

MA676: Bayesian Methods – Additional Exercises 1

1. A certain disease affects 0.1% of the population. A diagnostic test for the disease gives a positive response with probability 0.95 if the disease is present. It also gives a positive response with probability 0.02, however, if the disease is not present. If the test gives a positive response for a given person, what is the probability that the person has the disease?
2. The Japanese car company Maito make their Professor model in three countries, Japan, England and Germany, with one half of the cars being built in Japan, two tenths in England and three tenths in Germany. One percent of the cars built in Japan have to be returned to the dealer as faulty while the figures for England and Germany are four percent and two percent respectively. What proportion of Professors are faulty? If I buy a Professor and find it to be faulty, what is the chance that it was made in England?
3. Suppose that the number of defects on a roll of magnetic recording tape has a Poisson distribution for which the mean θ is unknown and that the prior distribution of θ is a gamma distribution with parameters $\alpha = 3$ and $\beta = 1$. When five rolls of this tape are selected at random and inspected, the number of defects found on the rolls are 2, 2, 6, 0, and 3. If the squared error loss function is used what is the Bayes estimate of θ ?
4. Suppose that X_1, \dots, X_n is a random sample from the distribution with pdf

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{if } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Suppose also that the value of the parameter θ is unknown ($\theta > 0$) and that the prior distribution of θ is a gamma distribution with parameters α and β ($\alpha > 0$ and $\beta > 0$). Determine the posterior distribution of θ and hence obtain the Bayes estimator of θ under a squared error loss function.

5. Suppose that X_1, \dots, X_n is a random sample of size n from a normal distribution with unknown mean θ and variance 1. Suppose also that the prior distribution of θ is also a normal distribution with mean 0 and given variance τ^2 . Derive the posterior distribution of θ given X_1, \dots, X_n . From the posterior distribution write the Bayes estimator under i) squared error loss and ii) absolute error loss, clearly stating the standard results you have used.