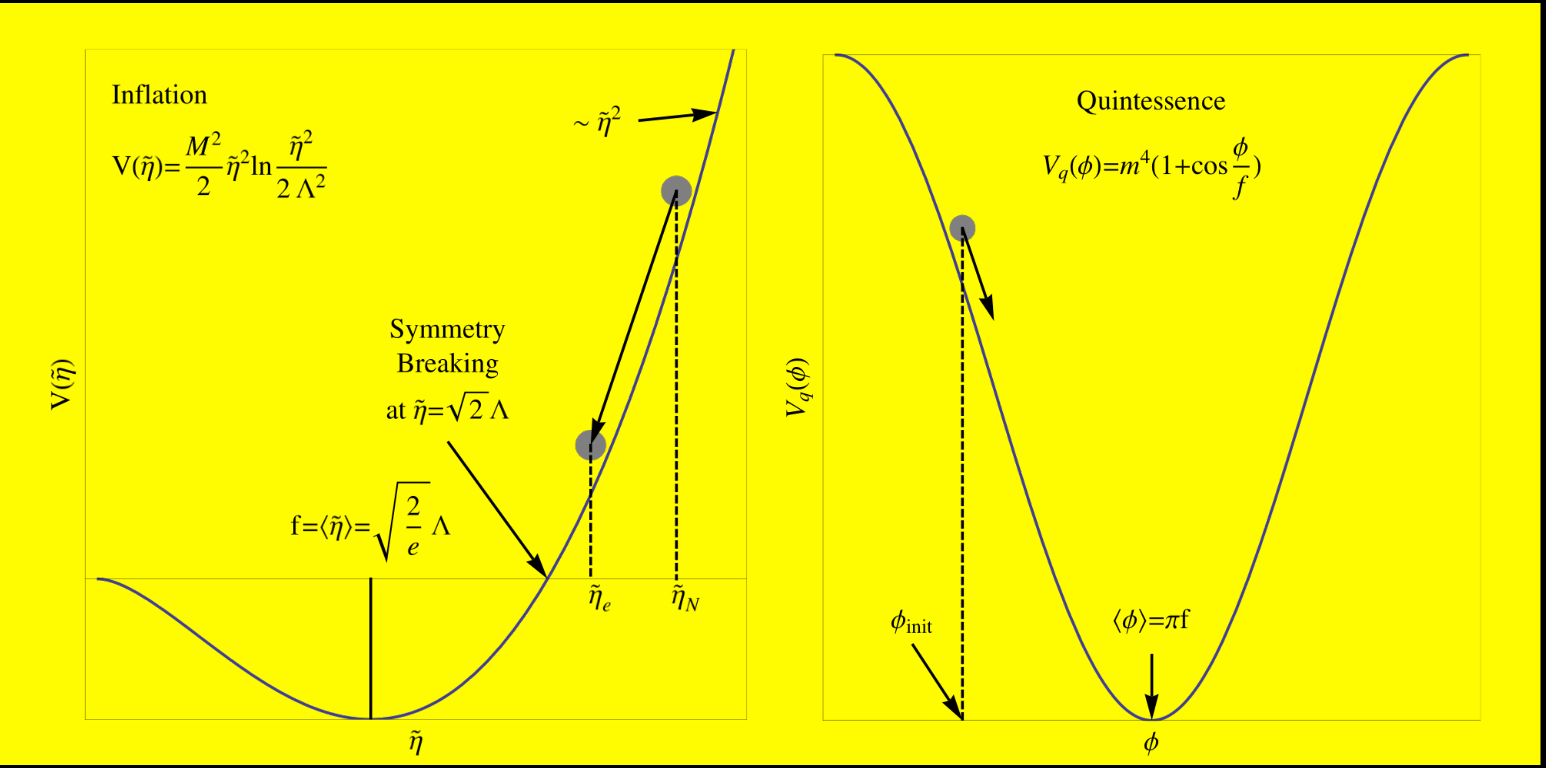


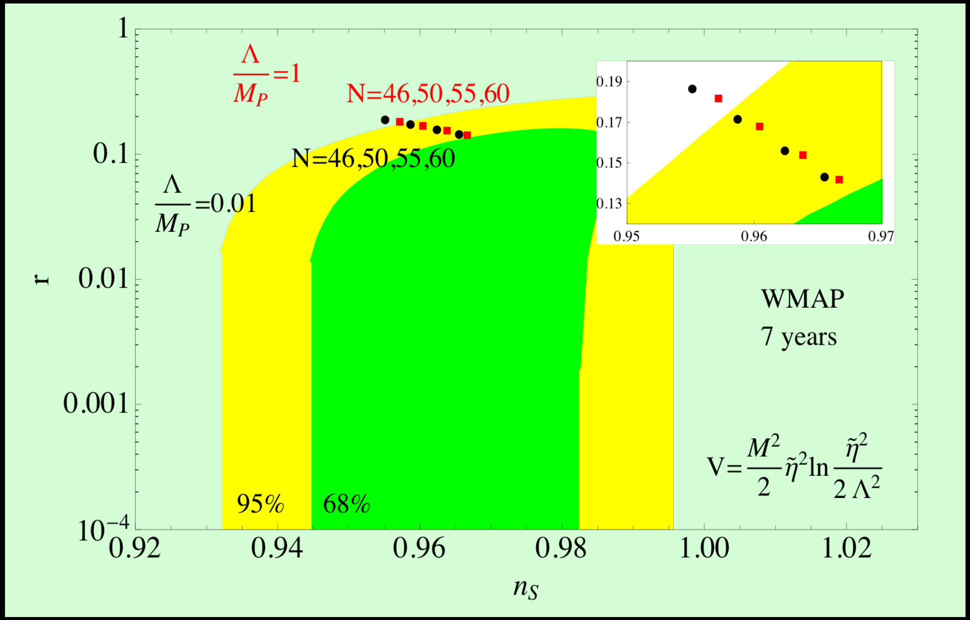
Di Bari, SFK, Luhn, Merle, Schmidt-May

Radiative Inflation and Dark Energy

$$V \approx M^2 \Phi^\dagger \Phi \ln\left(\frac{\Phi^\dagger \Phi}{\Lambda^2}\right) = \frac{M^2}{2}\tilde{\eta}^2 \ln\left(\frac{\tilde{\eta}^2}{2\Lambda^2}\right)$$

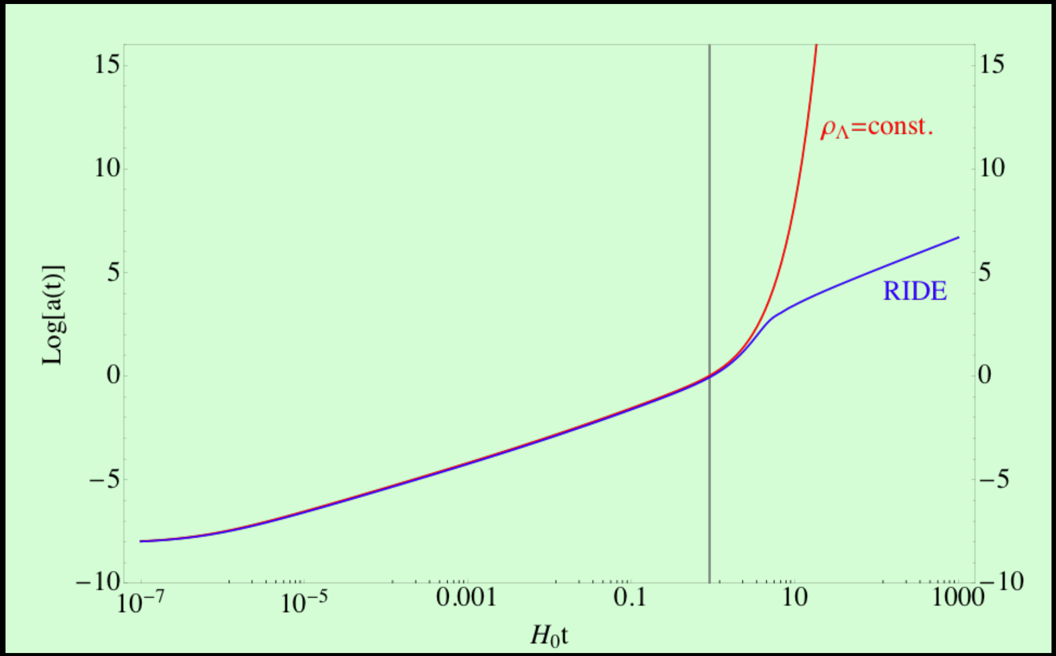
$$\Phi = \frac{1}{\sqrt{2}}\tilde{\eta}e^{i\phi/f} \; (\text{with } f = \langle\tilde{\eta}\rangle)$$





