SEMESTER 1 EXAMINATION 2020 - 2021

DIGITAL CODING AND TRANSMISSION

DURATION 240 MINS (Four Hours)

This paper contains 6 questions

Answer All the SIX questions

An outline marking scheme is shown in brackets to the right of each question.

This is an open-book exam

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper.

Section A

Question A1.

(a) In this course, you have learnt both the information theory and practical video source encoding techniques.

(i) According to information theory, what is the basic principle adopted in video source encoding techniques?

(ii) You have designed a video source coder. The output bit sequence of your video source coder contains 6.4067×10^{10} bits, in which the bit $\mathbf{0}$ has a 0.999 probability of occurrance and the bit $\mathbf{1}$ has a 0.001 probability of occurrance.

You decide to encode this bit sequence by the run length encoder (RLC) with a codeword length of $n = 10$ bits. What compression ratio can be achieved? Calculate the length of the bit sequence after the RLC encoding.

[4 marks]

(b) You are in charge of a mobile communication company. One of your engineers submits a 64 kbps speech coder design for the handset. In this 64 kbps speech coder design, the speech waveform is first filtered to limit its bandwidth to approximately 3.8 kHz and a sampling rate of 8 kHz is then applied. The resultant samples are first passed through an A-law compressor and the compressed samples are then quantised to 256 levels with each sample being represented by 8 bits.

(i) Why does the engineer apply the A-law compressor to speech samples?

(ii) Will you accept or reject this speech coder design? Explain your decision according to the information theory you have learnt.

[4 marks]

(c) A memoryless digital source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table Q-A1, at a rate $R_\mathrm{s} = 10^7$ Baud (Baud=symbol/s).

Table Q-A1.

X_i	$P(X_i)$	BCD word
A	0.27	000
B	0.20	001
C	0.17	010
D	0.16	011
E	0.07	100
F	0.06	101
G	0.04	110
H	0.03	111

(i) What is the data rate of the uncoded BCD signal and what is the information rate of this source?

(ii) Apply Shannon-Fano coding to the digital source characterised in Table Q-A1.

(iii) What is the original symbol sequence of the Shannon-Fano coded signal 0001100111111101101 (The left most bit is the first bit of the coded signal)?

[7 marks]

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Question A2.

(a) A binary symmetric channel (BSC) is depicted in Figure Q-A2,

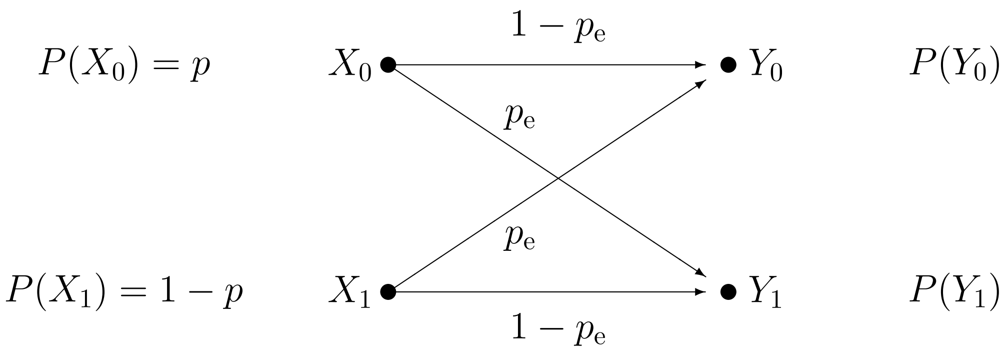

Figure Q-A2.

(i) For what values of p and p_e are the source and destination entropies, $H(X)$ and $H(Y)$, identical? In general, is $H(Y) \geq H(X)$ or $H(Y) \leq H(X)$ true? Justify your answer.

(ii) Given $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$, calculate all the probabilities $P(X_i, Y_j)$ as well as $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X, Y)$.

