Revision of Lecture Twenty-One

- FFT / IFFT most widely found operations in communication systems
- Important to know what are going on inside a FFT / IFFT algorithm
- With the aid of FFT / IFFT, this lecture looks into OFDM system and other multicarrier systems

OFDM Modem

- **OFDM basic concepts recall**: Let $\{S_k\}$ be the complex-valued symbol sequence (e.g. QAM symbols) transmitted at rate f_s or symbol period T_s
  - I have deliberately used the capital letter S_k to denote transmitted symbols (instead of the usual small letter s_k) and there is a good reason for it
  - Indeed transmitted symbols S_k are now viewed as "frequency"-domain samples
- At the OFDM transmitter: during the period $T = NT_s$, N symbols $S_0, S_1, \cdots S_{N-1}$ are transmitted, and complex baseband OFDM signal during period T is therefore

$$s(t) = \sum_{k=0}^{N-1} S_k e^{j2\pi \frac{k}{T} t}$$

- At the OFDM receiver: the received signal is multiple by $e^{-j2\pi \frac{n}{T} t}$ and integrated over T to obtain S_n, $n = 0, 1, \cdots N-1$

$$\frac{1}{T} \int_0^T s(t) e^{-j2\pi \frac{n}{T} t} \, dt = \frac{1}{T} \sum_{k=0}^{N-1} S_k \int_0^T e^{j2\pi \frac{k}{T} t} e^{-j2\pi \frac{n}{T} t} \, dt = S_n$$

- No one really implements OFDM Modem this way, as it needs N oscillators (N pairs of hardware modulators/demodulators)

DFT/FFT Implementation

- Sample complex-valued baseband signal $s(t)$ N times during a period T: i.e. $t = \frac{m}{N}T$

$$s_m = s\left(\frac{m}{N}T\right) = \sum_{k=0}^{N-1} S_k e^{j2\pi\frac{km}{N}}, \quad m = 0, 1, \cdots, N-1$$

  This is just IDFT formula multiplying by a factor N, thus one can view
  - Transmitted symbols $\{S_n\}_{n=0}^{N-1}$ as a set of N "frequency" samples (hence capital S)
  - Baseband signal samples $\{s_m\}_{m=0}^{N-1}$ as a set of "time" samples (hence small s)

- That is, from the set of N frequency samples to obtain the set of N time samples via IDFT:

$$s_m = N \cdot \mathrm{IDFT}\left(\{S_k\}_{k=0}^{N}\right), \quad m = 0, 1, \cdots, N-1$$

- At receiver, during a period T, the set of N transmitted symbols $\{S_n\}_{n=0}^{N-1}$ is recovered from the set of N time samples $\{s_m\}_{m=0}^{N-1}$ using DFT:

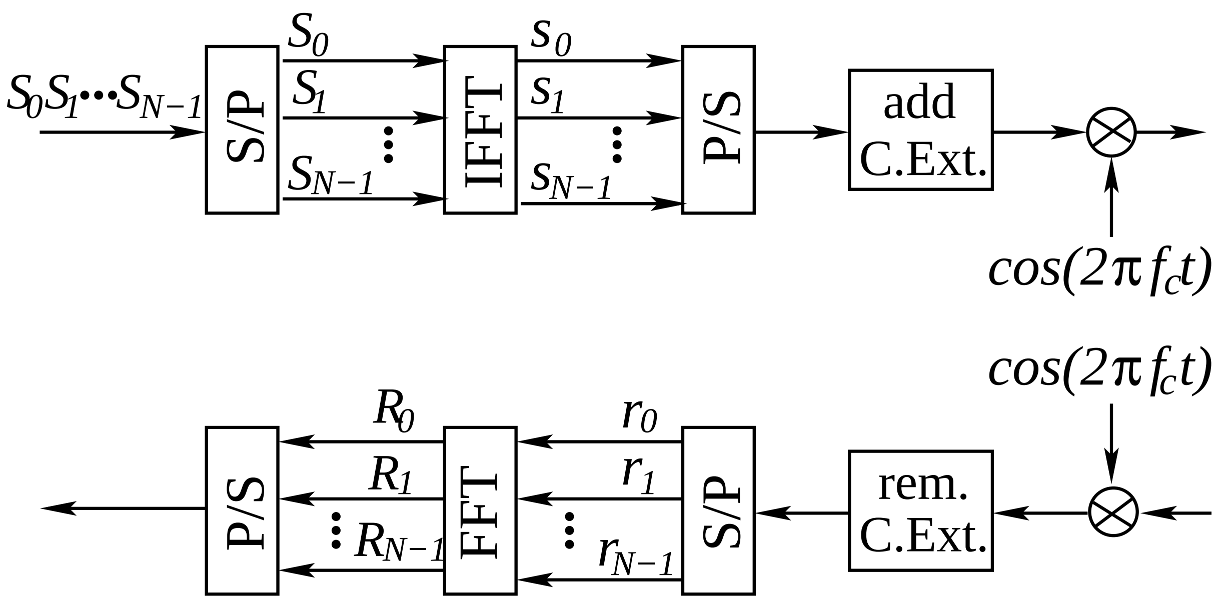
$$S_n = \frac{1}{N} \cdot \mathrm{DFT}\left(\{s_m\}_{m=0}^{N}\right), \quad n = 0, 1, \cdots, N-1$$