Short term predictions for Planck

Long term predictions for fate of Universe



Matter-antimatter asymmetry

Want to understand $\eta = \dfrac{n_b - n_{\bar{b}}}{n_\gamma} = (6.1 \pm 0.2) \times 10^{-10}$

q	$\bar{q}$
10,000,000,001	10,000,000,000

The Great Annihilation

Murayama

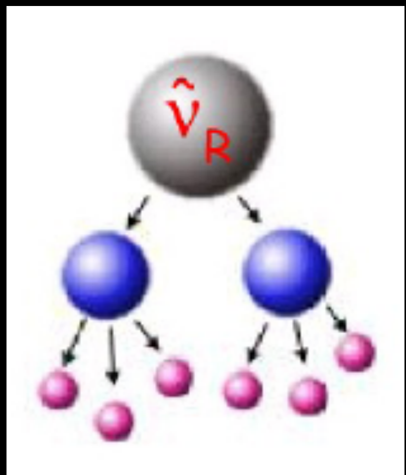
Leptogenesis

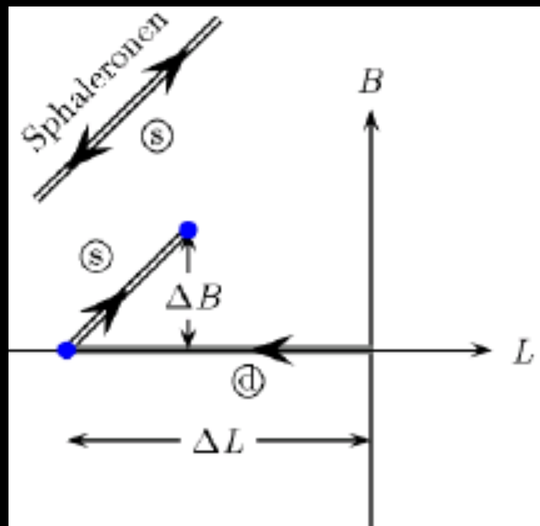
Di Bari and Marzola

- Right-handed neutrinos are produced in early universe and decay out of equilibrium giving net lepton numbers L_e , L_μ , L_τ
- CP violation from complex Yukawa couplings
- Out of equilibrium Boltzmann eqs lead to L_e , L_μ , L_τ partial washouts
- Surviving L_e, L_μ , L_τ are processed into B via B-L conserving sphalerons

Minimal model is 2RHN model

Antusch, Di Bari, SFK, Jones



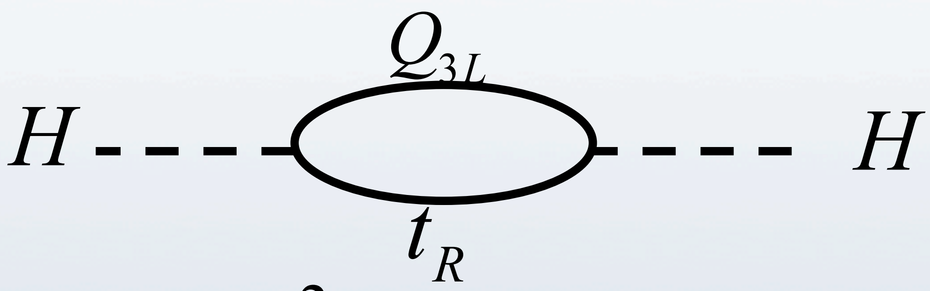


Origin of mass

$$V = m_H^2 |H|^2 + \tfrac{1}{2}\lambda |H|^4$$

Tree-level min cond $\quad m_H^2 = -\lambda v^2 = -\lambda (246\,GeV)^2$

Including rad corr $\quad m_H^2 + \delta m_H^2 = -\lambda (246\,GeV)^2$



$$\delta m_H^2 (top\,loop) = -\frac{3}{\sqrt{2}\pi^2} G_F m_t^2 \Lambda^2 = -(100\,GeV)^2 \left(\frac{\Lambda}{1\,TeV} \right)^2$$

Fine-tuning is required if the cut-off $\quad \Lambda \gg 1\,TeV$

Hierarchy problem → new physics at $\Lambda \sim$ TeV
e.g. Supersymmetry (SUSY) stop loops cancel

Minimal Supersymmetric Standard Model MSSM

Table 1: The MSSM Particle Spectrum

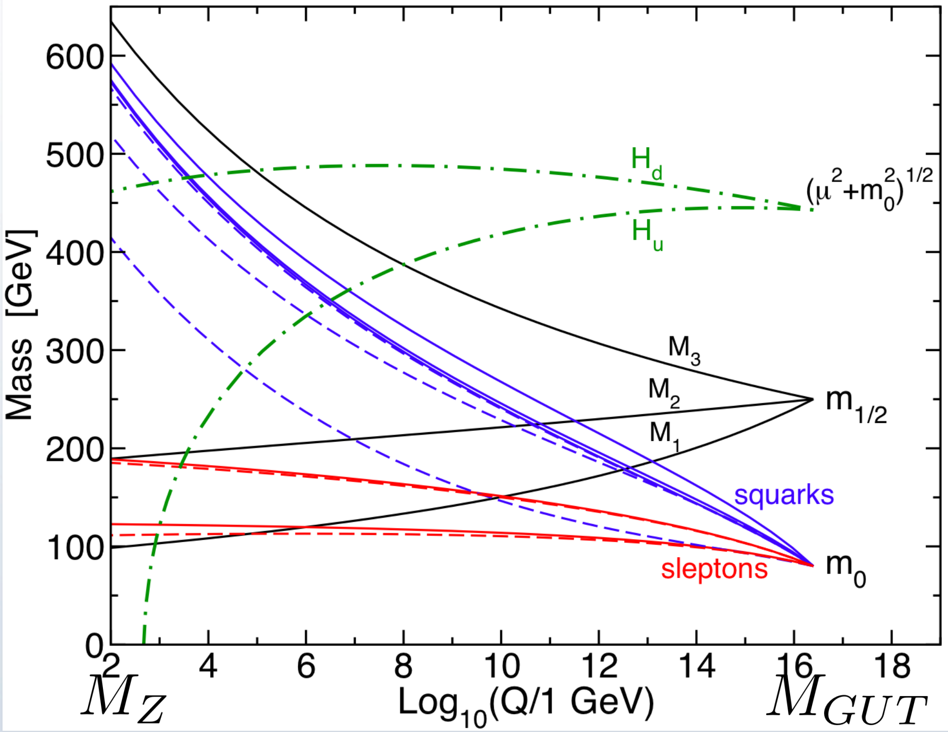
Superfield		Bosons	Fermions
Gauge			
$\widehat{G}$		g	$\widetilde{g}$
$\widehat{V}^a$		W^a	$\widetilde{W}^a$
$\widehat{V}'$		B	$\widetilde{B}$
Matter			
$\widehat{L}$	leptons	$\widetilde{L} = (\widetilde{\nu}, \widetilde{e}^-)_L$	$(\nu, e^-)_L$
$\widehat{E}^c$		$\widetilde{E} = \widetilde{e}^+_R$	e^c_L
$\widehat{Q}$	quarks	$\widetilde{Q} = (\widetilde{u}_L, \widetilde{d}_L)$	$(u, d)_L$
$\widehat{U}^c$		$\widetilde{U}^c = \widetilde{u}^*_R$	u^c_L
$\widehat{D}^c$		$\widetilde{D}^c = \widetilde{d}^*_R$	d^c_L
$\widehat{H}_d$	Higgs	H^i_d	$(\widetilde{H}^0_d, \widetilde{H}^-_d)_L$
$\widehat{H}_u$		H^i_u	$(\widetilde{H}^+_u, \widetilde{H}^0_u)_L$

Higgsino mass

$$\mu \tilde{H}_u \tilde{H}_d$$

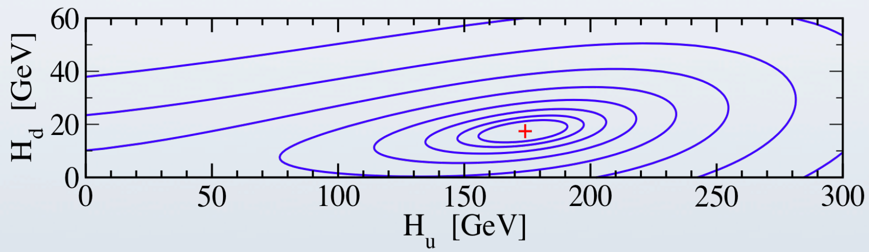
(What is the origin of this mu term?)

