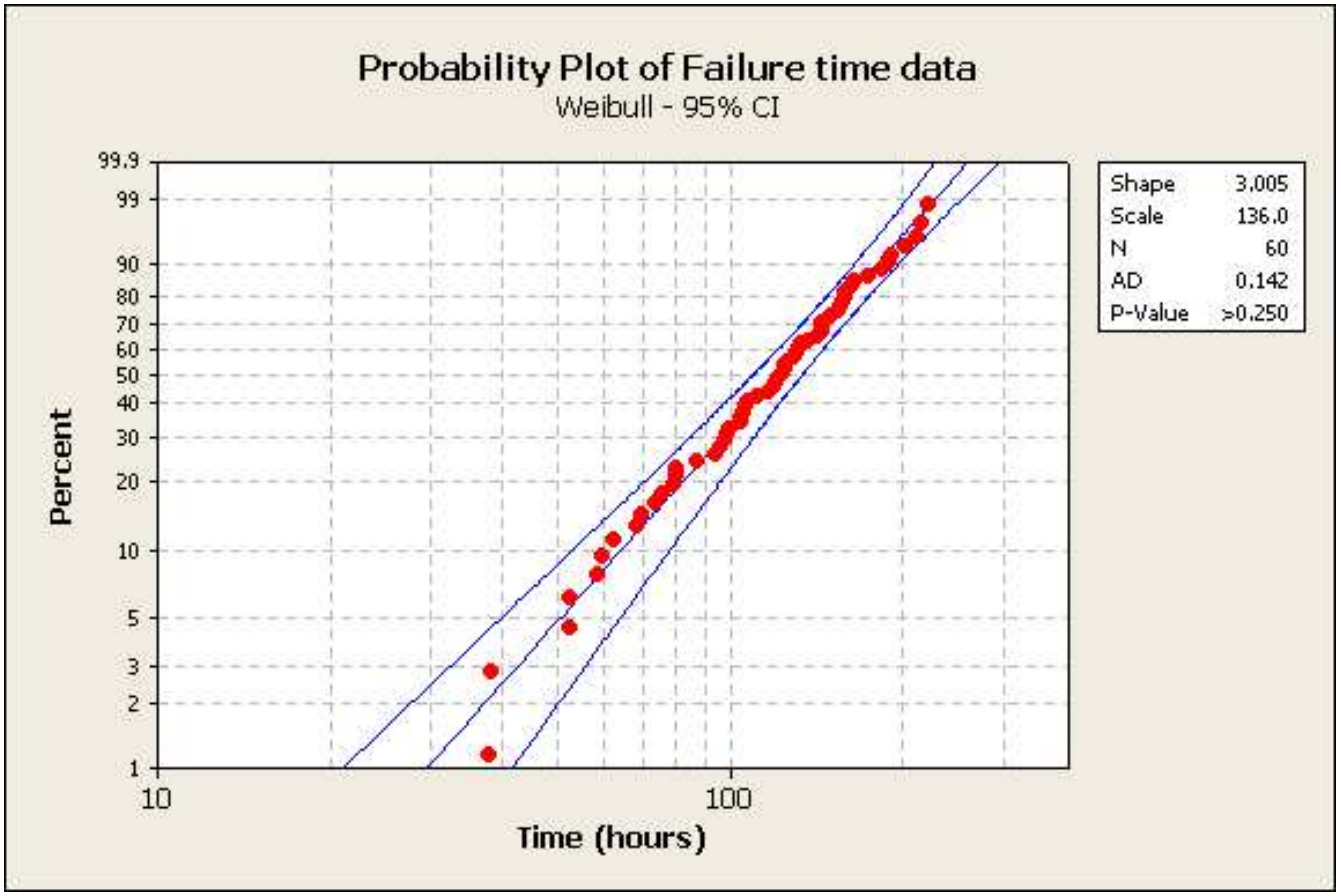
1. The time to failure T of a component in a mechanical system is assumed to follow a Weibull distribution with parameters α and β (see density function on the formula sheet).

