1. Sensor devices are placed on the bed of a river, in order to monitor the current. Initially, the devices are moored remotely, and the probability that this is achieved correctly is 0.75. If a device is not moored correctly, then a diver is sent to correct the mooring, with probability of success 0.90. The reliability of the recording equipment in a sensor is 0.99. A sensor is said to be operational if it is moored correctly, and recording data.

   (a) [7 marks] Use a tree-diagram to show that the probability that a sensor is operational is 0.965.

   (b) [12 marks] Four sensors are placed at a particular location. Calculate the probability that

      (i) all four are operational.

      (ii) at least one is operational.

   (c) [6 marks] Four more sensors are placed at the same location. What is the probability that exactly six of the eight sensors are operational?

2. A random set of 12 steel balls were assessed for the liability of the component steel to surface fatigue failure. As part of this investigation the lifetime of each ball (in minutes) was recorded. The results below were obtained using MINITAB.

```
Variable   N    Mean      SE Mean  StDev  Minimum  Q1      Median
C1         12   132.45    *        8.03   123.00   127.09  131.42
Variable   Q3       Maximum
C1         134.95   151.70
```

   (a) [7 marks] Draw a boxplot to summarise the distribution of the sample. [You may find it helpful to know the additional information that the second largest value in the sample was 142.01.]

   (b) [8 marks] Calculate the number which should appear as $*$ above. Hence, stating any assumptions you require, calculate a 95% confidence interval for the mean of the distribution of lifetime of a randomly selected ball.

   (c) [10 marks] Suppose that the maximum lifetime is modelled by an EVG1 distribution with $\alpha = 100$ and $\beta = 5$. Calculate the probability that the maximum lifetime is greater than 130. Hence, calculate the return period for a lifetime of 130 minutes.

3. The level of ozone concentration at a particular location on a randomly selected day is assumed to be normally distributed with mean 50 parts per billion, ppb and standard deviation 16.4 ppb.

   (a) [5 marks] What is the probability that a randomly selected day has ozone concentration greater than 80 ppb?

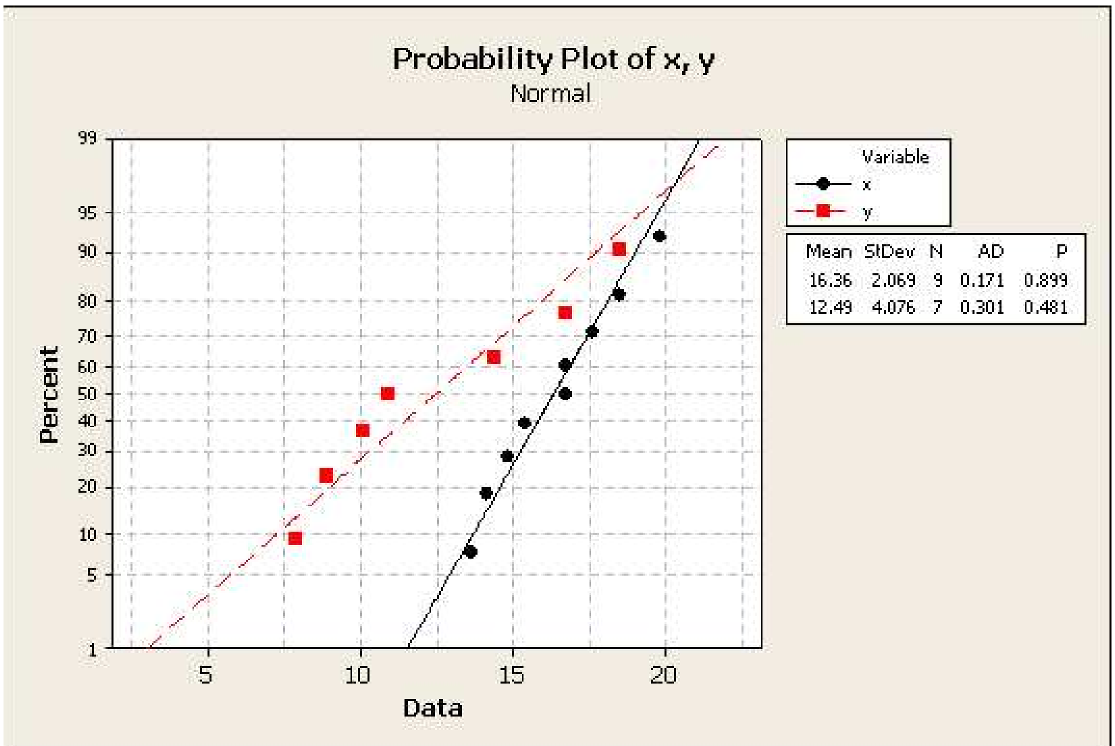
   (b) [5 marks] What proportion of days will have ozone concentration between 51 and 100 ppb?

