SEMESTER 1 EXAMINATION 2021 - 2022

DIGITAL CODING AND TRANSMISSION

DURATION 150 MINS (2.5 Hours)

This paper contains 6 questions

Answer FOUR questions, TWO from **Section A** and TWO from **Section B**.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper.

Section A

Question A1.

(a) In this course, you have learnt both the information theory and practical video source encoding techniques.

(i) According to the information theory, what is the basic principle adopted in video source encoding techniques?

(ii) You have designed a video source coder. The output bit sequence of your video source coder contains 6.4067×10^{12} bits, in which the bit $\mathbf{0}$ has the probability of occurring 0.999 and the bit $\mathbf{1}$ has the probability of occurring 0.001.

You decide to encode this bit sequence by the run length encoder (RLC) with a codeword length of $n = 10$ bits. What compression ratio can be achieved? Calculate the length of the bit sequence after the RLC encoding.

(iii) The RLC of the above video source coder produces the following bit sequence (The left most bit is the first bit of the sequence):

$$0000000000\ 1111111111\ 1111111110\ 1111111101\ \cdots$$

What is the input bit sequence to the RLC?

[8 marks]

(b) Two digital sources have: 1) The same source alphabet or symbol set of $\mathcal{S} = \{m_i, 1 \le i \le q\}$; 2) The same symbol probabilities of occurrence $\mathcal{P} = \{p_i, 1 \le i \le q\}$, with p_i being the probability of m_i occurring; 3) The same symbol rate of R_{s} [symbols/s]; and 4) One of the digital sources is memoryless, and the other has memory.

(i) Which digital source has a higher entropy? Furthermore, which digital source has a higher information rate?

(ii) What is the best strategy of encoding memoryless digital sources and why?

(iii) What is the best strategy of encoding the digital sources with memory and why?

[9 marks]

(c) A memoryless digital source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table Q-A1, at a rate $R_\mathrm{s} = 10^8$ Baud (Baud=symbol/s).

Table Q-A1.

X_i	$P(X_i)$	BCD word
A	0.27	000
B	0.20	001
C	0.17	010
D	0.15	011
E	0.07	100
F	0.06	101
G	0.05	110
H	0.03	111

(i) What is the data rate of the uncoded BCD signal and what is the information rate of this source?

(ii) Apply Huffman coding to the digital source characterised in Table Q-A1.

(iii) What is the original symbol sequence of the Huffman coded signal 0111000101110101001? (The left most bit is the first bit of the coded signal)

[8 marks]

TURN OVER

Question A2.

(a) A binary symmetric channel (BSC) is depicted in Figure Q-A2,

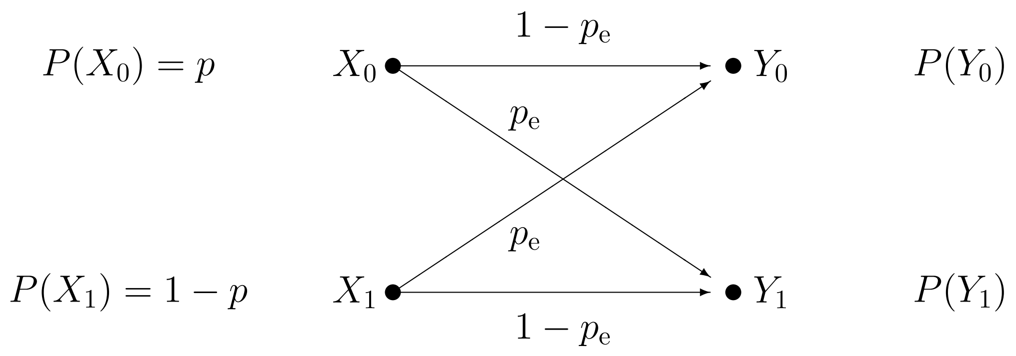

Figure Q-A2.

(i) For what values of p and p_e are the source and destination entropies, $H(X)$ and $H(Y)$, identical? In general, is $H(Y) \geq H(X)$ or $H(Y) \leq H(X)$ true? Justify your answer.

(ii) Given $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$, calculate all the probabilities $P(X_i, Y_j)$ as well as $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X, Y)$.

(iii) The source of this BSC of $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$ emits the binary symbols with the rate of 10^7 Baud (Baud=symbol/s). What is the maximum achievable rate of this BSC for achieving error-free transmission?

