SEMESTER 1 EXAMINATION 2022 - 2023

DIGITAL CODING AND TRANSMISSION

DURATION 150 MINS (2.5 Hours)

This paper contains 6 questions

Answer FOUR questions, TWO from **Section A** and TWO from **Section B**.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper.

Section A

Question A1.

(a) You are in charge of a team of engineers to design a handset for the fifth generation mobile network and a team member submits the proposal of a 64 kbps speech coder design for the handset. In this 64 kbps speech coder design, the speech waveform is first filtered to limit its bandwidth to approximately 3.8 kHz and a sampling rate of 8 kHz is then applied. The resultant samples are first passed through an A-law compressor and the compressed samples are then quantised to 256 levels with each sample being represented by 8 bits.

(i) Explain the reason of applying the A-law compressor.

(ii) Will you accept or reject this speech coder design? Explain your decision clearly according to the information theory you have learnt in this course.

[7 marks]

(b) In this course, you have learnt both the information theory and practical video source encoding techniques.

(i) According to the information theory, what is the basic principle adopted in video source encoding techniques?

(ii) You have designed a video source coder. The output bit sequence of your video source coder contains 6.4067×10^{12} bits, in which the bit $\mathbf{0}$ has the probability of occurring 0.999 and the bit $\mathbf{1}$ has the probability of occurring 0.001.

You decide to encode this bit sequence by the run length encoder (RLC) with a codeword length of $n = 10$ bits. What compression ratio can be achieved? Calculate the length of the bit sequence after the RLC encoding.

(iii) The RLC of the above video source coder produces the following bit sequence (The left most bit is the first bit of the sequence):

$$1111111101\ 0000000000\ 1111111111\ 1111111110\ 1111111101\cdots$$

What is the input bit sequence to the RLC?

[9 marks]

(c) A memoryless digital source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table Q-A1, at a rate $R_{\mathrm{s}} = 10^8$ Baud (Baud=symbol/s).

Table Q-A1.

X_i	$P(X_i)$	BCD word
A	0.27	000
B	0.20	001
C	0.17	010
D	0.16	011
E	0.07	100
F	0.06	101
G	0.04	110
H	0.03	111

(i) What is the data rate of the uncoded BCD signal? What is the source entropy and what is the information rate of this source?

(ii) Apply Shannon-Fano coding to the digital source characterised in Table Q-A1.

(iii) Derive the coding efficiency of both the uncoded BCD signal and the Shannon-Fano coded signal.

[9 marks]

TURN OVER

Question A2.

(a) A binary symmetric channel (BSC) is depicted in Figure Q-A2,

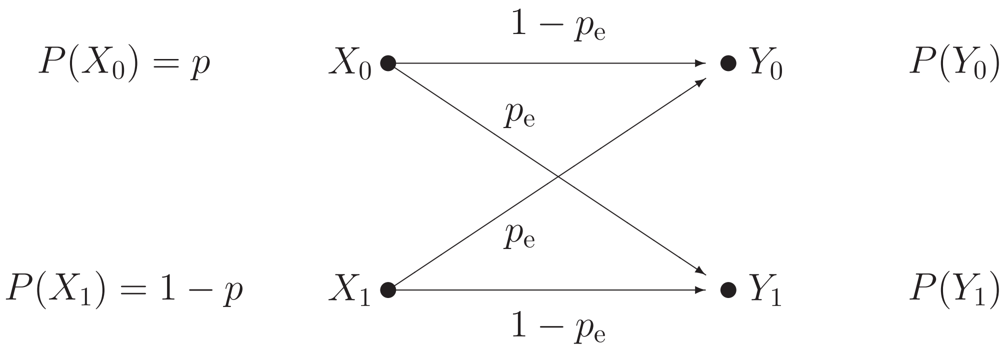

Figure Q-A2.

where the source is transmitting at the bit rate of R_b [bits/second], and its channel capacity can be defined as the maximum of the average mutual information $I(X, Y)$ given by

$$C = \max\{I(X, Y)\} \qquad \text{[bits/symbol]}$$

(i) Derive the channel capacity of this BSC in terms of [bits/second]. You should express the capacity as the function of the bit rate R_b and the channel error probability p_e.

(ii) What are the best-case and worst-case channel capacities of this BSC in terms of [bits/second]?

