SEMESTER 1 EXAMINATIONS 2023 - 2024

DIGITAL CODING AND TRANSMISSION

DURATION 150 MINS (2.5 Hours)

This paper contains 6 questions

Answer FOUR questions, TWO from **Section A** and TWO from **Section B**.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct ‘Word to Word’ translation dictionary AND it contains no notes, additions or annotations.

10 page examination paper.

Section A

Question A1.

(a) A 64 kbps speech coder was designed in 1970s. In this 64 kbps speech coder design, the speech waveform was filtered to limit its bandwidth to approximately 3.8 kHz and a sampling rate of 8 kHz was then applied. The resultant samples were first passed through an A-law compressor and the compressed samples were then quantised to 256 levels with each sample being represented by 8 bits.

(i) Will you use this speech coder design for mobile handset? Explain your answer clearly according to the information theory you have learnt in this course.

(ii) What is the basic principle adopted in today's speech source encoding techniques?

[6 marks]

(b) (i) According to the information theory, what is the basic principle adopted in video source encoding techniques?

(ii) You have designed a video source coder. The output bit sequence of your video source coder contains 6.4067×10^{14} bits, in which the bit $\mathbf{0}$ has the probability of occurring 0.999 and the bit $\mathbf{1}$ has the probability of occurring 0.001.

You decide to encode this bit sequence by the run length encoder (RLC) with a codeword length of $n = 10$ bits. What compression ratio can be achieved? Calculate the length of the bit sequence after the RLC encoding.

(iii) The RLC of the above video source coder produces the following bit sequence (The left most bit is the first bit of the sequence):

$$1111111101\ 0000000000\ 1111111111\ 1111111110\ 1111111101 \cdots$$

What is the input bit sequence to the RLC?

[9 marks]

(c) A memoryless digital source emits symbols X_i, $1 \leq i \leq 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table Q-A1, at a rate $R_\mathrm{s} = 10^8$ Baud (Baud=symbol/s).

Table Q-A1.

X_i	$P(X_i)$	BCD word
A	0.27	000
B	0.20	001
C	0.17	010
D	0.15	011
E	0.07	100
F	0.06	101
G	0.05	110
H	0.03	111

(i) What is the data rate of the uncoded BCD signal and what is the information rate of this source?

(ii) Apply Huffman coding to the digital source characterised in Table Q-A1.

(iii) Derive the coding efficiency of both the uncoded BCD signal and the Huffman coded signal.

(iv) What is the original symbol sequence of the Huffman coded signal 0111000101110101001 (The left most bit is the first bit of the coded signal)?

[10 marks]

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Question A2.

(a) A binary symmetric channel (BSC) is depicted in Figure Q-A2,

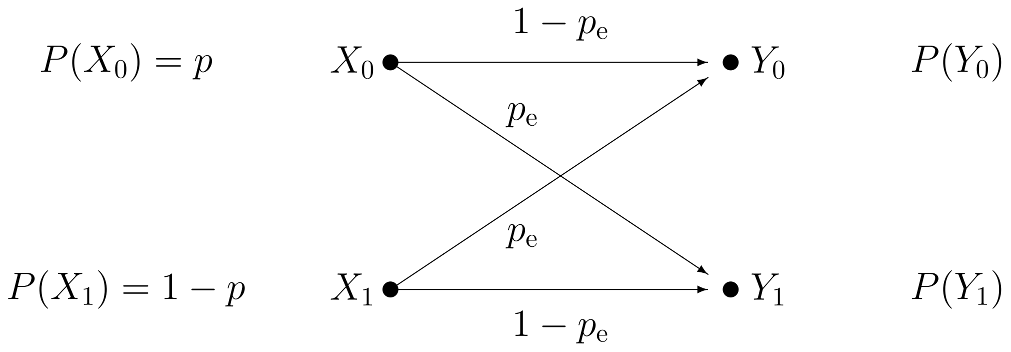

Figure Q-A2.

(i) For what values of p and p_e are the source and destination entropies, $H(X)$ and $H(Y)$, identical? In general, is $H(Y) \geq H(X)$ or $H(Y) \leq H(X)$ true? Justify your answer.

(ii) Given $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$, calculate all the probabilities $P(X_i, Y_j)$ as well as $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X, Y)$.

(iii) The source of this BSC of $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$ emits the binary symbols with the rate of 10^8 Baud (Baud=symbol/s). What is the maximum achievable rate of this BSC for achieving error-free transmission?

[10 marks]

(b) (i) State the Shannon-Hartley channel capacity formula, and define clearly each variable in the formula.

