ELEC6014 RCNSs: Brief Notes

Coherent and Non-coherent Receivers

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Coherent Receiver

(a) Carrier recovery for demodulation

– Receiver signal $\hat{S}(t) = A\cos\left(\omega_c t + \varphi\right) + N(t)$
– Local carrier $\cos\left(\omega_c t + \bar{\varphi}\right)$
– Carrier recovery (e.g. phase lock loop) circuit

$$\Delta\varphi = \varphi - \bar{\varphi} \rightarrow 0 \quad \text{i.e.} \quad \bar{\varphi} \rightarrow \varphi$$

– Demodulation leads to recovered baseband signal

$$Y(t) = X(t+\tau) + N(t)$$

where $X(t)$ is transmitted baseband signal

(b) Timing recovery for sampling

– Align receiver clock with transmitter clock, so that sampling $\Rightarrow$ no ISI

$$Y_k = X_k + N_k$$

where X_k are transmitted symbols, and N_K noise samples

Non-coherent Receiver

(a) No carrier recovery for demodulation

- Receiver signal $\hat{S}(t) = A\cos\left(\omega_c t + \varphi\right) + N(t)$
- Local carrier $\cos\left(\omega_c t + \bar{\varphi}\right)$
- No carrier recovery,

$$\phi = \Delta\varphi = \varphi - \bar{\varphi} \neq 0 \quad \text{i.e.} \quad \bar{\varphi} \neq \varphi$$

- Demodulation leads to recovered baseband signal

$$Y(t) = X(t+\tau)e^{j\phi} + N(t)$$

(b) Timing recovery for sampling

- Align receiver clock with transmitter clock, sampling results in

$$Y_k = X_k e^{j\phi} + N_k$$

Could not recover transmitted symbols properly from Y_k !

Differential Detection

(a) Differential encoding for transmission

– Symbols $\{C_k\} \Rightarrow \{X_k\}$ for transmission

$$X_k = C_k \cdot X_{k-1}$$

– As $X_k \cdot X_{k-1}^* = C_k \cdot \left(X_{k-1} \cdot X_{k-1}^*\right)$,

$$C_k = \frac{X_k \cdot X_{k-1}^*}{|X_{k-1}|^2} \quad (1)$$

(b) Non-coherent detection

– Receiver samples

$$Y_k = X_k \cdot |H| \cdot e^{j\phi} + N_k$$

$|H|$: magnitude of combined channel tap, $\phi \neq 0$: unknown phase

– Differential decoding

$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|Y_{k-1}|^2} \quad (2)$$

Differential Detection (derivation)

$$\begin{aligned} Y_k \cdot Y_{k-1}^* &= \left(X_k \cdot |H| \cdot e^{j\phi} + N_k\right) \cdot \left(X_{k-1}^* \cdot |H| \cdot e^{-j\phi} + N_{k-1}^*\right) \\ &= X_k \cdot X_{k-1}^* \cdot |H|^2 \cdot e^{j(\phi-\phi)} \\ &\quad + N_k \cdot N_{k-1}^* + X_k \cdot |H| \cdot e^{j\phi} \cdot N_{k-1}^* + N_k \cdot X_{k-1}^* \cdot |H| \cdot e^{-j\phi} \end{aligned}$$

$$\begin{aligned} |Y_{k-1}|^2 &= X_{k-1} \cdot X_{k-1}^* \cdot |H|^2 + N_{k-1} \cdot N_{k-1}^* \\ &\quad + X_{k-1} \cdot |H| \cdot e^{j\phi} \cdot N_{k-1}^* + N_{k-1} \cdot X_{k-1}^* \cdot |H| \cdot e^{-j\phi} \end{aligned}$$

When noise N_k is very small

$$Y_k \cdot Y_{k-1}^* \approx X_k \cdot X_{k-1}^* \cdot |H|^2 \quad \text{and} \quad |Y_{k-1}|^2 \approx |X_{k-1}|^2 \cdot |H|^2$$

- Thus,

$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|Y_{k-1}|^2} \approx C_k + \bar{N}_k$$

  where power of enhanced noise $\bar{N}_k$ is **larger** than that of N_k

- Note that influence of channel phase ϕ has been removed

Comparison

- Coherent detection
  - Require expensive and complex carrier recovery circuit
  - Better bit error rate of detection

$$\hat{X}_k = X_k + N_k$$

- Non-coherent detection
  - Do not require expensive and complex carrier recovery circuit
  - Poorer bit error rate of detection

$$\hat{C}_k = C_k + \bar{N}_k$$

- Differential systems have important advantages and are widely used in practice

Differential PSK

(a) For differential **phase shift keying**, $|X_{k-1}|^2 = \text{con}$ and $C_k = \frac{X_k \cdot X_{k-1}^*}{\text{con}}$

- $C_k \leftarrow$ phase of $X_k \cdot X_{k-1}^*$
- $\hat{C}_k \leftarrow$ phase of $Y_k \cdot Y_{k-1}^*$

(b) At receiver, **differential decoding** (2) becomes

$$\hat{C}_k = \frac{Y_k \cdot Y_{k-1}^*}{|X_{k-1}|^2} = \frac{Y_k \cdot Y_{k-1}^*}{\text{con}} \tag{3}$$