Constrained Minimal Supersymmetric Standard Model CMSSM



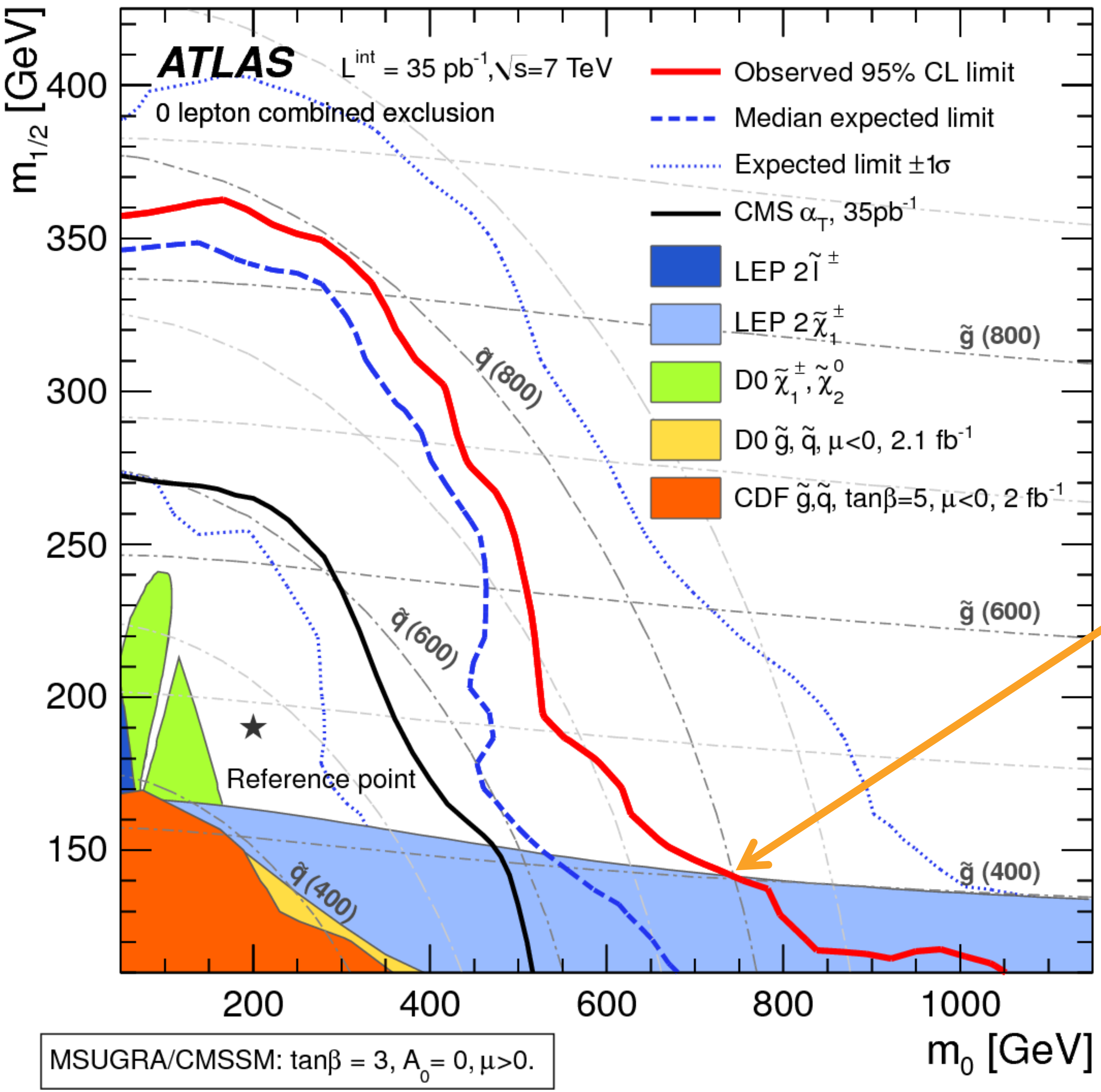
Two Higgs doublets get VEVs

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}$$



$$\tan\beta = \frac{v_u}{v_d}$$

CMSSM under intense scrutiny



800 GeV squarks
400 GeV gluinos
on the borderline
of exclusion

Beyond the MSSM

To provide an origin of μ term and reduce fine tuning consider a term λSHuHd   where singlet $<S> \sim \mu$

But leads to weak scale axion due to global U(1) PQ symmetry

Need to remove axion somehow

In NMSSM we add S^3 to break U(1) PQ to Z_3 – but this results in cosmological domain walls (or tadpoles if broken)

In USSM we gauge the U(1) PQ symmetry to eat the axion resulting in a massive Z' gauge boson – but not anomaly free

In E_6SSM the anomalies of the USSM are cancelled by three complete 27's of E_6 at the TeV scale with U(1) PQ $\in E_6$

The Exceptional Supersymmetric Standard Model E$_6$SSM

SFK, Moretti, Nevzorov

$$E_6 \to SO(10) \times U(1)_\psi \qquad SO(10) \to SU(5) \times U(1)_\chi$$

E$_6$ broken via SU(5) chain

Right handed neutrinos are neutral under: $U(1)_N = \frac{\sqrt{15}}{4} U(1)_\psi + \frac{1}{4} U(1)_\chi \longrightarrow Z'(N)$

M_{GUT} — $E_6 \to SM \times U(1)_N$

$$27 \to \underbrace{10 + \bar{5}}_{} + \underbrace{5 + \bar{5}}_{} + \underbrace{S + N}_{} \qquad \underbrace{L_4 + \bar{L}_4}_{}$$

- $10 + \bar{5}$: Quarks, leptons
- $5 + \bar{5}$: Exotics and Higgs
- $S + N$: Singlets and RHν s
- $L_4 + \bar{L}_4$: Extra Leptons or not (Howl, SFK)

RH ν masses: M_3, M_2, M_1

TeV — $U(1)_N$ broken, Z' & exotics get mass, μ term generated

M_W — $SU(2)_L \times U(1)_Y$ broken

Matter content of E_6SSM at TeV

Quarks and Leptons

EXOTIC D,D-bar

$$27_i \longrightarrow 3\left[(Q_i,\, u_i^c,\, d_i^c,\, L_i,\, e_i^c)\right] + 3(D_i,\, \overline{D}_i) + $$
$$+3(H_{2i}) + 3(H_{1i}) + 3(S_i) + 3(N_i^c) + L_4 + \overline{L}_4{}'$$

Three families of Higgs
= one active family i=3
+ 2 inert families (no VEV)

Singlets

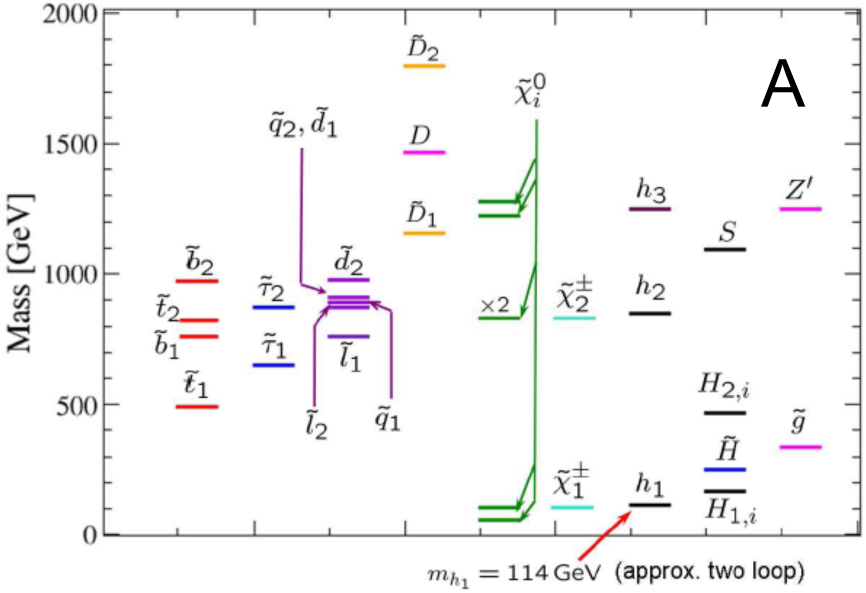
Right-handed neutrinos (superheavy)

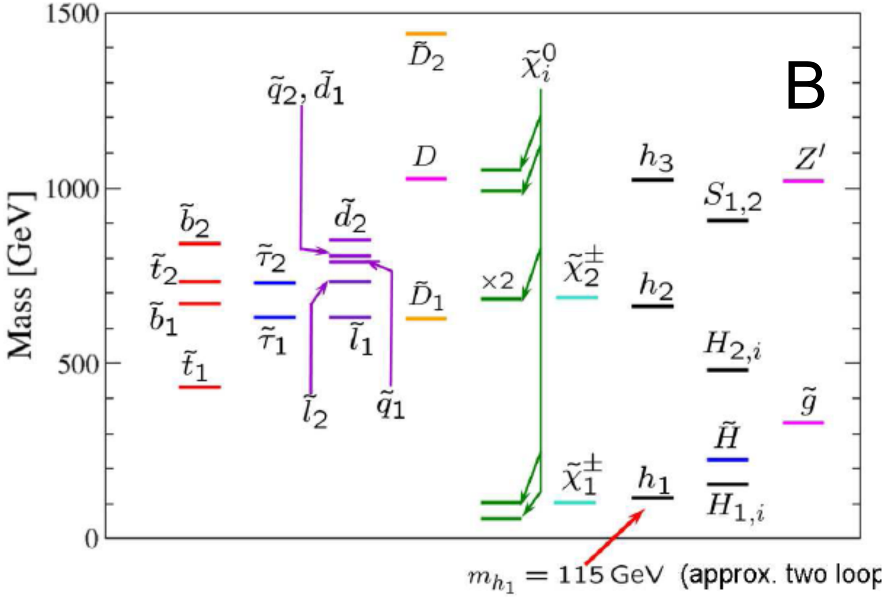
EXOTIC lepton doublets mass μ'

Plus a TeV scale Z' from $U(1)_N$ where gauge anomalies are cancelled due to complete 27 reps