[10 marks]

(b) (i) State the famous Shannon-Hartley channel capacity formula, and define clearly each variable in the formula.

According to this channel capacity formula, what are the two basic resources for digital communications? Discuss how you can trade off these two resources to achieve the required channel capacity.

(ii) The signal-to-noise ratio (SNR) of a wireless channel is $\text{SNR} = 63$, and the bandwidth of this wireless channel is $10\,$MHz. What is the capacity of this channel?

[6 marks]

(c) A digital communication system uses an 8-ary signalling scheme with the transmission rate of 100 MBaud (10^8 [symbols/s]). The probabilities of occurrence for the eight symbols at the transmitter are respectively

$$P(X_1) = 0.2,\ P(X_2) = 0.1,\ P(X_3) = 0.1,\ P(X_4) = 0.1,$$
$$P(X_5) = 0.1,\ P(X_6) = 0.1,\ P(X_7) = 0.1,\ P(X_8) = 0.2.$$

It is known that this 8-ary symbol source is a first-order Markov process with the known transition probability matrix

$$\mathbf{\Gamma} = [p_{ij}] = \begin{bmatrix} 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 3.0 \\ 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 & 0.2 & 0.2 \end{bmatrix},$$

where $p_{ij} = P(X_j|X_i)$, $1 \leq i, j \leq 8$, are the transition probabilities.

(i) Determine the information rate of the source at the transmitter.

(ii) The channel is an ideal additive white Gaussian noise (AWGN) channel, and the channel's signal to noise ratio is known to be 127. Determine the minimum channel bandwidth required to achieve error-free transmission.

[9 marks]

TURN OVER

Question A3.

(a) Figure Q-A3 depicts schematic of digital communication modem.

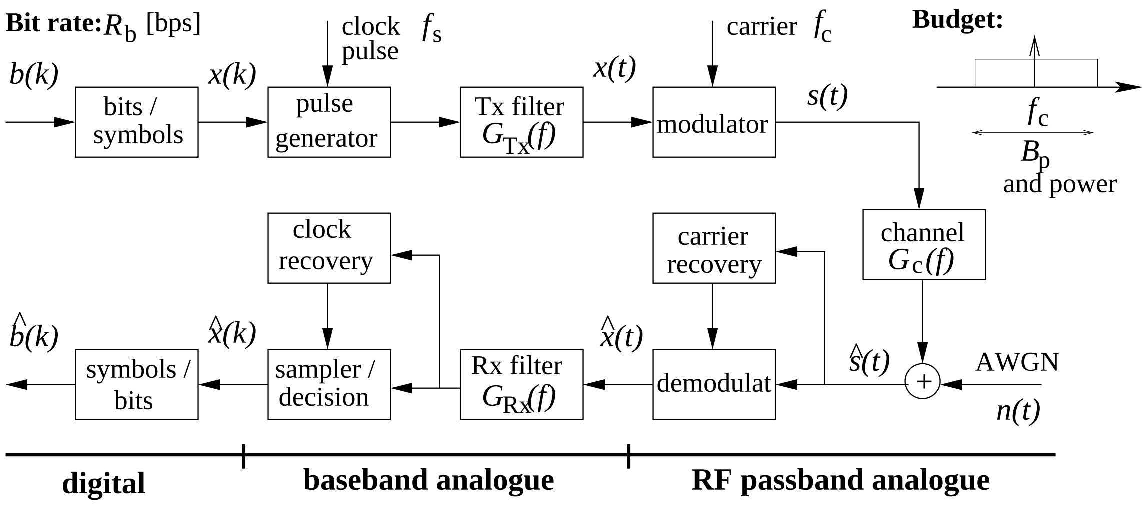

Figure Q-A3.

(i) Briefly explain the purpose or design framework of a modem in the context of wireless communication.

(ii) Briefly explain the two key objectives that the transmit and receive pulse shaping filter pair are designed to achieve.

(iii) Briefly explain the function of the carrier recovery circuit at the receiver. Why is it necessary to have accurate carrier recovery?

(iv) Briefly explain the function of the clock recovery circuit at the receiver. Why it is necessary to obtain an accurate timing information?

[15 marks]

(b) Receiver with carrier recovery is called coherent receiver.

