SEMESTER 1 EXAMINATIONS 2024 - 2025

DIGITAL CODING AND TRANSMISSION

DURATION 150 MINS (2.5 Hours)

This paper contains 6 questions

Answer FOUR questions, TWO from **Section A** and TWO from **Section B**.

An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper.

Section A

Question A1.

(a) A 64 kbps speech coder was designed in the 1970s. In this 64 kbps speech coder design, the speech waveform was filtered to limit its bandwidth to approximately 3.8 kHz and a sampling rate of 8 kHz was then applied. The resultant samples were first passed through an A-law compressor and the compressed samples were then quantised to 256 levels with each sample being represented by 8 bits.

(i) Explain the reason for sampling at 8 kHz.

(ii) Explain the reason for applying the A-law compressor.

(iii) Will you use this speech coder design for a mobile handset? Explain your answer clearly according to the information theory you have learnt in this course.

(iv) What is the basic principle adopted in today's speech source encoding techniques?

[8 marks]

(b) (i) According to the information theory, what is the basic principle adopted in video source encoding techniques?

(ii) You have designed a video source coder. The output bit sequence of your video source coder contains 6.4067×10^{15} bits, in which the bit $\mathbf{0}$ has the probability of occurring 0.999 and the bit $\mathbf{1}$ has the probability of occurring 0.001.

You decide to encode this bit sequence by the run length encoder (RLC) with a codeword length of $n = 10$ bits. What compression ratio can be achieved? Calculate the length of the bit sequence after the RLC encoding.

(iii) The RLC of the above video source coder produces the following bit sequence (The left most bit is the first bit of the sequence):

$$1111111101 \; 0000000000 \; 1111111111 \; 1111111110 \; 1111111101 \cdots$$

What is the input bit sequence to the RLC?

[8 marks]

(c) A memoryless digital source emits symbols X_i, $1 \le i \le 8$, in binary coded decimal (BCD) format with probabilities $P(X_i)$ as given in Table Q-A1, at a rate $R_{\mathrm{s}} = 10^8$ Baud (Baud=symbol/s).

Table Q-A1.

X_i	$P(X_i)$	BCD word
A	0.27	000
B	0.20	001
C	0.17	010
D	0.16	011
E	0.07	100
F	0.06	101
G	0.04	110
H	0.03	111

(i) What is the data rate of the uncoded BCD signal and what is the information rate of this source?

(ii) Apply Shannon-Fano coding to the digital source characterised in Table Q-A1.

(iii) Derive the coding efficiency of both the uncoded BCD signal and the Shannon-Fano coded signal.

(iv) What is the original symbol sequence of the Shannon-Fano coded signal $0001100101110011011101111$ (The left most bit is the first bit of the coded signal)?

[9 marks]

TURN OVER

Question A2.

(a) A binary symmetric channel (BSC) is depicted in Figure Q-A2-1:

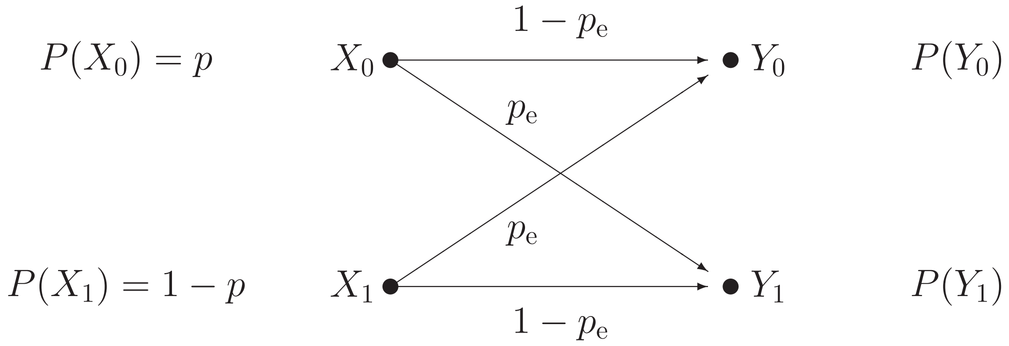

Figure Q-A2-1.

(i) What are the assumptions made for BSC?

(ii) For what values of p and p_e are the source and destination entropies, $H(X)$ and $H(Y)$, identical? In general, is $H(Y) \geq H(X)$ or $H(Y) \leq H(X)$ true? Justify your answer.

(iii) Given $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$, calculate all the joint probabilities $P(X_i, Y_j)$ as well as the conditional probabilities $P(X_i|Y_j)$, and derive the numerical value for the mutual information $I(X, Y)$.