Predicts heavy squarks and sleptons and light gluinos

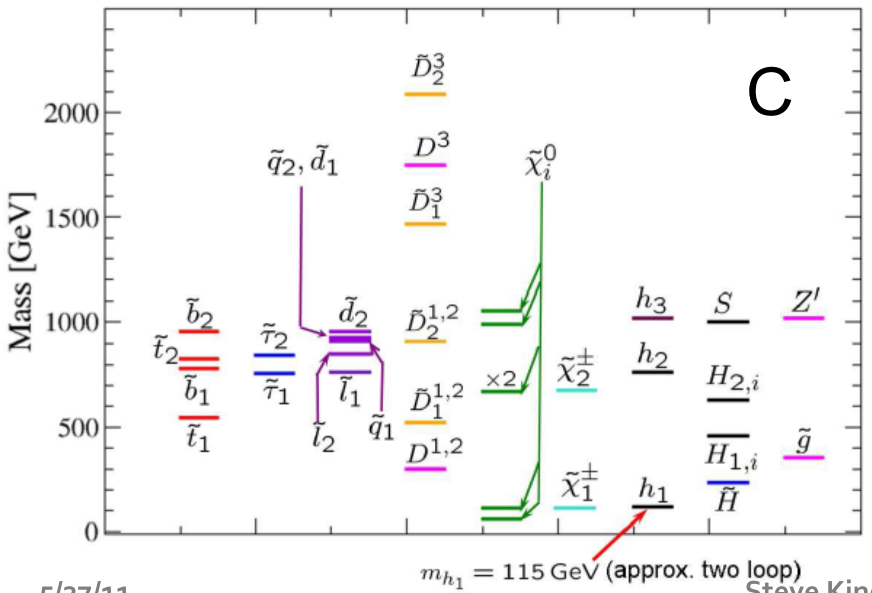
5/27/11 66

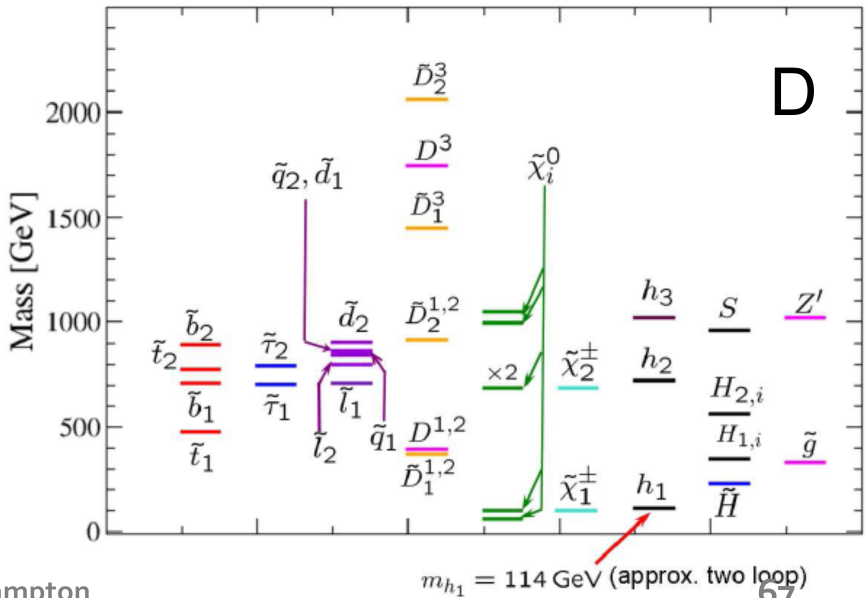




Constrained E_6SSM Benchmarks

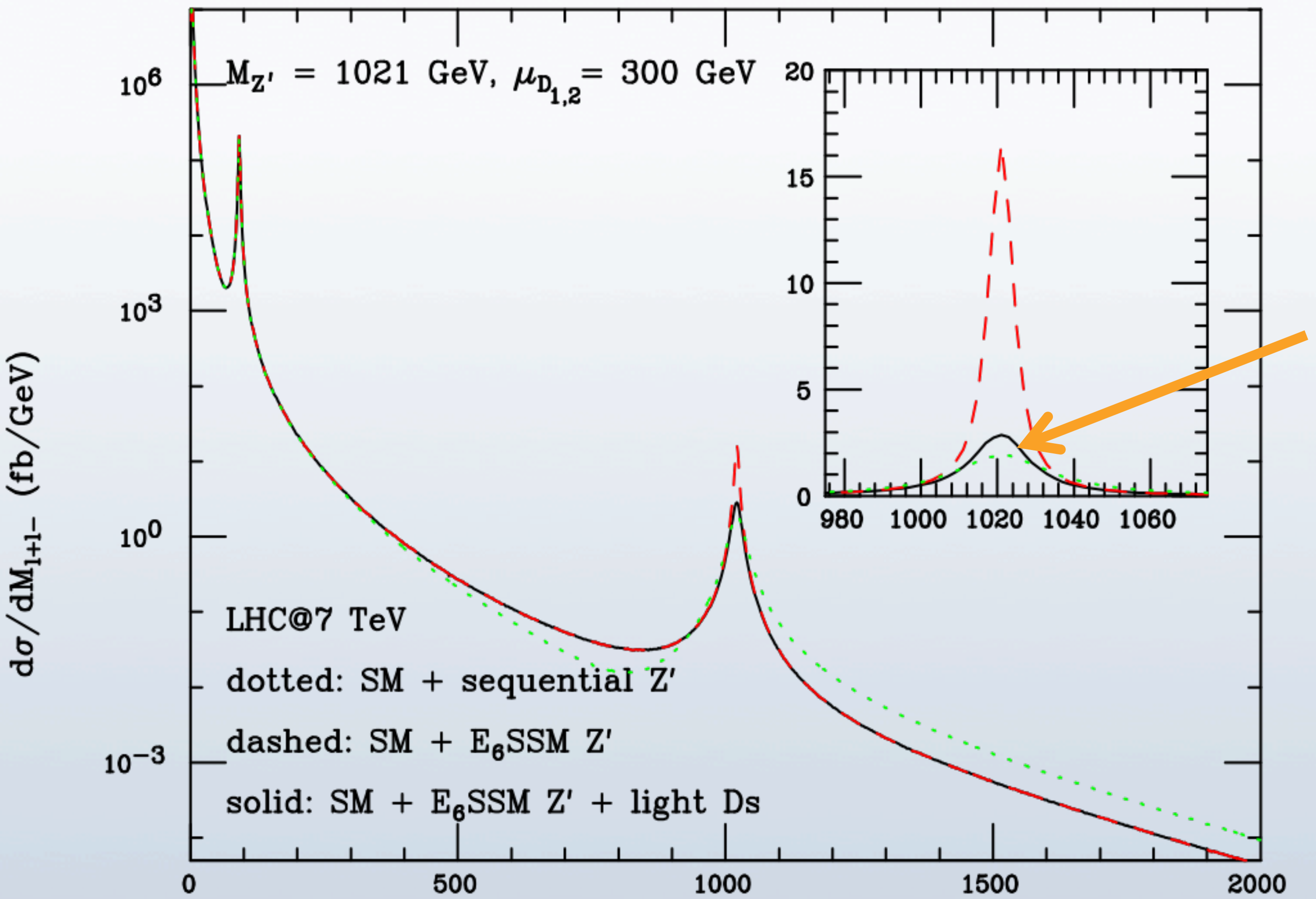
Athron, SFK, Miller, Moretti, Nevzorov





Z’ in benchmark C

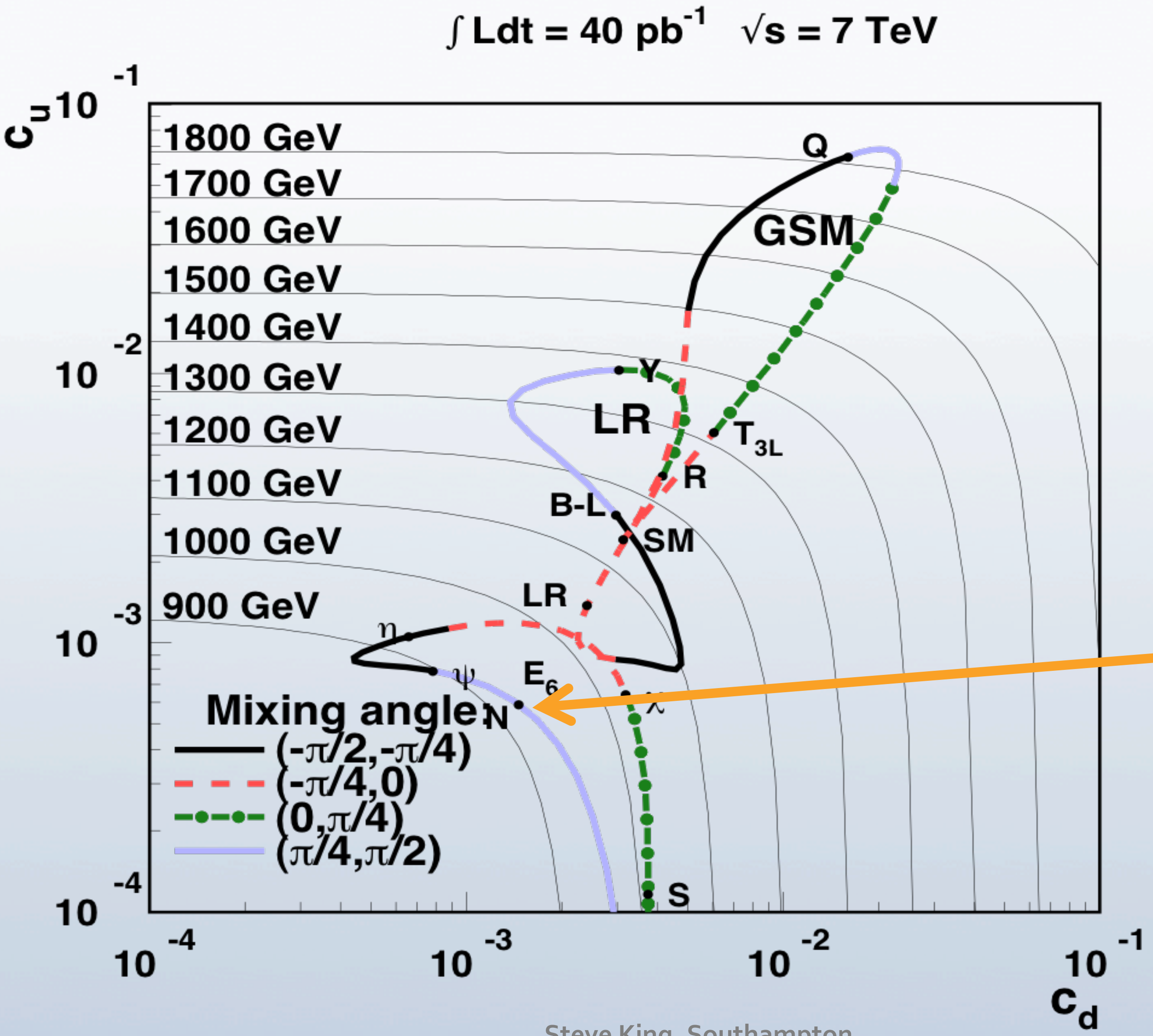
Athron, SFK, Miller, Moretti, Nevzorov



Exotic decays makes Z’ peak smaller and harder to discover

CMS limits on Z' masses

Accomando, Belyaev, SFK, Shepherd et al



U(1)$_N$ mass limit 930 GeV will be reduced to 860 GeV including exotic decays

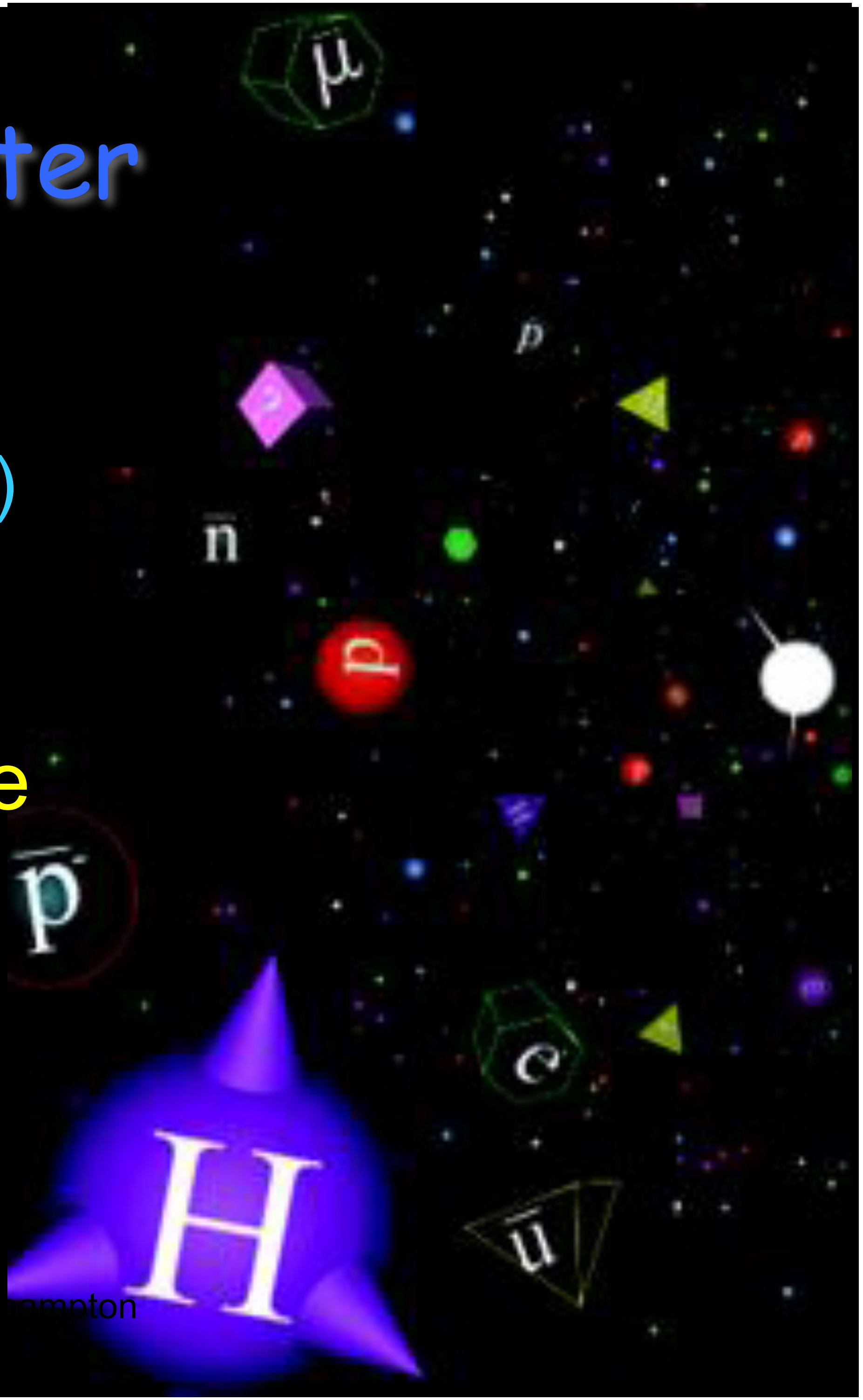
Origin of dark matter

SM candidates include

- Neutrino (hot or warm only)
- Axion

SUSY candidates include

- Neutralino
- Gravitino



Belyaev, Svantesson

MSSM Neutralino Dark Matter

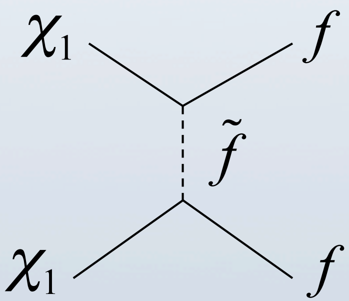
Neutralino mass matrix

$$\begin{array}{cccc} \tilde{B} & \tilde{W}_3 & \tilde{H}_d & \tilde{H}_u \end{array}$$

$$\begin{pmatrix} M_1 & & & \\ & M_2 & & \\ & & 0 & -\mu \\ & & -\mu & 0 \end{pmatrix}$$

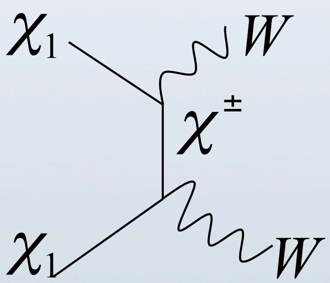
$$\chi_1 = N_1\tilde{B} + N_2\tilde{W} + N_3\tilde{H}_d + N_4\tilde{H}_u$$

$$\Omega_{DM}h^2 = C\frac{T_0^3}{M_P^2}\frac{1}{\langle\sigma v\rangle}$$



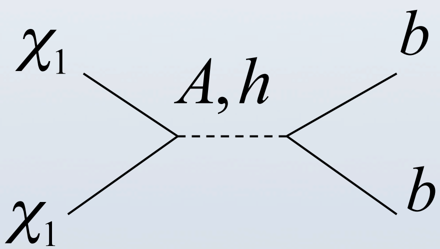
Bulk

$$m_{\tilde{f}} \approx m_{\chi_1}$$



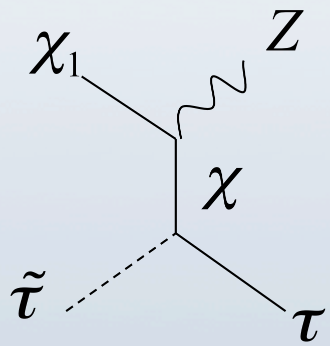
Focus

Higgsino LSP



Funnel