[8 marks]

(b) State the famous Shannon-Hartley channel capacity formula, and define clearly each variable in the formula. According to this channel capacity formula, what are the two basic resources for digital communications? Please plot the channel capacity normalized to the bandwidth versus the Signal-to-Noise Ratio (SNR) in dB. Furthermore, please discuss, when it can and cannot be used.

[3 marks]

(c) A digital communication system uses a 4-ary signalling scheme with the transmission rate of 10^8 [symbols/s]. The probabilities of occurrence for the four symbols at the transmitter are respectively

$$P(X_1) = 0.2, \;\; P(X_2) = 0.3, \;\; P(X_3) = 0.2, \;\; P(X_4) = 0.3.$$

It is known that this 4-ary symbol source is a first-order Markov process with the known transition probability matrix

$$\mathbf{\Gamma} = [p_{i,j}] = \begin{bmatrix} 0.6 & 0.1 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.1 & 0.2 \\ 0.1 & 0.1 & 0.8 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.8 \end{bmatrix},$$

where $p_{i,j} = P(X_j|X_i)$, $1 \leq i, j \leq 4$, are the transition probabilities.

The channel is an ideal additive white Gaussian noise (AWGN) channel and the channel's signal to noise ratio is known to be 63.

Determine the minimum channel bandwidth required to achieve error-free transmission.

How would the bandwidth change, if the SNR is increased to 255?

[4 marks]

Question A3.

(a) Figure Q-A3 depicts a digital communication modem.

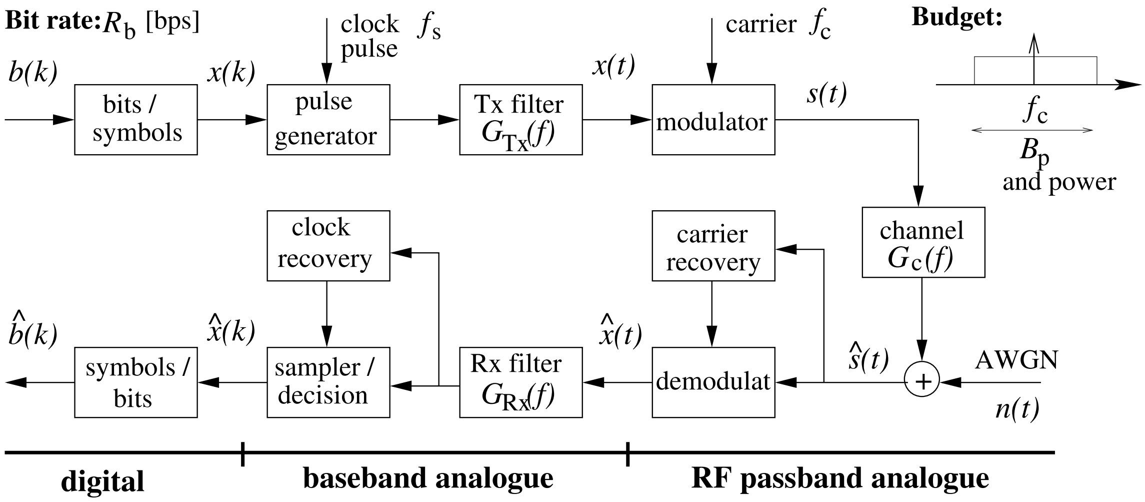

Figure Q-A3.

(i) Briefly explain the two key objectives that the transmit and receive pulse shaping filter pair are designed to achieve.

(ii) Briefly explain the function of the carrier recovery circuit at the receiver. Why is it necessary to obtain an accurate carrier recovery?

(iii) Briefly explain the function of the clock recovery circuit at the receiver. Why is it necessary to obtain accurate timing information?

[12 marks]

(b) A communication channel has a passband bandwidth $B_p = 110$ MHz. The combined transfer function of the transmit and receive filters has a raised-cosine characteristic with a roll-off factor of 0.1. Design a modulation scheme so that you can transmit at the bit rate of $R_b = 400$ Mbits/s over this channel.