[10 marks]

(b) (i) State the famous Shannon-Hartley channel capacity formula, and define clearly each variable in the formula. According to this channel capacity formula, what are the two basic resources for digital communications?

(ii) The signal-to-noise ratio (SNR) of a wireless channel is $\mathrm{SNR} = 63$, and the bandwidth of this wireless channel is $10\,$MHz. What is the capacity of this channel?

[5 marks]

(c) A digital communication system uses an 8-ary signalling scheme with the transmission rate of 100 MBaud (10^8 [symbols/s]). The probabilities of occurrence for the eight symbols at the transmitter are respectively

$$P(X_1) = 0.2,\ P(X_2) = 0.1,\ P(X_3) = 0.1,\ P(X_4) = 0.1,$$
$$P(X_5) = 0.1,\ P(X_6) = 0.1,\ P(X_7) = 0.1,\ P(X_8) = 0.2.$$

It is known that this 8-ary symbol source is a first-order Markov process with the known transition probability matrix

$$\mathbf{\Gamma} = [p_{ij}] = \begin{bmatrix} 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 \\ 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 & 0.2 & 0.2 \end{bmatrix},$$

where $p_{ij} = P(X_j|X_i)$, $1 \leq i, j \leq 8$, are the transition probabilities.

(i) Determine the information rate of the source at the transmitter.

(ii) The channel is an ideal additive white Gaussian noise (AWGN) channel, and the channel's signal to noise ratio is known to be 127. Determine the minimum channel bandwidth required to achieve error-free transmission.

[10 marks]

TURN OVER

Question A3.

(a) Figure Q-A3 depicts schematic of digital communication modem.

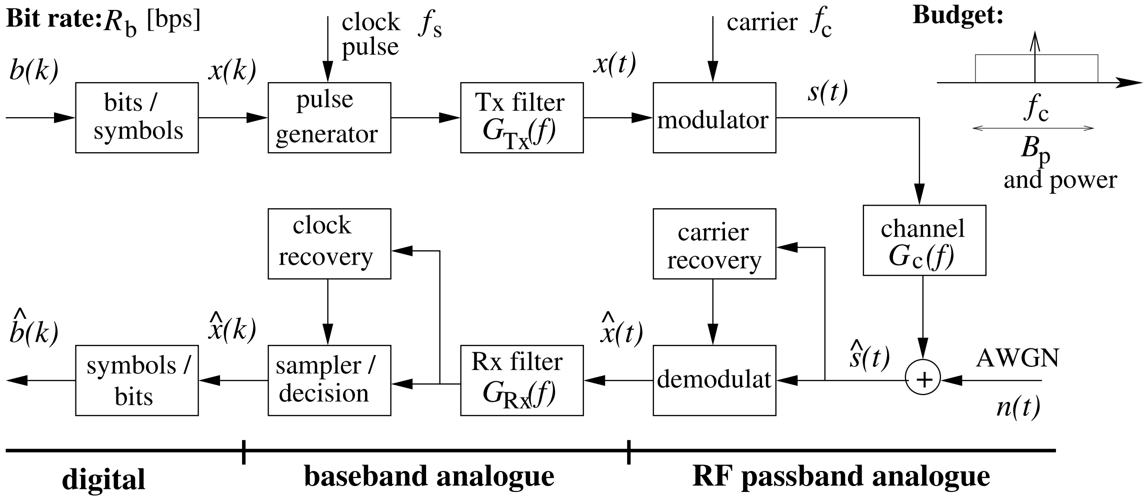

Figure Q-A3.

(i) Briefly explain the purpose or design framework of a modem in the context of wireless communication.

(ii) Briefly explain the two key objectives that the transmit and receive pulse shaping filter pair are designed to achieve.

(iii) Briefly explain the function of the carrier recovery circuit at the receiver. Why is it necessary to have accurate carrier recovery?

(iv) Briefly explain the function of the clock recovery circuit at the receiver. Why is it necessary to obtain an accurate timing information?

[15 marks]

(b) (i) An additive white Gaussian noise (AWGN) channel has a passband bandwidth $B_p = 110$ MHz. The combined transfer function of the transmit and receive filters has a raised-cosine characteristic with a roll-off factor of 0.1. Design a modulation scheme so that you can transmit at the bit rate of $R_b = 400$ Mbits/s over this channel.