$$m_{A,h} \approx 2m_{\chi_1}$$



Co-annihilation

$$m_{\tilde{\tau}} \approx m_{\chi_1}$$

Hall, SFK

Almost decoupled inert sector could be responsible for dark matter

$$A_{\alpha\beta} = -\frac{1}{\sqrt{2}} \begin{pmatrix} & \tilde{H}^0_{d\beta} & \tilde{H}^0_{u\beta} & \tilde{S}_\beta \\ \tilde{H}^0_{d\alpha} & 0 & \lambda_{\alpha\beta} s & \tilde{f}_{\beta\alpha} v \sin\beta \\ \tilde{H}^0_{u\alpha} & \lambda_{\beta\alpha} s & 0 & f_{\beta\alpha} v \cos\beta \\ \tilde{S}_\alpha & \tilde{f}_{\alpha\beta} v \sin\beta & f_{\alpha\beta} v \cos\beta & 0 \end{pmatrix}$$

$$m_{\chi_1^0} \approx \frac{f^2}{\lambda}\frac{v^2}{s}\sin 2\beta$$

- LSP is naturally light $\sim$ v^2 /s
- LSP is inert Higgsino/singlino

Dark matter benchmark in the E_6SSM

Hall, SFK, Pakvasa
Nevzorov, Sher

$\tan(\beta)$	1.5
$m_{H^\pm} \simeq m_A \simeq m_{h_3}/\mathrm{GeV}$	1977
m_{h_1}/GeV	135.4
λ_{22}	0.001
λ_{21}	0.077
λ_{12}	0.077
λ_{11}	0.001
f_{22}	0.001
f_{21}	0.61
f_{12}	0.6
f_{11}	0.001
$\tilde{f}_{22}$	0.001
$\tilde{f}_{21}$	0.426
$\tilde{f}_{12}$	0.436
$\tilde{f}_{11}$	0.001
$m_{\tilde{\chi}_1^0}/\mathrm{GeV}$	41.91
$m_{\tilde{\chi}_2^0}/\mathrm{GeV}$	-42.31
$m_{\tilde{\chi}_3^0}/\mathrm{GeV}$	-129.1
$m_{\tilde{\chi}_4^0}/\mathrm{GeV}$	132.4
$m_{\tilde{\chi}_5^0}/\mathrm{GeV}$	171.4
$m_{\tilde{\chi}_6^0}/\mathrm{GeV}$	-174.4
$m_{\tilde{\chi}_1^\pm}/\mathrm{GeV}$	129.0
$m_{\tilde{\chi}_2^\pm}/\mathrm{GeV}$	132.4

