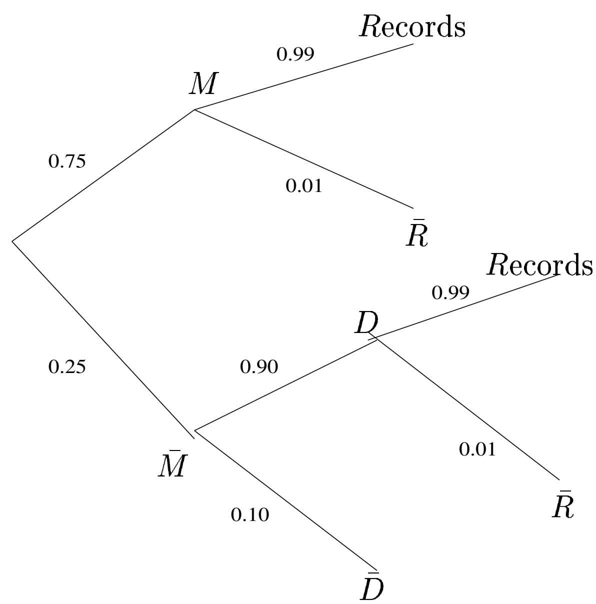
MATH2019–Statistics for Civil and Envrionmental Engineering

Sketch Solutions: 2005-2006

1. Unseen, but similar problem seen



(a) [7 marks] Using the above tree diagram we find that

$$P(\text{A sensor is operational}) = 0.75 \times 0.99 + 0.25 \times 0.90 \times 0.99 = 0.965$$

(b) (i) [6 marks]

$$P(\text{All four are operational}) = (0.965)^4 = 0.868079.$$

(b) (ii) [6 marks]

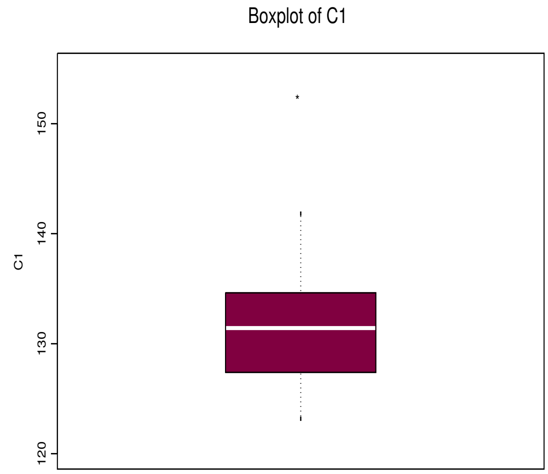
$$\begin{aligned} P(\text{At least one is operational}) &= 1 - P(\text{None is operational}) \\ &= 1 - (1 - 0.965)^4 \\ &= 0.9999985 \end{aligned}$$

(c) [6 marks] The required probability is:

$$\begin{aligned} P(\text{Exactly 6 are operational}) &= \binom{8}{6}(0.965)^6(1 - 0.965)^2 \\ &= 28(0.965)^6(1 - 0.965)^2 \\ &= 0.0273468. \end{aligned}$$

2. Unseen, similar problem done in class

(a) [7 marks] Here $IQR = 134.95 - 127.09 = 7.86$ and $H = 1.5 \times IQR = 11.79$ and $127.09 - 11.79 = 115.30$ and $134.95 + 11.79 = 146.74$ So the whiskers end at 123 and at 142.01. Here is the boxplot.



(b) [8 marks] The SE Mean $= \frac{8.03}{\sqrt{12}} = 2.32$. We assume that the population distribution is normal. Hence, a 95% confidence interval for the mean is given by:

$$132.45 \pm 2.2 \times 2.32 = (127.35, 137.55).$$

(c) [10 marks]

The required probability is

$$\begin{aligned} 1 - P(X \leq x) &= 1 - \exp\left(-e^{\frac{x-\alpha}{\beta}}\right) \\ &= 1 - \exp\left(-e^{\frac{130-100}{5}}\right) \\ &= 1 - 0.9975 = 0.0025 \end{aligned}$$

Now, the return period for a lifetime of $130 = \frac{1}{0.0025} = 400$.

3. Similar problem seen

(a) [5 marks]

$$\begin{aligned} P(\text{Ozone level greater than 80}) &= P(X > 80) \\ &= P\left(Z > \tfrac{80-50}{16.4}\right). \\ &= P\left(Z > 1.83\right) \\ &= 1 - 0.966 = 0.034. \end{aligned}$$

(b) [5 marks]

$$\begin{aligned} P(\text{Ozone level between 51 and 100}) &= P(51 < X < 100) \\ &= P\left(\tfrac{51-50}{16.4} < Z < \tfrac{100-50}{16.4}\right). \\ &= P\left(0.06 < Z < 3.05\right) \\ &= 0.9988 - 0.5239 = 0.4749. \end{aligned}$$

(c) [7 marks] The required concentration level is $= 50 - 2.33 \times 16.4 = 11.78$.

(d) [8 marks] A 95% confidence interval for the mean is given by:

$$56.95 \pm 2.13 \times \frac{23.69}{\sqrt{16}} = (44.33, 69.56).$$

The data do not provide significant evidence (at 5% level of significance) to reject the hypothesis that the mean ozone level is equal to 50 `ppb` since the hypothesised value of the mean, 50 is included in the 95% confidence interval.

4. Unseen, ideas discussed in class

(a) [8 marks] The plot do not suggest equality of variances as the slopes of the two lines are different.

(b) [17 marks] We assume that the two populations are independently normally distributed and they have unequal variances. The 95% confidence interval is given by:

$$\overline{x} - \overline{y} \pm c\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}} = 16.36 - 12.49 \pm 2.3 \times \sqrt{2.07^2/9 + 4.08^2/7)} = (-0.02, 7.76)$$

where $c = 2.3$ is derived from a t distribution with k degrees of freedom and

$$\begin{aligned} k &= \frac{\left(\frac{s_x^2}{n} + \frac{s_y^2}{m}\right)^2}{\frac{(s_x^2/n)^2}{n-1} + \frac{(s_y^2/m)^2}{m-1}} \\ &= 8.39 \approx 8. \end{aligned}$$

Since the 95% confidence interval includes zero we fail to reject the null hypothesis and conclude the data do not provide any significance evidence to reject the hypothesis that there is no difference in mean failure stress.

5. Similar problem done in class

(a) [8 marks] We recall that

$$\hat{\beta} = \frac{\sum_{i=1}^n y_i(x_i - \bar{x})}{\sum_{i=1}^n (x_i - \bar{x})^2} \text{ and } \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}.$$

Here we find that

$$\hat{\beta} = -3.5086 \text{ and } \hat{\alpha} = 234.07.$$

(b) [4 marks] The proportion of variation in wear volume explained by the regression is 87.9% as given by the R^2 statistic. Yes, the regression model is a good fit since it explaining such a high percentage of data variability.

(c) [4 marks] A 95% confidence interval for the slope of the regression line is given by

$$-3.5086 \pm 0.4911 \times 2.365 = (-4.67, -2.35).$$

There a significant relationship between wear volume and oil viscosity since the 95% confidence interval excludes the value zero.

(d) [9 marks] The value of oil viscosity for which the wear volume is 128.81 is 30. The 95% CI given by (111.84, 145.79) is the 95% confidence interval for the mean wear volume at oil viscosity equal to 30 whereas the wider 95% PI given by (78.66, 178.97) is the predictive interval for a new observation (wear volume) at the same oil viscosity.