According to this channel capacity formula, what are the two basic resources for digital communications? Discuss how you can trade off these two resources to achieve the required channel capacity.

(ii) The signal-to-noise ratio (SNR) of a wireless channel is $\mathrm{SNR} = 127$, and the bandwidth of this wireless channel is $10\,$MHz. What is the capacity of this channel?

[6 marks]

(c) A digital communication system uses an 8-ary signalling scheme with the transmission rate of 100 MBaud (10^8 [symbols/s]). The probabilities of occurrence for the eight symbols at the transmitter are respectively

$$P(X_1) = 0.2,\ P(X_2) = 0.1,\ P(X_3) = 0.1,\ P(X_4) = 0.1,$$
$$P(X_5) = 0.1,\ P(X_6) = 0.1,\ P(X_7) = 0.1,\ P(X_8) = 0.2.$$

It is known that this 8-ary symbol source is a first-order Markov process with the known transition probability matrix

$$\mathbf{\Gamma} = [p_{ij}] = \begin{bmatrix} 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 & 0.3 \\ 0.3 & 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 & 0.3 \\ 0.3 & 0.3 & 0.2 & 0.2 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.4 & 0.3 & 0.2 & 0.1 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.4 & 0.3 & 0.1 & 0.2 \end{bmatrix},$$

where $p_{ij} = P(X_j|X_i)$, $1 \leq i, j \leq 8$, are the transition probabilities.

(i) Determine the information rate of the source at the transmitter.

(ii) The channel is an ideal additive white Gaussian noise (AWGN) channel, and the channel's signal to noise ratio is known to be 63. Determine the minimum channel bandwidth required to achieve error-free transmission.

[9 marks]

Question A3.

(a) Figure Q-A3 depicts schematic of digital communication modem.

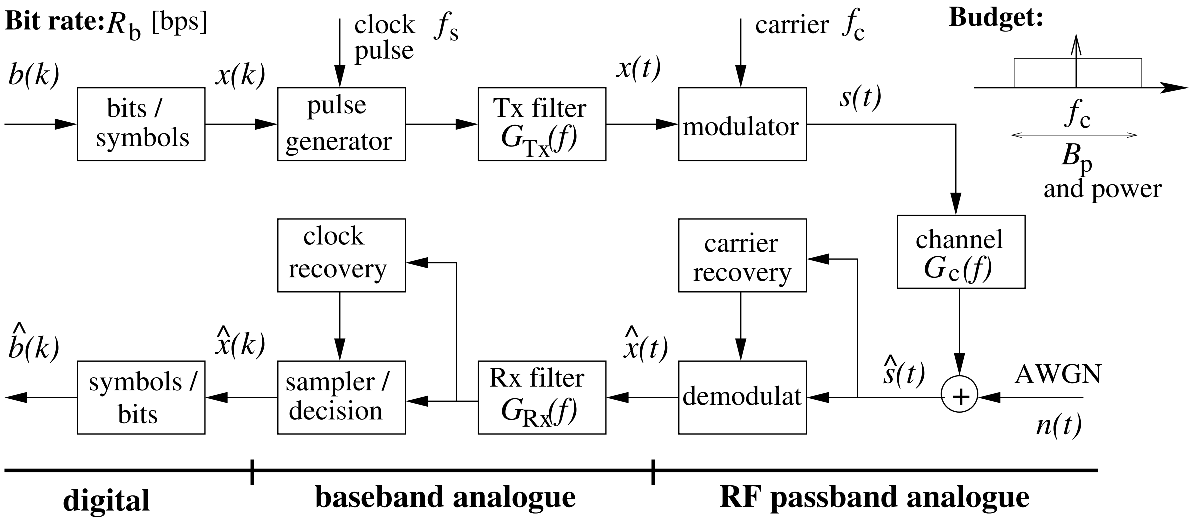

Figure Q-A3.

(i) Briefly explain the purpose or design framework of a modem in the context of wireless communication.

(ii) Briefly explain the two key objectives that the transmit and receive pulse shaping filter pair are designed to achieve.

(iii) Briefly explain the function of the carrier recovery circuit at the receiver. Why is it necessary to have accurate carrier recovery?

(iv) Briefly explain the function of the clock recovery circuit at the receiver. Why it is necessary to obtain an accurate timing information?

[15 marks]

(b) Receiver with carrier recovery is called coherent receiver.

(i) With the key signal equations, explain how the transmitter and receiver operate based on the differential phase shift keying (DPSK).

(ii) What is the assumption made in order for this DPSK based design to operate properly?