$\Omega_\chi h^2$	0.096
R_{Z11}	-0.0250
R_{Z12}	0.0040
R_{Z22}	-0.0257
ΔN_ν^{eff}	0.000090
D	2.011
$X_{11}^{h_1}$	0.137
$X_{12}^{h_1} + X_{21}^{h_1}$	-1.9×10^{-6}
$X_{22}^{h_1}$	-0.138
$\sigma_{SI}/10^{-44}\ \mathrm{cm}^2$	2.6-10.5

$\mathrm{Br}(h \to \tilde{\chi}_1^0\tilde{\chi}_1^0)$	49.5%
$\mathrm{Br}(h \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$	7.9×10^{-11}
$\mathrm{Br}(h \to \tilde{\chi}_2^0\tilde{\chi}_2^0)$	49.0%
$\mathrm{Br}(h \to b\bar{b})$	1.36%
$\mathrm{Br}(h \to \tau\bar{\tau})$	0.142%
$\Gamma(h \to \tilde{\chi}_1^0\tilde{\chi}_1^0)/\mathrm{MeV}$	98.3
$\Gamma^{tot}/\mathrm{MeV}$	198.7

XENON100 limit
$3.4 \times 10^{-44}\ \mathrm{cm}^2$

Higgs decays 98.5% of time into inert neutralinos !

Mainly invisible Higgs decays at LHC!!

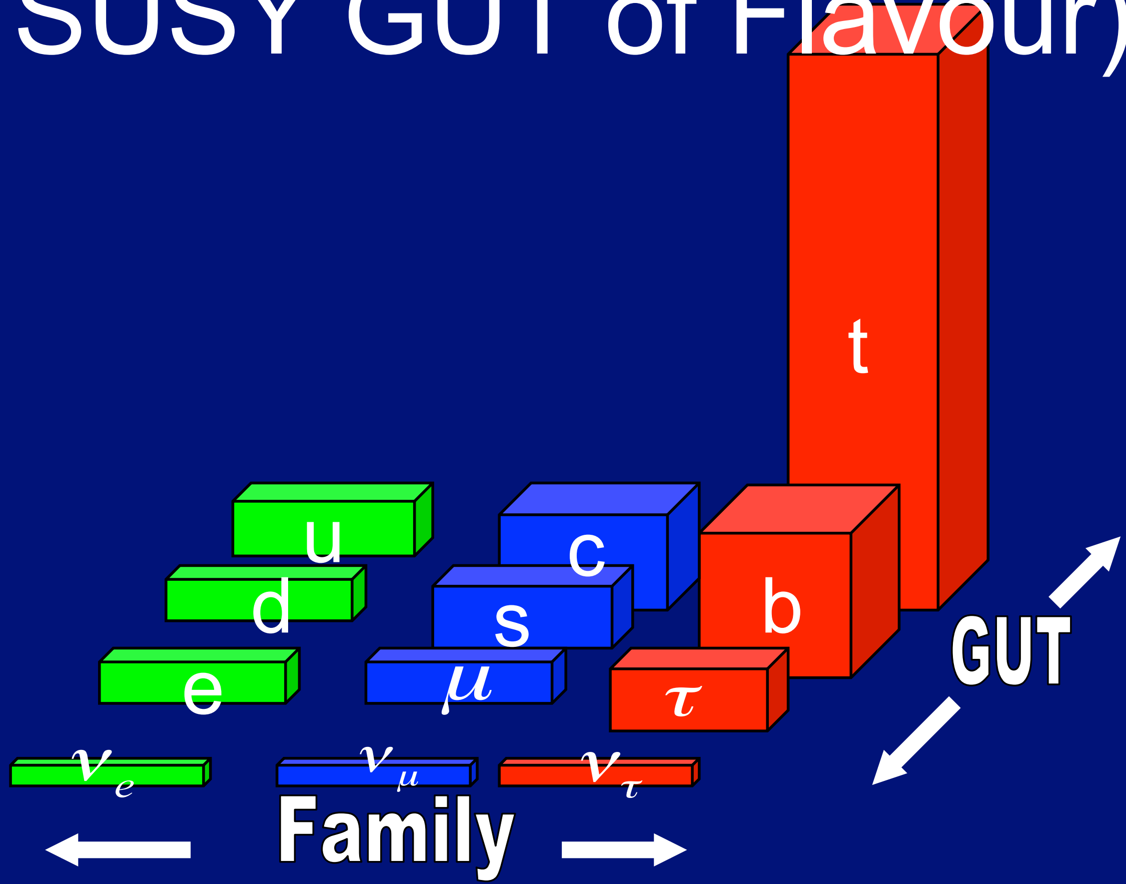
Only hope is: $h \to \chi_2\chi_2$

$$\chi_2 \to \chi_1\mu^+\mu^-$$

Soft lepton pairs ~1 GeV + missing E_T

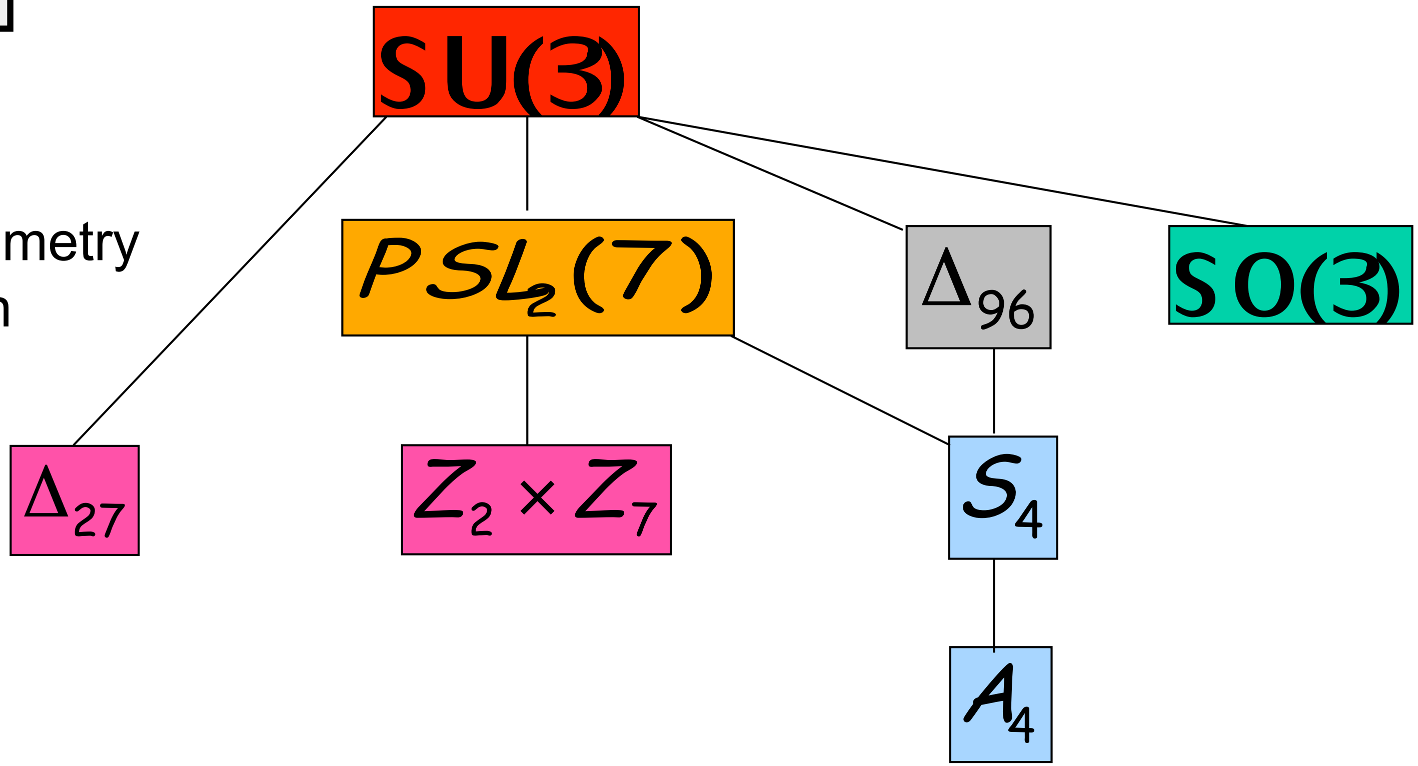
N.B. Large LSP direct detection cross-sections close to current limits → XENON100 will test