(ii) If the channel is a fading channel, what will be your choice of modulation scheme?

[5 marks]

(c) If the channel is dispersive, what is the additional device required at the receiver? With the aid of a schematic diagram, clearly explain the purpose of this additional device as well as its design trade offs.

[5 marks]

TURN OVER

Section B

Question B1.

(a) Please draw the schematic of a video codec relying on block-based motion-compensation as well as motion vectors (MV) and explain the operation of all constituent blocks.

[5 marks]

(b) Assuming that 8 bit/pixel gray-scale resolution is required and the video frame size is 400 x 500 pixels, please calculate the uncompressed stereoscopic bitrate at a frame-scanning rate of 50 frames/s.

[2 marks]

(c) Calculate the total number of bits required for the transmission of the MVs for the entire frame, if the motion-compensated blocks have a size of 10 x 10 pixels and the motion-search scope is [-4 ... +3] in both the horizontal and vertical directions. Then evaluate the total stereoscopic bitrate.

[3 marks]

(d) Calculate the total number of multiplications and additions per video frame required by the motion-compensation using the mean-squared error (MSE) matching criterion, when carrying out a full search over every position of the entire search-scope of [-4 ... +3] pixel positions. Then contrast this complexity to that when using the mean absolut difference (MAD) criterion.

[3 marks]

(e) Now compare the resultant computational complexity to that, when using tree-search based motion compensation and recalculate the bitrate and the complexity. Then discuss the design trade-offs of full-scale vs. tree-search based motion compensation.

[3 marks]

(f) Please assume that 2bit/sample differential pulse code modulation (DPCM) is used for quantizing the motion compensated error residual (MCER) and calculate the total number of bits required for encoding the MCER in a frame. Then calculate the total bitrate of this stereoscopic video codec using full-scale motion compensation and 2bit/sample DPCM MCER coding at 50 frames/s.

[3 marks]

(g) Please proceed by recalculating both the number of MCER encoding bits/frame using 20 bits/block quad-tree (QT) coding and the total stereoscopic bitrate at 50 Frames/s employing full-scale motion compensation.

[3 marks]

(h) Finally, recalculate the total stereoscopic bitrate under the assumption that only 10 % of the blocks are motion-active and 10 % of them are MCER-active, accounting for any additional side-information requirements.

[3 marks]

TURN OVER

Question B2.

(a) You are tasked with the characterization of a speech codec's signal-to-noise ratio (SNR) under the assumption that the input signal is uniformly distributed across the linear quantiser's entire dynamic range and no quantiser overload is encountered. Please derive an equation for the input signal's variance.

[5 marks]

(b) Furthermore, under the assumption that the quantization error $e(n)$ is also uniformly distributed across the quantization interval, derive a formula for the variance, ie. the power of $e(n)$.

[4 marks]

(c) Based on these two formulae, express the speech codec's SNR in terms of dB as a function of the number of quantization bits b.

[4 marks]

(d) Draw the schematic of an analysis-by-synthesis (ABS) speech codec and detail the task of each of its constituent blocks with the aid of sketching both the short-term and long-term residuals.

[4 marks]

(e) Formulate an equation for the average mean-square short-term residual as a function of a single predictor tap, followed by deriving the formula of the optimal one-tap predictor coefficient.

[4 marks]

(f) Formulate an equation for the prediction gain attained by this optimal one-tap predictor and calculate its numerical value in dB, when the adajent sample correlation is as high as 0.8.

[4 marks]

Question B3.

(a) Given the generator polynomials of $g_1 = [1, 0, 1]$, $g_2 = [1, 1, 0]$ and $g_3 = [1, 1, 1]$, draw the corresponding convolutional encoder's schematic.

[4 marks]

(b) Draw the encoder's state transition diagram, indicating the resultant encoded bits for all transitions.

[7 marks]

(c) Consider the received sequence of $\mathbf{001}$, 010, 000, 100, 000, 000, where the left-most three-bit codeword printed in bold font appears at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

[7 marks]

(d) Consider the soft-decision demodulator output sequence of $+\mathbf{2}+\mathbf{1}-\mathbf{1}$, $-3-4-2$, $-3+2-1$, $-4-3-2$, $-1-1-2$, $-3-4-3$, where the left-most three soft demodulator output values printed in bold font appear at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

[7 marks]

END OF PAPER