   (c) [7 marks] What concentration is exceeded by exactly 99% of the days?

   (d) [8 marks] A sample of 16 days is observed to have a mean concentration level of 56.95 ppb and a standard deviation of 23.69 ppb. Using these data, calculate a 95% confidence interval for mean level of ozone concentration and hence determine if the data provide significant evidence to reject the hypothesis that the mean ozone level is equal to 50 ppb.

4. An experiment was carried out to determine failure mechanisms of plasma-sprayed barrier coatings. The failure stress ($\times 10^6$Pa) for one particular coating under two different test conditions produced the following resuts.

Test 1:	n =9, $\bar{x}$ = 16.36, s_x = 2.07
Test 2:	m =7, $\bar{y}$ = 12.49, s_y = 4.08



(a) [8 marks] A normal probability plot of the results from the two test conditions is given above. Does the plot suggest equality of variance for failure stress under the two tests? Explain your answer.

(b) [17 marks] Stating any assumptions you require, calculate a 95% confidence interval for the difference in the mean failure stress under the two tests. Hence, test the hypothesis that there is no difference in mean failure stress.

5. Consider the linear regression model

$$y_i = \alpha + \beta x_i + \epsilon_i \qquad i = 1, \ldots n,$$

for the following data on the fretting wear of mild steel and oil viscosity. Let $x =$ oil viscosity and $y =$ wear volume $(10^{-4} mm^3)$.

y	240	181	193	155	172	110	113	75	94
x	1.6	9.4	15.5	20	22	35.5	43	40.5	33

The following was produced in MINITAB as part of a regression analysis

```
Predictor     Coef   SE Coef      T      P
Constant       *      13.75       *   0.000
x              *     0.4911       *   0.000

S = 19.9570   R-Sq = 87.9%   R-Sq(adj) = 86.2%

Predicted Values for New Observations

New
Obs      Fit  SE Fit        95% CI              95% PI
  1   128.81    7.18  (111.84, 145.79)  (78.66, 178.97)
```

(a) [8 marks] Show that the least squares estimates of α and β are: $\hat{\alpha} = 234.07$ and $\hat{\beta} = -3.5086$.

(b) [4 marks] What is the proportion of variation in wear volume explained by the regression? Does the regression model fit well?

(c) [4 marks] Calculate a 95% confidence interval for the slope of the regression line. [4 marks] Is there a significant relationship between wear volume and oil viscosity?

(d) [9 marks] Calculate the value of oil viscosity for which the wear volume is predicted in the MINITAB output. Explain how you would interpret the two intervals provided with this prediction.

END OF PAPER

SEMESTER 2 EXAMINATION 2006

Statistics for Civil and Environmental Engineering

Duration: 120 mins

Full marks may be obtained for complete answers to FOUR questions.

Only your best FOUR answers will be taken into account.

A copy of formula sheet FS/MATH2019/2006 will be provided.

Graph paper will be provided.

The Official University Calculator MAY be used

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