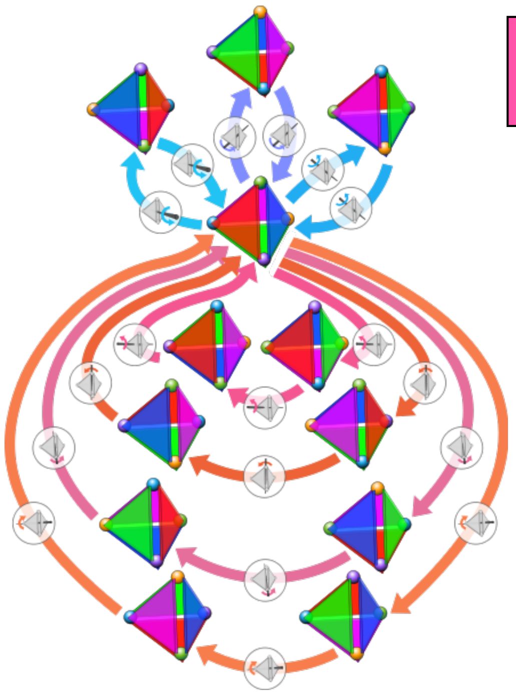
SUSY GUTs and Family Symmetry (aka SUSY GUT of Flavour)



G_{Family}

e.g. A_4 is the symmetry of the tetrahedron

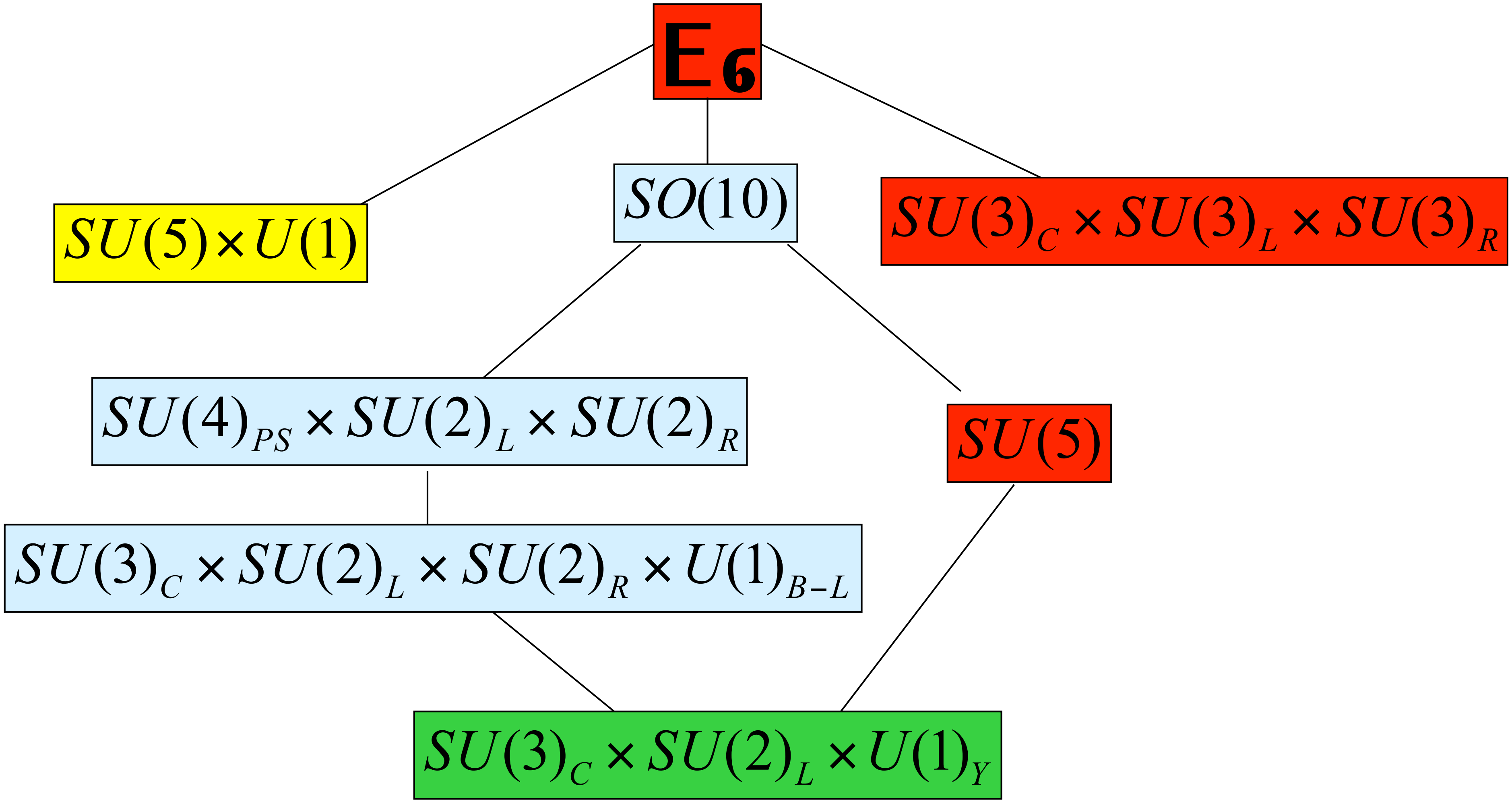




Discrete family symmetry suggested by TB mixing

Cooper, SFK, Luhn

G_{GUT}



E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C\times SU(3)_L\times SU(3)_R$
$SU(4)_{PS}\times SU(2)_L\times SU(2)_R$
$SU(5)$
$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$
$SU(3)_C\times SU(2)_L\times U(1)_Y$

S_4xSU(5) SUSY GUT of Flavour

$$F = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \sim \mathbf{3}\,, \qquad T = \begin{pmatrix} T_1 \\ T_2 \end{pmatrix} \sim \mathbf{2}\,, \quad T_3 \sim \mathbf{1}\,.$$

Antusch, SFK, Luhn, Spinrath

Purely real or imaginary vacuum alignments

Fritzsch type quark mass matrices

$$M_u \sim \begin{pmatrix} \lambda^8 & \lambda^6 & 0 \\ \lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} v_u\,, \qquad M_d \sim \begin{pmatrix} 0 & \mathrm{i}\lambda^5 & 0 \\ \mathrm{i}\lambda^5 & \lambda^4 & 2\lambda^4 \\ 0 & 0 & \lambda^2 \end{pmatrix} v_d\,,$$

Charged lepton mass matrix

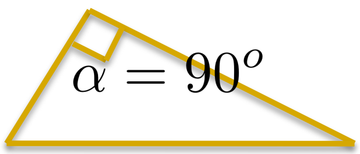
$$M_e \sim \begin{pmatrix} 0 & \mathrm{i}\lambda^5 & 0 \\ \mathrm{i}\lambda^5 & -3\lambda^4 & 0 \\ 0 & -6\lambda^4 & \lambda^2 \end{pmatrix} v_d$$

Neutrino mass matrices

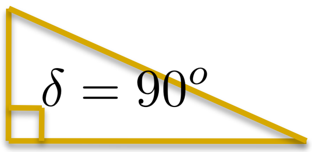
$$M_D \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} v_u\,, \qquad M_R \sim \begin{pmatrix} \alpha+2\gamma & \beta-\gamma & \beta-\gamma \\ \beta-\gamma & \beta+2\gamma & \alpha-\gamma \\ \beta-\gamma & \alpha-\gamma & \beta+2\gamma \end{pmatrix} \lambda^4 M$$