[3 marks]

(c) If the channel is dispersive, what is the additional device required at the receiver? With the aid of a schematic diagram, clearly explain the purpose of this additional device as well as its design trade offs.

[5 marks]

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Section B

Question B1.

(a) You are tasked with designing a high-resolution video codec for telemedicine applications, where 10 bit/pixel gray-scale resolution is required and the video frame size is 1000 x 1000 pixels. Calculate the uncompressed bitrate required at a frame-scanning rate of 50 frames/s. Then list the bitrate reduction techniques known to you.

[2 marks]

(b) Draw the schematic of this video codec and explain its operation, assuming that it uses motion vector search and motion compensation.

[3 marks]

(c) Explain, how the motion vectors (MV) are calculated, discussing the video quality, bitrate and complexity trade-offs of **1**/ different search scope sizes; **2**/ different block sizes; **3**/ different MV search.

[3 marks]

(d) Calculate the total number of bits required for the transmission of the MVs for the entire frame, if the motion-compensated blocks have a size of 10 x 10 pixels and the scope of the motion-search is [-4 ... +3] in both the horizontal and vertical directions.

[2 marks]

(e) Calculate the total number of multiplications and additions per video frame required by the motion-compensation, when carrying out a full search over every position of the entire search-scope of [-4 ... +3] pixel positions.

[2 marks]

(f) Compare the resultant computational complexity to that, when using a subsampled search, visiting every other pixel position and contrast the design trade-offs of full-scale vs. subsampled MV-search.

[2 marks]

(g) Explain, how vector quantisation (VQ) of the MCER is carried out. Then, assuming that a 256-entry VQ codebook is used for encoding the 10 x 10 pixel MCER blocks, calculate the total number of bits required for encoding the MCER.

[2 marks]

(h) Based on the above-mentioned 10 x 10 pixel blocks, calculate the total bitrate generated at 50 frames/s, when relying on full-scale MV-aided motion compensation.

[2 marks]

(i) Then recalculate the total bitrate, if simple frame-differencing is used instead of MV-aided motion compensation. Contrast these solutions in terms of their video quality, bit-rate, complexity etc.

[2 marks]

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Question B2.

(a) Sketch the block diagram of a Differential Pulse Code Modulated (DPCM) speech codec and explain the operation of each constituent block with the aid of time- and/or frequency-domain sketches.

[3 marks]

(b) Derive an equation for the predictor coefficient value of the optimum one-tap predictor.

[2 marks]

(c) Assuming that the one-step correlation of the input speech signal is known, derive an expressions for the prediction gain achieved by the one-tap predictor.

[3 marks]

(d) Substitute $r = 0.8$ in the expression derived and calculate the prediction gain in dB.

[2 marks]

(e) Sketch the schematic of the low-rate Vocoder and explain, how it exploits the specific characteristics of voiced and unvoiced speech with reference to their stylised Autocorrelation Function as well as their Power Spectral Density.

[3 marks]

(f) Discuss the pros and cons of opting for a DPCM versus a Vocoder scheme in terms of the associated design trade-offs. [2 marks]

Question B3.

(a) Given the generator polynomial of $g(z) = z^4 + z + 1$, draw the corresponding BCH[15,11,3] encoder's schematic.

[3 marks]

(b) Describe the operation of the BCH[15,11,3] by a numbered list of actions.

[2 marks]

(c) Assuming that the original information stream to be BCH[15,11,3]-encoded is $i(x) = 1 + x + x^4 + x^5 + x^6 + x^{10}$, create a table having the following four columns and 15 rows: Clock Index; Input Bit; Shift-register State; Output Bit; Then list the bits in all four colums and 15 rows of the table, as the shift-register is clocked 15 times.

[5 marks]

(d) Draw the state transition diagram of the BCH[15,11,3] encoder;

[5 marks]

END OF PAPER