(iii) Compared with the coherent design, what are the advantage gained as well as the penalty paid by this DPSK design?

[10 marks]

Section B

Question B1.

(a) Please design a low-rate motion-compensation based video codec for a smart-phone, where 8bit/pixel gray-scale resolution is required and the video frame size is 100 x 100 pixels. Calculate the uncompressed bitrate required at a frame-scanning rate of 20 frames/s.

[2 marks]

(b) Draw the schematic of this motion-compensation based video codec and explain its operation, with particular emphasis on the motion vector search and motion compensation operations.

[4 marks]

(c) Explain, how the motion vectors (MV) are calculated, discussing the video quality, bitrate and complexity trade-offs of **1**/ different MV search scope sizes; **2**/ different block sizes; **3**/ different MV search techniques.

[4 marks]

(d) Calculate the total number of bits required for the transmission of the MVs for a video frame, if the motion-compensated blocks have a size of 10 x 10 pixels and the scope of the motion-search is [-4 ... +3] in both the horizontal and vertical directions.

[2 marks]

(e) Calculate the total number of multiplications and additions per video frame required by the motion-compensation, when carrying out a full search over every position of the entire search-scope of [-4 ... +3] pixel positions.

[2 marks]

(f) Compare the resultant computational complexity to that, when using a subsampled search, visiting every other pixel position and contrast the design trade-offs of full-scale vs. subsampled MV-search.

[2 marks]

(g) Explain with the aid of sketches, how vector quantisation (VQ) of the MCER is carried out. Then, assuming that a 128-entry VQ codebook

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is used for encoding the 10 x 10 pixel MCER blocks, calculate the total number of bits required for encoding the MCER.

[4 marks]

(h) Based on the above-mentioned 10 x 10 pixel blocks, calculate the total bitrate generated at 20 frames/s, when relying on full-scale MV-aided motion compensation.

[2 marks]

(i) Then recalculate the total bitrate, if simple frame-differencing is used instead of MV-aided motion compensation. Contrast the frame-differencing and full-search MV-based solutions in terms of their video quality, bit-rate and complexity by describing the associated design trade-offs.

[3 marks]

Question B2.

(a) Assuming that the input signal of a speech encoder is uniformly distributed across the linear quantiser's entire dynamic range and no quantiser overload is encountered, please derive an equation for the input signal's variance.

[5 marks]

(b) Furthermore, under the assumption that the quantization error $e(n)$ is also uniformly distributed across the quantization interval, derive a formula for the variance, ie. the power of $e(n)$.

[4 marks]

(c) Based on these two formulae, express the speech codec's Signal-to-Noise Ratio (SNR) in terms of dB as a function of the number of quantization bits b.

[4 marks]

(d) Draw the schematic of a compander-expander based speech codec and explain its operation with the aid of signal sketches.

[4 marks]

(e) You are now tasked with the design of a compander-expander based speech codec, which has the SNR calculation formula of

$$SNR = \frac{\sigma_x^2}{\sigma_e^2} = \frac{\int_{-x_{max}}^{x_{max}} x^2 p(x) dx}{\frac{q^2}{12} \int_{-x_{max}}^{x_{max}} \left(p(x)/|\dot{C}(x)|^2 \right) dx}. \tag{1}$$

where σ_x^2 and σ_e^2 represent the variance of the signal and of the noise, $p(x)$ is the input signal's distribution and $\dot{C}(x)$ is the companding characteristic.

Please derive a formula for $\dot{C}(x)$, which results in a SNR that is independent of $p(x)$.

[4 marks]

(f) Draw the schematic of a DPCM speech encoder and decoder and explain its operation with the aid of signal sketches.

[4 marks]

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Question B3.

(a) Given the generator polynomials of $g_1 = [1, 0, 0]$, $g_2 = [1, 0, 1]$ and $g_3 = [1, 1, 1]$, draw the corresponding convolutional encoder's schematic.

[4 marks]

(b) Draw the encoder's state transition diagram, indicating the resultant encoded bits for all transitions.

[7 marks]

(c) Consider the received sequence of $\mathbf{001}$, 101, 000, 100, 000, 000, where the left-most three-bit codeword printed in bold font appears at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

[7 marks]

(d) Consider the soft-decision based demodulator output sequence of $\mathbf{+2+1-1}$, $-2-3-4$, $-3+2-1$, $-4-3-2$, $-1-1-2$, $-4-3-1$, where the left-most three soft demodulator output values printed in bold font appear at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

[7 marks]

END OF PAPER