Phase sum rule $\alpha \approx \delta^d_{12} - \delta^u_{12}$

Quark unitarity triangle



Lepton unitarity triangle

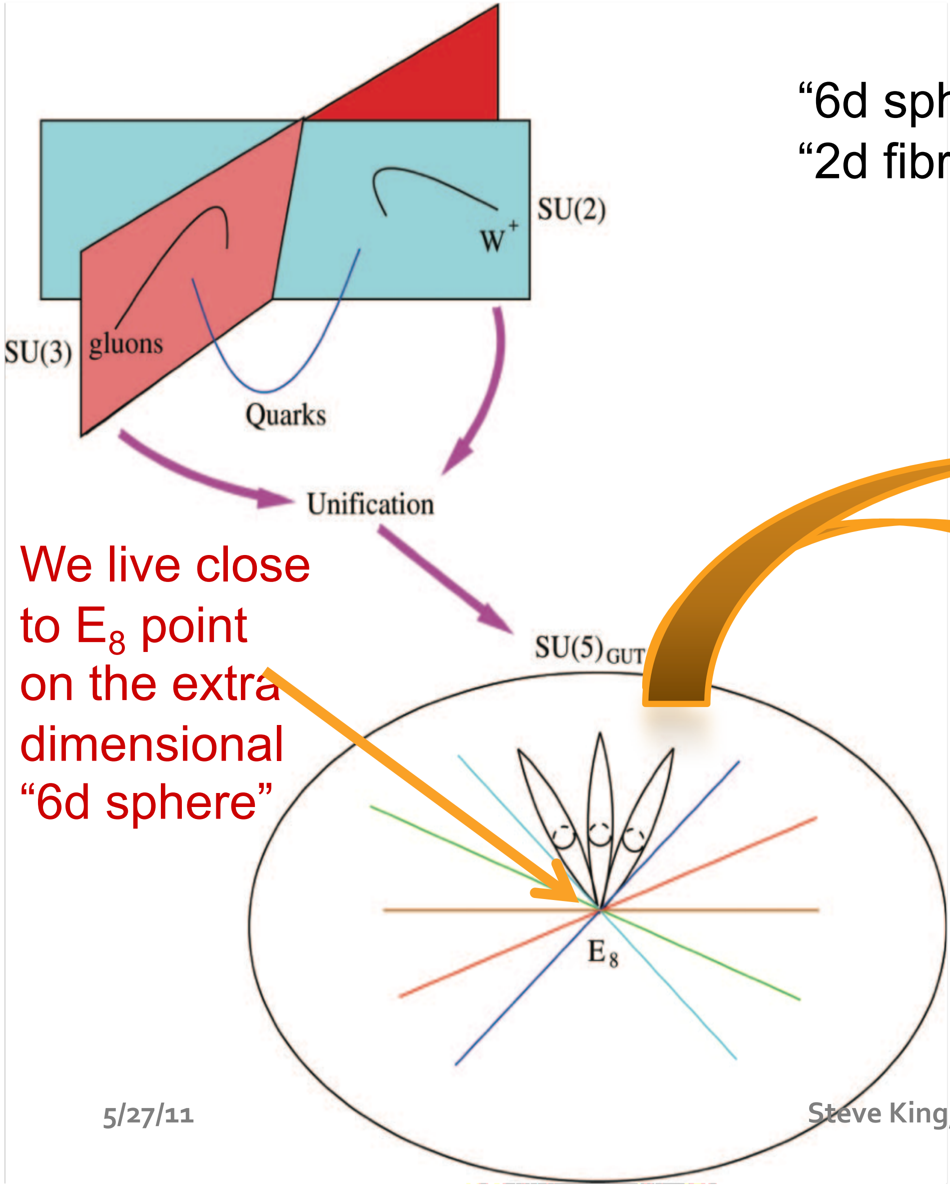


Approximate TB mixing

$$U_{TB} = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

F-Theory GUTs: a 12d string theory

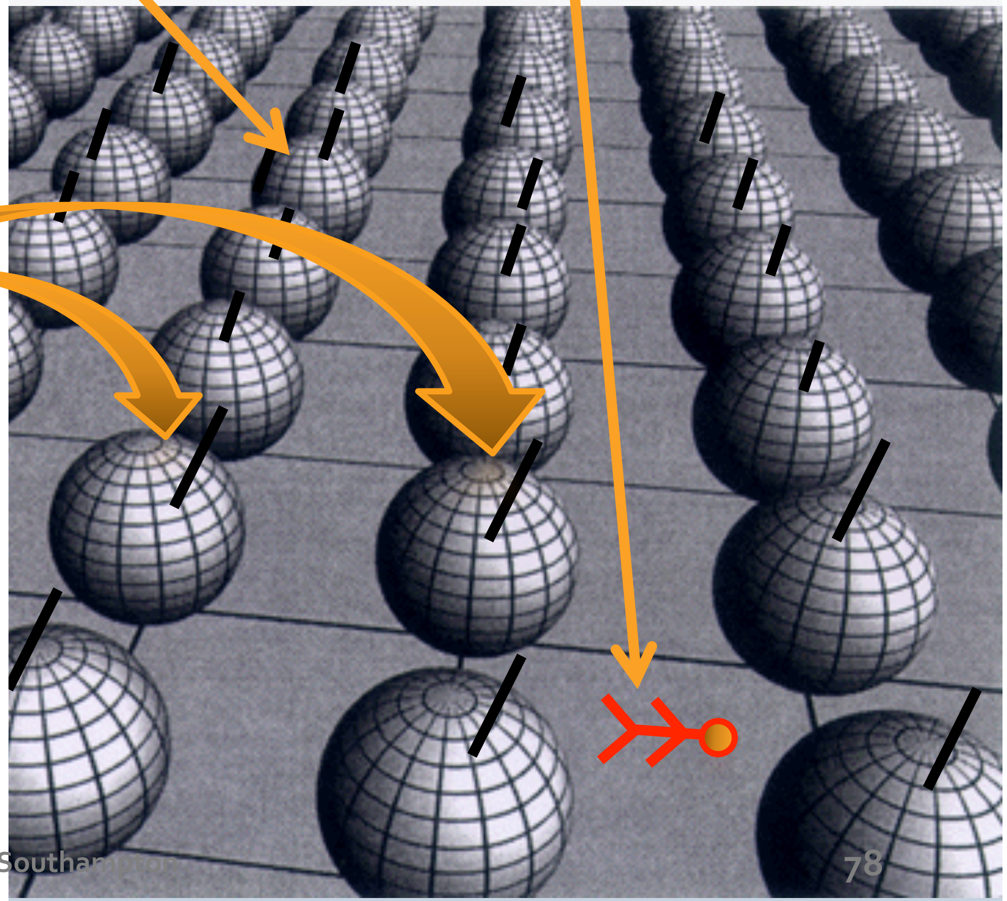
Heckman and Vafa



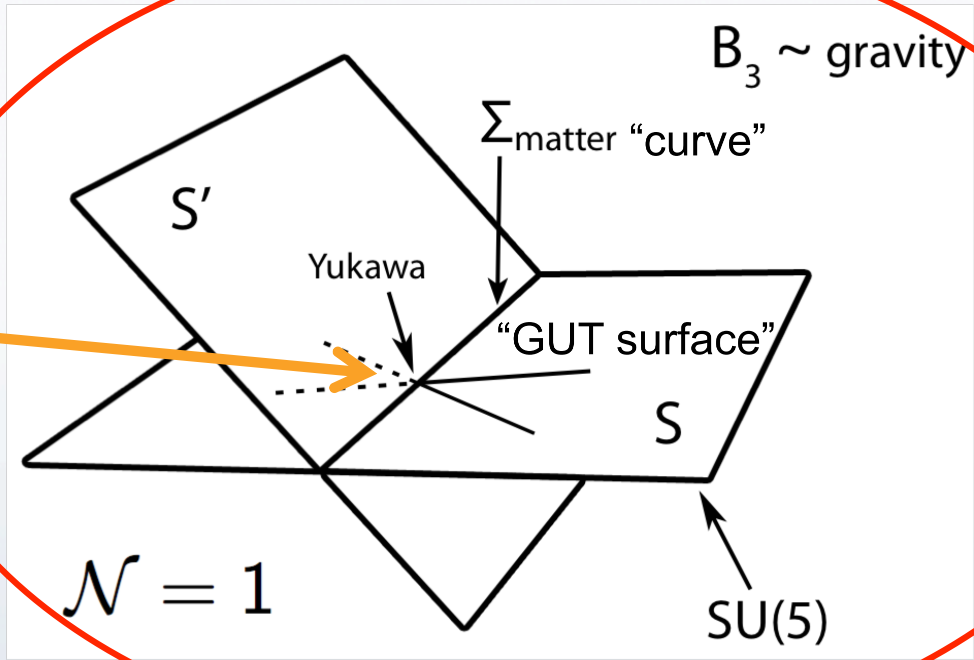
We live close to E_8 point on the extra dimensional "6d sphere"

"6d spheres" with "2d fibres"

"4d Flatlander"



The "6d sphere"



We live close to here

dim.	internal dim.	feature
10	$6 = \dim(B_3)$	gravity
8	$4 = \dim(S)$	gauge fields
6	$2 = \dim(S \cap S')$	matter
4	$0 = \dim(S \cap S' \cap S'')$	interactions

"gravity bulk"
"GUT surface S"
"matter curve Σ"
"Yukawa point"

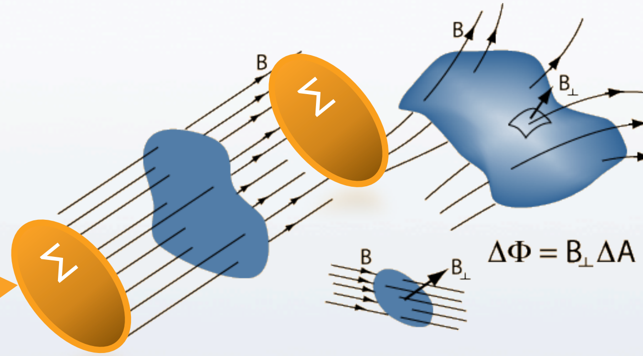
Figure 1: The structure of an F-theory GUT

GUT breaking is achieved not with Higgs but with Hypercharge Flux

$$SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$5 \to (1,2)_{1/2} + (3,1)_{-1/3}.$$

2-d Matter curve Σ



Think of magnetic flux PHYS2001

Index theorem gives number of chiral doublets and triplets (think of Gauss's law):

$$(1,2)_{1/2} : n_L - n_R = 3\int_\Sigma F_{U(1)_Y} + q\int_\Sigma F_{U(1)_\perp}$$

$$(3,1)_{-1/3} : n_L - n_R = -2\int_\Sigma F_{U(1)_Y} + q\int_\Sigma F_{U(1)_\perp}$$

Doublet-triplet Higgs splitting requires:

$$\text{Higgs:} \int_\Sigma F_{U(1)_Y} \neq 0$$

$$\text{Matter:} \int_\Sigma F_{U(1)_Y} = 0.$$

Typically predicts exotics

Callaghan, SFK, Leontaris, Ross

Conclusion

- Standard Models are highly successful but leave puzzles in their wake
- Inflation and Dark energy may be linked
- Minimal model of Leptogenesis is 2RHN model
- SUSY provides answers to origin of mass and unification but MSSM is under threat
- E_6SSM predicts heavy squarks/sleptons and light gluinos plus Z′ and exotics
- Z′ may be harder to spot due to exotic decays
- Dark matter may be inert singlino/higgsino → signal at XENON100
- SUSY GUTs of Flavour address Flavour problem and Unification
- F-theory may be origin of SUSY GUTs but predicts exotics (E_6SSM ?)
- LHC could discover SUSY and exotics in 2011/2012