- For convenience, assuming $|H|^2 = 1$ (or $|H|^2$ is known), then

$$\frac{Y_k \cdot Y_{k-1}^*}{\text{con}} = \frac{X_k \cdot X_{k-1}^*}{\text{con}} + \frac{N_k \cdot N_{k-1}^*}{\text{con}} + \frac{X_k}{\text{con}} \cdot e^{j\phi} \cdot N_{k-1}^* + N_k \cdot e^{-j\phi} \cdot \frac{X_{k-1}^*}{\text{con}}$$

- Noting magnitudes of $\frac{X_k}{\text{con}}$ and $\frac{X_{k-1}^*}{\text{con}}$ are 1, $\frac{N_k \cdot N_{k-1}^*}{\text{con}}$ is much smaller than the last two terms, while $e^{j\phi} \cdot N_{k-1}^*$ and $N_k \cdot e^{-j\phi}$ have the same variance as N_k,

$$\hat{C}_k \approx C_k + 2N_k$$

- Compared with coherent detection, noise is **doubled** or 3 dB worse off

Differential BPSK

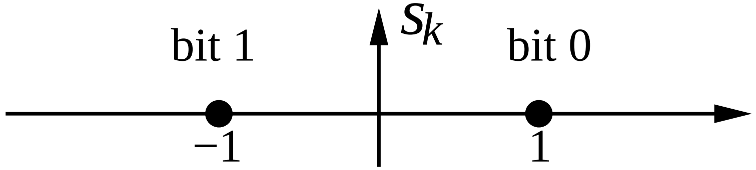
(a) For differential **binary phase shift keying**, at transmitter

– Bit sequence $\{b_k\}$ with $b_k \in \{0, 1\}$ are encoded by

$$d_k = \overline{b_k \oplus d_{k-1}}$$

Initial condition d_0 is given and known to both transmitter and receiver

– Encoded bit sequence $\{d_k\}$ is then BPSK modulated into BPSK symbol sequence $\{s_k\}$



– Transmitted signal

$$x(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \frac{1}{2}(1 - s_k)\pi\right), \ kT_b \le t < (k+1)T_b$$

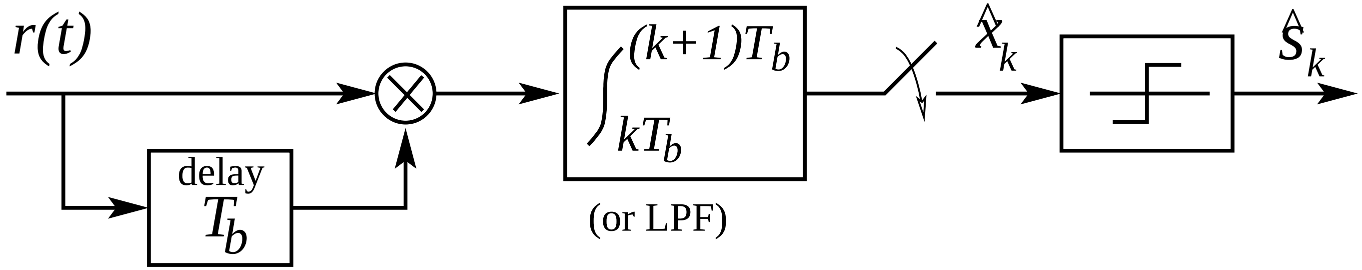
where f_c is carrier frequency, E_b bit energy, and T_b bit duration

(b) At **coherent demodulator**, from sampled received signal r_k

$$\{r_k\} \rightarrow \{\hat{s}_k\} \rightarrow \{\hat{d}_k\} \rightarrow \{\hat{b}_k\} \text{ with } \hat{b}_k = \overline{\hat{d}_k \oplus \hat{d}_{k-1}}$$

DBPSK (Continue)

(c) For **noncoherent demodulator**, no local carrier is needed



– Let $A^2 = \frac{2E_b}{T_b}$ and $\frac{1}{2}(1 - s_k)\pi = \varphi_k$, then received RF signal

$$r(t) = A\cos\left(2\pi f_c t + \varphi_k + \vartheta\right), \;\; kT_b \le t < (k+1)T_b$$

– Noting $\cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta)$, input to integrator

$$A^2\cos\left(4\pi f_c t + 2\vartheta + \varphi_k + \varphi_{k-1}\right) + A^2\cos\left(\varphi_k - \varphi_{k-1}\right)$$

– First term average over one period is zero, and $\cos\left(\varphi_k - \varphi_{k-1}\right) = 1$ if $s_k = s_{k-1}$; $\cos\left(\varphi_k - \varphi_{k-1}\right) = -1$ if $s_k = -s_{k-1}$

$$\hat{x}_k = \begin{cases} E_b, & s_k = s_{k-1} \\ -E_b, & s_k = -s_{k-1} \end{cases}$$

– Detection is achieved by $\{\hat{x}_k\} \rightarrow \{\hat{s}_k\} \rightarrow \{\hat{d}_k\} \rightarrow \{\hat{b}_k\}$