MATH2019 – Statistics for Civil and Environmental Engineering Formula Sheet/Statistical Tables

Normal distribution with mean μ and standard deviation σ has density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2\sigma^2}(x-\mu)^2\right).$$

For a standard normal ($\mu = 0$, $\sigma = 1$) variable Z, the distribution function $F(z) = P(Z \leq z)$ is given below, for various values of $z \geq 0$.

	2nd decimal place of z									
z	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

Weibull distribution with parameters α and β has density function

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \exp\left\{-\left(\frac{x}{\beta}\right)^\alpha\right\}$$

and its mean and standard deviation are given by $\mu = \beta\Gamma\left(1+\frac{1}{\alpha}\right)$ and $\sigma^2 = \beta^2\left\{\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma\left(1+\frac{1}{\alpha}\right)^2\right\}$ where $\Gamma(t) = \int_0^\infty x^{t-1}e^{-x}dx$. The Weibull distribution with $\alpha = 1$ is the **exponential distribution** which has density function

$$f(x) = \frac{1}{\beta}e^{-\frac{x}{\beta}}$$

and distribution function

$$F(x) = P(X \leq x) = 1 - e^{-\frac{x}{\beta}}$$

Its mean and standard deviation are given by $\mu = \beta$ and $\sigma = \beta$.

Extreme value distribution (EVG1) with parameters α and β has density function

$$f(x) = \frac{1}{\beta} \exp\left(-\frac{x-\alpha}{\beta} - e^{-\frac{x-\alpha}{\beta}}\right)$$

and its mean and standard deviation are given by $\mu = \alpha + \beta\gamma$ and $\sigma = \pi\beta/\sqrt{6}$ where $\pi \approx 3.1416$ and $\gamma \approx 0.5772$. Its distribution function is

$$F(x) = P(X \leq x) = \exp\left(-e^{-\frac{x-\alpha}{\beta}}\right).$$

t distribution

This table allows you to calculate c for various $p = P(t_k \leq c)$ for different degrees of freedom k.

	$P(t_k \leq c)$						$P(t_k \leq c)$				
k	0.9	0.95	0.975	0.99	0.995	k	0.9	0.95	0.975	0.99	0.995
1	3.08	6.31	12.71	31.82	63.66	11	1.36	1.80	2.20	2.72	3.11
2	1.89	2.92	4.30	6.96	9.92	12	1.36	1.78	2.18	2.68	3.05
3	1.64	2.35	3.18	4.54	5.84	15	1.34	1.75	2.13	2.60	2.95
4	1.53	2.13	2.78	3.75	4.60	20	1.33	1.72	2.09	2.53	2.85
5	1.48	2.02	2.57	3.36	4.03	25	1.32	1.71	2.06	2.49	2.79
6	1.44	1.94	2.45	3.14	3.71	30	1.31	1.70	2.04	2.46	2.75
7	1.41	1.89	2.36	3.00	3.50	40	1.30	1.68	2.02	2.42	2.70
8	1.40	1.86	2.31	2.90	3.36	50	1.30	1.68	2.01	2.40	2.68
9	1.38	1.83	2.26	2.82	3.25	60	1.30	1.67	2.00	2.39	2.66
10	1.37	1.81	2.23	2.76	3.17	100	1.29	1.66	1.98	2.36	2.63
						∞	1.28	1.64	1.96	2.33	2.58

Two sample t test

Suppose that we observe a sample of n observations $x_1, \ldots, x_n$ (with sample mean $\overline{x}$ and sample standard deviation s_x) from the distribution of variable X and a sample of m observations $y_1, \ldots, y_m$ (with sample mean $\overline{y}$ and sample standard deviation s_y) from the distribution of variable Y.

If the distribution of X has mean μ_x and the distribution of Y has mean μ_y then the end points of a confidence interval for the difference in the means $\mu_x - \mu_y$ are given by

$$\overline{x} - \overline{y} \pm c\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}$$

where c is derived from a t distribution with k degrees of freedom and $k = \dfrac{\left(\dfrac{s_x^2}{n} + \dfrac{s_y^2}{m}\right)^2}{\dfrac{(s_x^2/n)^2}{n-1} + \dfrac{(s_y^2/m)^2}{m-1}}$.

Regression

Suppose that, for each of a sample of n experimental units, two variables X and Y are measured. The samples are denoted by $x_1, \ldots, x_n$ and $y_1, \ldots, y_n$, with corresponding sample means $\overline{x}$ and $\overline{y}$.

For the linear regression model

$$y_i = \alpha + \beta x_i + \epsilon_i \qquad i = 1, \ldots n,$$

the least squares estimates of α and β are given by

$$\hat{\beta} = \frac{\sum_{i=1}^n x_i y_i - n\overline{x}\,\overline{y}}{\sum_{i=1}^n x_i^2 - n\overline{x}^2}. \qquad \text{and} \qquad \hat{\alpha} = \overline{y} - \hat{\beta}\overline{x}.$$