(iv) The source of this BSC with $p = \frac{1}{2}$ and $p_e = \frac{1}{16}$ emits the binary symbols at the rate of 10^9 Baud (Baud=symbol/s). What is the maximum achievable rate of this BSC for achieving error-free transmission?

[13 marks]

(b) (i) State the Shannon-Hartley channel capacity formula, and define clearly each variable in the formula.

According to this channel capacity formula, what are the two basic resources for digital communications? Discuss how you can trade off these two resources to achieve the required channel capacity.

(ii) The signal-to-noise ratio (SNR) of a wireless channel is $\mathrm{SNR} = 1023$, and the bandwidth of this wireless channel is 1 GHz. What is the capacity of this channel?

[5 marks]

(c) In the late 1980s, the author of *ISDN Explained* (first edition), who was a chief engineer of British Telecoms (BT), presented the following figure in the book:

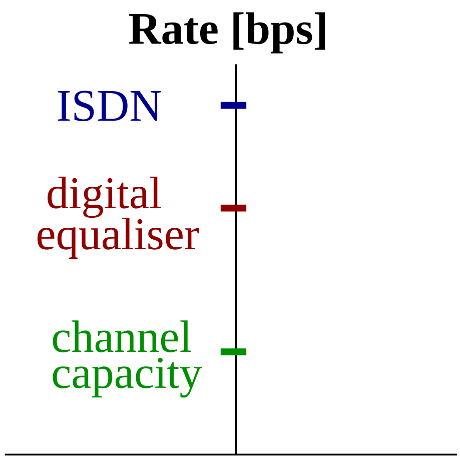

Figure Q-A2-2. A figure from *ISDN Explained* (first edition).

This figure seemed to suggest that in the late 1980s after digitization, the BT telephone line with equalization achieves a higher rate than the original telephone line's channel capacity, and the BT's future ISDN line attains an even higher rate.

Explain what is wrong with this figure. Justify your explanation according to Shannon's information theory. [7 marks]

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Question A3.

(a) Figure Q-A3 depicts the schematic of a digital communication modem.

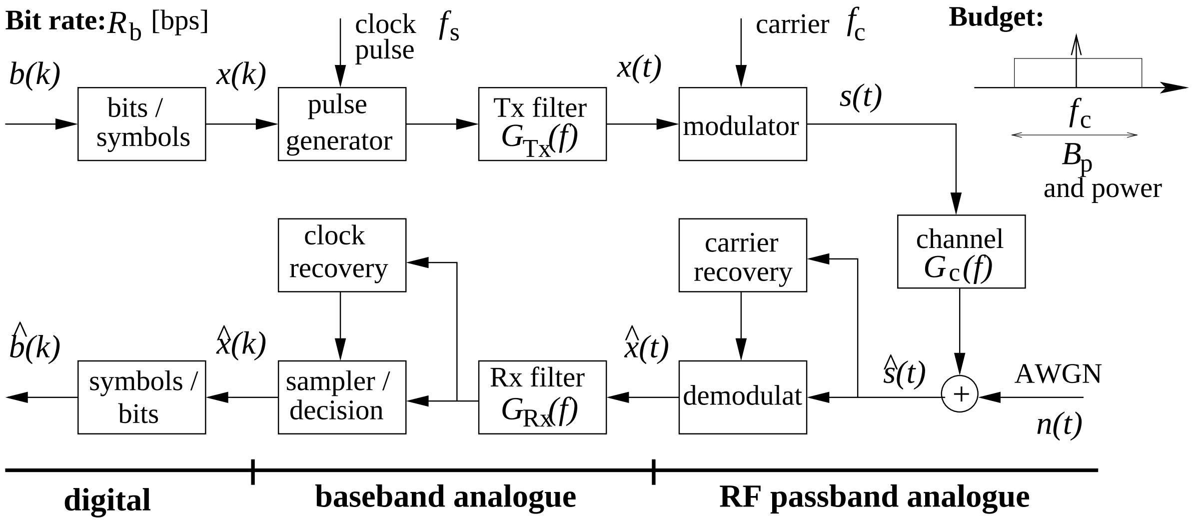

Figure Q-A3.

(i) Briefly explain the purpose or design framework of a modem in the context of wireless communication.

(ii) Briefly explain the two key objectives that the transmit and receive pulse shaping filter pair are designed to achieve.

(iii) Briefly explain the function of the carrier recovery circuit at the receiver. Why is it necessary to have accurate carrier recovery?

(iv) Briefly explain the function of the clock recovery circuit at the receiver. Why is it necessary to obtain accurate timing information?

[15 marks]

(b) A receiver with carrier recovery is called a coherent receiver, and a receiver without carrier recovery is called a non-coherent receiver. A non-coherent design employs the differential phase shift keying (DPSK).