a) Show that the probability that failure occurs within time x is given by

$$P(T \leq x) = 1 - \exp\left\{-\left(\frac{x}{\beta}\right)^{\alpha}\right\}.$$

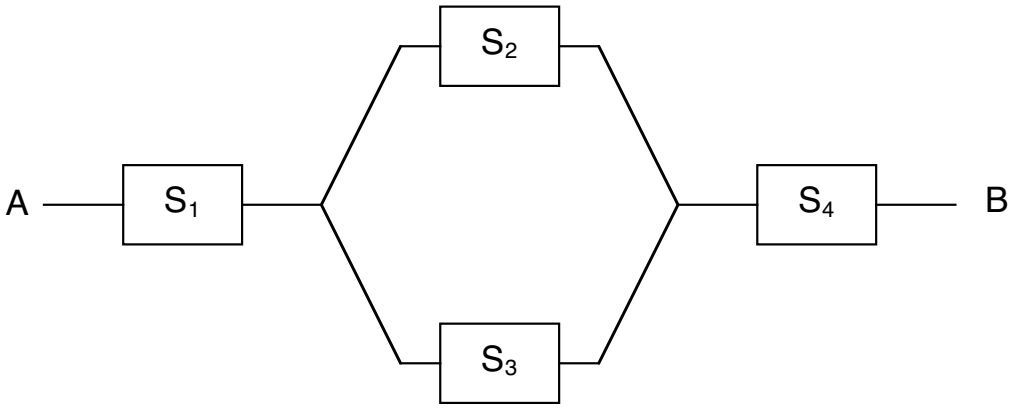
b) Data are collected on the lifetimes of 20 such components, and the Weibull probability plot below obtained using MINITAB.



Does the Weibull model fit these data? Write down the estimates of the parameters α and β and hence show that the probability of failure occuring before 100 hours is estimated as 0.328 and the probability of failure occuring before 200 hours is estimated as 0.959.

c) Would an exponential distribution provide a plausible alternative model for these lifetimes?

d) A system is being constructed with four such components S_1, S_2, S_3, S_4 arranged as in the diagram below. The system functions correctly if a path exists from A to B via correctly functioning components.



Calculate the probability that the system is still functioning correctly after (i) 100 hours, (ii) 200 hours.

2. The diameter of holes for cable harness is assumed to be normally distributed with mean 4cm and standard deviation 0.024cm.

a) What is the probability that the diameter of a hole is less than 3.97cm?

b) What proportion of holes have diameters between 3.98cm and 4.04cm?

c) What diameter is exceeded by only 10% of holes?

d) In order to validate the model, data are collected on a sample of 8 holes. The sample has mean diameter 4.012cm and standard deviation 0.022cm. Calculate a 95% confidence interval for the population mean diameter μ and hence test the hypothesis that $\mu = 4$cm.

e) The sample data will also be used to check the assumption that the holes are normally distributed. Sketch the normal probability plot which will be obtained if (i) the normal model is good, and (ii) the normal model is suspect and a lognormal model is more appropriate.

3. Suppose that the maximum annual sea levels at a particular location on the South Coast of England are assumed to follow an EVG1 distribution with parameters $\alpha = 3.6$ and $\beta = 0.8$ (see formula sheet).

a) Calculate the probability that, in any given year, the maximum sea level exceeds 5m. Hence calculate the probability that, in a ten year period, the maximum sea level exceeds 5m, (i) in exactly one of the years, (ii) in exactly two of the years, (iii) in at least one of the years.

b) It is proposed to build sea defences to withstand a specified maximum sea level x for which the probability of exceedance is p. Explain what is meant by the *design life* and *reliability* of a structure, and derive an equation relating p to design life and reliability. Hence show that if the sea defences are required to have a design life of 25 years, with reliability 99%, then $p = 0.0042$.

c) Using the values of α and β given above, calculate the sea level x which has the required exceedance probability $p = 0.0042$

(i) exactly, using the distribution function of the EVG1 distribution.

(ii) approximately, using the fundamental formula for flood control.

4. Two different analytical tests, A and B, can be used to determine the impurity level in steel alloys. The tests were each applied to the same 16 steel specimens and the difference between the results of test A and test B calculated and entered into column C1 in MINITAB. The following descriptive statistics were obtained:

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median
C1	16	0	0.1366	0.0842	0.3369	-0.2889	-0.1112	0.1003

Variable	Q3	Maximum
C1	0.3443	1.0884

a) Draw a boxplot of these data. [You may find it helpful to know the additional information that the second largest value in the sample was 0.4246.]

b) Stating any assumptions you require, calculate a 95% confidence interval for the mean difference between the methods. Hence, test the hypothesis that the two methods are equivalent.

c) Why isn't a two-sample t-test appropriate here?

d) For 10 of the 16 specimens, test A registered a higher impurity level than test B, with test B producing the higher level for the remaining 6. Calculate the probability that, in a random sample of 5 of these 16 specimens, all give a higher measurement with method A.

