How to Model Spatial Inhomogeneity

Draft by Paul Blackwell, using notes by Chris Glasbey

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This is a summary of a roundtable discussion at the Workshop on Recent Advances in Modelling Spatio-Temporal Data, University of Southampton, May 2005 (see `http://www.maths.soton.ac.uk/s3riwshop/`).

List of discussants (unverified): Paul Blackwell, Caitlin Buck, Richard Chandler, Ian Dryden, Carmen Fernandez, Chris Glasbey, Keith Harris, John Haslett, Chris Holmes, Ron Smith, Olivier Thas, Verena Trenkel, Christina Yap, Kamila Żychaluk.

1 Empirical methods

Deformation of space, or space-time. The classic reference is Sampson & Guttorp (1992); a Bayesian variant is given by O'Hagan & Schmidt (2003).

Spatially varying kernel methods. If a dependent process is formed by kernel smoothing of some underlying independent random field, then the use of a spatially varying kernel leads to inhomogeneity in the dependent process. See Higdon et al. (1999).

Splines: I have a note about the work of Matt Wand, Harvard/Macquarie, on splines/kriging; I'm not sure exactly what the point was.

2 Use of Covariates

The use of covariates builds into a model information that is important or useful in its own right; also, spatial inhomogeneity in the covariates induces inhomogeneity in the response i.e. the variable primarily being modelled.

3 Discontinuities

Some approaches explicitly build spatial discontinuities into the variable being modelled. One way is to partition the space into "tiles", defined for example by a Voronoi (a.k.a. Dirichlet or Thiessen) tessellation (Denison et al., 2002) or a generalization (Blackwell & Møller, 2003). Different tiles may have different levels or, for the appropriate types of process, correlation structures or intensity functions.

Such models also give a way of modelling *smooth* heterogeneity, by mixing over different parameters and/or different tilings.

4 Mechanistic Modelling

Mechanistic models have some level of internal structure that is interpretable in terms of the underlying science.

One category involves taking standard spatial models in which properties are distance-based, and redefining distance in a way that is scientifically meaningful. For example, the distance between two points may depend on the local topography (c.f. Sampson & Guttorp 1992). Distances may be limited to lengths of paths through some network, e.g. a river network (Campbell et al., 2002). The 'distances' may be abstract, and separate from spatial location, e.g. distances in a social network, or distances in genetic space.

Spatial heterogeneity can also occur in fully mechanistic models, whether inherently stochastic or defined through sets of differential equations.

References

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