(i) With the key signal equations, explain how the transmitter and receiver operate based on the DPSK. What is the assumption made in order for this DPSK based design to operate properly?

(ii) Compared with the coherent design, what are the advantages gained as well as the penalty paid by this DPSK design?

[10 marks]

Section B

Question B1.

(a) (i) An Internet-based gaming console uses a [1000 x 1000]-pixel video resolution at a frame scanning rate of 100 frames/sec (fps) and 10 bits/pixel. Calculate the total uncompressed bit rate.

(ii) The uncompressed bitrate has to be reduced for efficient low-delay communications with a remote player. Using low-complexity frame-differencing and 4-bit Differential Pulse Code Modulation (DPCM) of the resultant Motion Compensated Error Residual (MCER), calculate the bitrate of this video compression codec.

(iii) Now upgrade this codec design by using full motion-compensation relying on [10 x 10] tiles and a motion-vector search-scope of [+8; -7] both in the horizontal and vertical direction.

Describe the operation of a specific motion-compensation scheme with the aid of sketches and explain how the motion vectors (MVs) are determined. Then recalculate the bitrate using the same 4-bit DPCM scheme.

(iv) Calculate the motion-compensation complexity of the codec in terms of the number of Mean Absolute Difference (MAD) metric evaluations.

(v) Now increase the size of the tiles to [20 x 20] pixels without modifying the motion-compensation search-scope and then recalculate the bitrate.

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(vi) Recalculate the motion-compensation complexity and explain with reference to your quantitative results, how the bit-rate, video quality and complexity are affected by reducing the tile-size.

[15 marks]

(b) (i) Now upgrade the DPCM coding of the MCER to quad-tree (QT) coding and explain its operation with the aid of carefully labelled sketches. Then using a total of 30 bits per tile calculate the bitrate required for representing the MCER of the video codec using both [20 x 20] as well as [10 x 10] tiles.

(ii) Calculate the **total** bitrate of the video codec using 4-bit DPCM and 1/ low-complexity frame-differencing; 2/ full motion-compensation relying on [20 x 20] tiles; 3/ full motion-compensation relying on [10 x 10] tiles;

(iii) Calculate the total bitrate of the video codec using 30-bit QT coding per tile and 1/ low-complexity frame-differencing; 2/ full motion-compensation relying on [20 x 20] tiles; 3/ full motion-compensation relying on [10 x 10] tiles;

[10 marks]

Question B2.

(a) Sketch the ‘speech-quality versus bitrate plane’ of speech codecs and contrast the speech codec families in terms of their speech quality, complexity and delay.

[5 marks]

(b) Sketch the schematic of an ‘analysis-by-synthesis’ (ABS) speech codec and detail the role of its specific components with the aid of the corresponding input and output waveforms.

[5 marks]

(c) Sketch a carefully scaled diagram of a typical voiced speech segment as well as the corresponding short-term and long-term residual segment in the time-domain. With careful reference to this diagram discuss, how any residual ‘redundancy’ or predictability manifests itself in these time-domain signals.

[5 marks]

(d) Based on the equation representing short-term prediction residual derive the optimum one-tap predictor’s formula.

[3 marks]

(e) Define the prediction gain and derive an equation characterizing the prediction gain of the optimum one-tap predictor.

[3 marks]

(f) Finally, formulate an equation for deriving the optimal long-term predictor and its gain by minimizing the mean squared long-term prediction residual.

[4 marks]

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Question B3.

(a) Given the generator polynomials of g_1=[1,0,1], g_2=[1,1,1] and g_3=[1,1,1], sketch the corresponding convolutional encoder's schematic.

[3 marks]

(b) Sketch the encoder's state transition diagram, indicating the resultant encoded bits for all transitions.

[6 marks]

(c) Then consider the received sequence of $\mathbf{001}$, 010, 000, 100, 000, 000, where the left-most three-bit codeword printed in bold font was received last and hence appears at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis.

[10 marks]

(d) Assuming that the decoded bit sequence is identical to the transmitted one, determine the number of transmission errors induced by the channel during the specified received sequence.

[1 marks]

(e) Finally, consider the soft-decision based demodulator output sequence of $+\mathbf{2}+\mathbf{1}-\mathbf{1}$, $-3-4-2$, $-3+2-1$, $-4-3-2$, $-1-1-2$, $-4-3-1$, where the left-most of the three demodulator output values printed in bold font were received last and hence appear at the left hand side of the corresponding decoding trellis. Determine the transmitted information sequence by drawing the trellis diagram and clearly indicate the "winning" path through the trellis, including the branch metrics and path metrics.

[5 marks]

END OF PAPER