(i) With the key signal equations, explain how the transmitter and receiver operate based on the differential phase shift keying (DPSK).

(ii) What is the assumption made in order for this DPSK based design to operate properly?

(iii) Compared with the coherent design, what are the advantage gained as well as the penalty paid by this DPSK design?

[10 marks]

TURN OVER

Section B

Question B1.

(a) You are asked to design a high-resolution video codec for gaming applications relying on 1000 x 2000 pixels/frame and 10 bits/pixel gray-scale representation as well as a 100 frames/sec frame scanning rate. Please calculate the associated bitrate without the employment of any source-compression.

[2 marks]

(b) Please draw the schematic of a suitable encloder/decoder pair using low-complexity frame-differencing and recalculate the bitrate by assuming that the frame-difference signal is limited to the range of [-8 ... +7] 99 % of the time.

[3 marks]

(c) Explain the concept of motion-compensation as well as motion vectors (MV) and incorporate them into the encoder/decoder schematic. Describe the operation of all constituent blocks with the aid of carefully labelled sketches.

[4 marks]

(d) Calculate the total number of bits required for the transmission of the MVs for a video frame, if the motion-compensated blocks have a size of 20 x 20 pixels and the motion-search scope is [-4 ... +3] in both the horizontal and vertical directions. Then evaluate the total bitrate assuming that the motion-compensated error residual (MCER) is limited to the range of [-8 ... +7] 99 % of the time.

[4 marks]

(e) Let us now introduce vector quatization (VQ) for the 20 x 20 MCER blocks and assume that its codebook has 1024 blocks. Recalculate the total motion-compensated bitrate.

[3 marks]

(f) Calculate the total number of multiplications and additions per video frame required by the motion-compensation using the mean-squared error (MSE) matching criterion, when carrying out a full search over every position of the entire search-scope of [-4 ... +3] x and y pixel positions. Then contrast this complexity to that when using the mean absolut difference (MAD) criterion.

[3 marks]

(g) Now compare the resultant computational complexity to that, when the motion compensation only considers every other pixel-position and recalculate both the bitrate and the complexity. Then discuss the design trade-offs of full-scale vs. subsampled motion compensation.

[3 marks]

(h) Finally, recalculate the total bitrate under the following assumptions:

- full-scale motion-search is used without any subsampling;
- the MCER is compressed using the above-mentioned 1024-entry VQ codeboook;
- only 10 % of the blocks are motion-active and 10 % of them are MCER-active;

Please account for any additional side-information requirements in your calculations owing to the active/passive classification.

[3 marks]

TURN OVER

Question B2.

(a) Please draw the stylized speech-quality vs. bit-rate plane of the speech codecs known to you and highlight their engineering trade-offs.

[3 marks]

(b) Draw the schematic of a vocoder and describe its operation with the aid of carefully labelled time- and frequency-domain signal plots, paying particular attention to the classification and representation of the different classes of input speech.

[5 marks]

(c) Draw the schematic of an analysis-by-synthesis (ABS) speech codec and detail the task of each of its constituent blocks with the aid of sketching both the short-term and long-term residuals. Please detail the differences between vocoders and ABS codecs.

[8 marks]

(d) Formulate an equation for the average mean-square short-term residual as a function of a single predictor tap, followed by deriving the formula of the optimal one-tap predictor coefficient based on minimizing the mean-squared prediction error.

[5 marks]

(e) Now formulate an equation for deriving the optimal long-term prediction gain by minimizing the mean-squared long-term prediction residual.

[4 marks]

Question B3.

(a) Given the generator polynomial of $g(z) = z^4 + z + 1$, draw the corresponding BCH[15,11,3] encoder's schematic.

[3 marks]

(b) Describe the operation of the BCH[15,11,3] encoder by a numbered list of actions.

[4 marks]

(c) Assuming that the original information stream to be BCH[15,11,3]-encoded is $i(x) = 1 + x + x^4 + x^5 + x^6 + x^{10}$, please create a table having the following four columns and 15 rows: Clock Index; Input Bit; Binary Shift-register State; Output Bit; Then list the bits in all four colums and 15 rows of the table, as the shift-register is clocked 15 times.

[10 marks]

(d) Draw the state transition diagram of the BCH[15,11,3] encoder;

[8 marks]

END OF PAPER