5. In an investigation of a particular wind turbine, observations of the wind speed (X in kph) and the DC output (Y) were made on 25 consecutive days. It is thought that Y might be linearly related to $1/X$. The following output was produced in MINITAB as part of a regression analysis.

```
The regression equation is
Y = 2.98 - 6.93 1/X

Predictor      Coef  SE Coef       T      P
Constant    2.97886  0.04490   66.34  0.000
1/X         -6.9345   0.2064  -33.59  0.000

S = 0.0941714   R-Sq = 98.0%   R-Sq(adj) = 97.9%

Predicted Values for New Observations

New
Obs     Fit  SE Fit       95% CI            95% PI
  1  1.5920  0.0188  (1.5530, 1.6309)  (1.3933, 1.7906)
```

a) What is the proportion of variation in DC output explained by the regression? Does the regression model fit well?

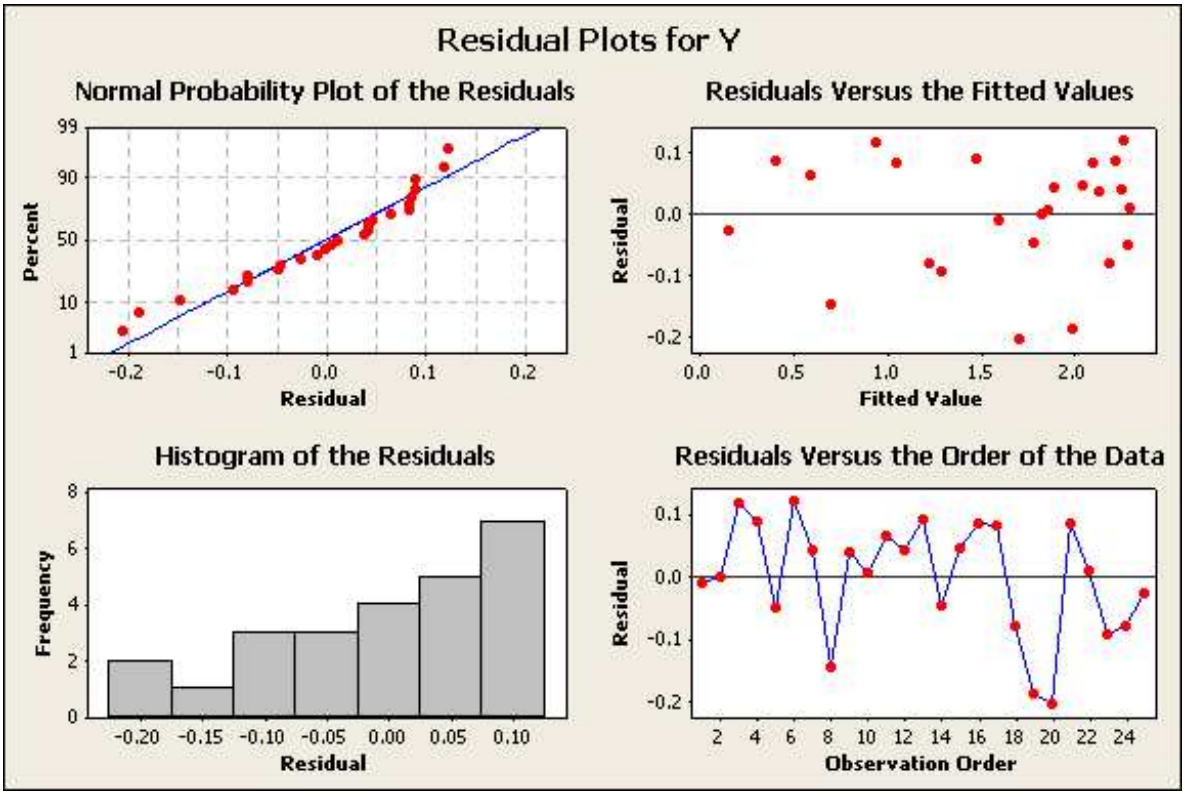
b) Using the regression analysis, what would you estimate to be the mean DC output for a wind velocity of 10kph? At what wind speed is the DC output predicted to be zero? How might you interpret this value and what would you predict the DC output to be for wind speeds less than this?

c) Calculate a 95% confidence interval for the slope of the regression line. Hence test the hypothesis that the the slope is equal to zero. What is the practical significance of this hypothesis?

QUESTION 5 CONTINUED ON NEXT PAGE

d) Calculate the value of wind velocity for which the DC output is predicted in the MINITAB output. Explain how you would interpret the two intervals provided with this prediction.

e) The residual plots below were produced by MINITAB. Explain the relevance of each of these, and whether or not they provide cause for concern about the model.



END OF PAPER

Full marks may be obtained for complete answers to FOUR questions.

Only your best FOUR answers will be taken into account

A copy of formula sheet FS/MATH2019/2005 will be provided.

Graph paper will be provided.

The University Approved Calculator MAY be used