- IDFT/DFT are of course implemented by IFFT/FFT

OFDM Transceiver

- OFDM transmitter/receiver:

  The carrier modulation and demodulation is standard. Also clock recovery, not shown, is standard. New components are the **cyclic extension** add and remove



- Let transmitted frequency frame be $[S_0 \ S_1 \cdots S_{N-1}]$ and transmitted time frame be $[s_0 \ s_1 \cdots s_{N-1}]$

- From discrete Fourier theory

$$\{s_m\}_{m=0}^{N-1} \leftrightarrow \{S_n\}_{n=0}^{N-1}$$

  – Actually, from finite N frequency samples, time domain signal has infinite duration, but this time domain signal is periodic with a period of N samples

Cyclic Extension

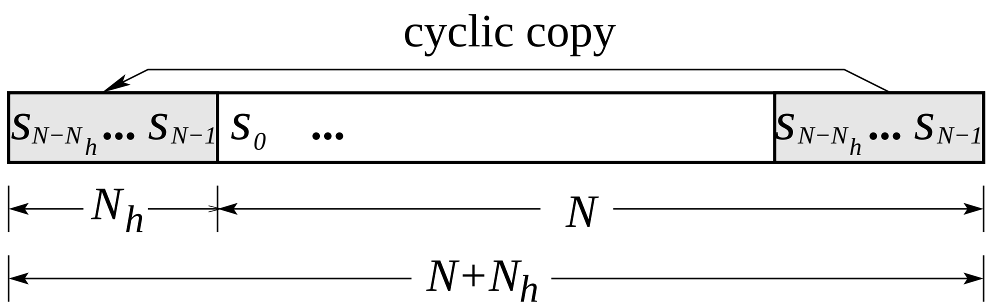
- Why **cyclic extension**
  - Thus if the channel is ideal, at receiver, from N time samples $\{r_k\}_{k=0}^{N-1}$, N frequency samples $\{R_k\}_{k=0}^{N-1}$, i.e. the transmitted symbols can be recovered via FFT
  - If the channel is dispersive, say, the CIR length is $N_h T_s$, then the transmitted length of N time symbols will spread to a length of $N_h + N$, and N frequency samples is insufficient

- A solution is to add some dummy symbols to make it $N + N_h$ frequency samples or cyclic extension

- An equivalent and more efficient alternative is to add cyclic extension in a transmitted time frame

- Add cyclic extension at transmitter:

  The last N_h time samples is copied back to the beginning of the frame, and transmitted samples are $N + N_h$



Cyclic Extension (continue)

- Remove cyclic extension at receiver: number the current frame as i and the $N + N_h$ times samples as $-N_h, -N_h + 1, \cdots, -1, 0, 1, \cdots, N - 1$
  – Let the CIR be $h_0, \cdots, h_{N_h}$ and ignore noise for simplicity, we have:

$$
\begin{aligned}
r_{i,-N_h} &= h_0 s_{i,N-N_h} + h_1 s_{i-1,N-1} + h_2 s_{i-1,N-2} + \cdots + h_{N_h} s_{i-1,N-N_h} \\
r_{i,-N_h+1} &= h_0 s_{i,N-N_h+1} + h_1 s_{i,N-N_h} + h_2 s_{i-1,N-1} + \cdots + h_{N_h} s_{i-1,N-N_h+1} \\
&\vdots \\
r_{i,-1} &= h_0 s_{i,N-1} + h_1 s_{i,N-2} + h_2 s_{i,N-3} + \cdots + h_{N_h} s_{i-1,N-1} \\
r_{i,0} &= h_0 s_{i,0} + h_1 s_{i,N-1} + h_2 s_{i,N-2} + \cdots + h_{N_h} s_{i,N-N_h} \\
&\vdots \\
r_{i,N_h} &= h_0 s_{i,N_h} + h_1 s_{i,N_h-1} + h_2 s_{i,N_h-2} + \cdots + h_{N_h} s_{i,0} \\
&\vdots \\
r_{i,N-1} &= h_0 s_{i,N-1} + h_1 s_{i,N-2} + h_2 s_{i,N-3} + \cdots + h_{N_h} s_{i,N-N_h-1}
\end{aligned}
$$

- Inter-frame interference: transmitted samples from the previous $(i - 1)$th frame spread into the first N_h time samples of i frame. Thus, the first N_h time samples are discarded

Cyclic Extension (continue)

- Remaining N samples are used to generate $R_{i,k}$, $0 \le k \le N-1$
- Noting the cyclic extension: $s_{-1} = s_{N-1}, \cdots, s_{-N_h} = s_{N-N_h}$, and the last N time samples in the current i frame can be written as:

$$r_{i,n} = \sum_{j=0}^{N_h} h_j s_{i,n-(j \bmod N)}, \; 0 \le n \le N-1$$

- The N frequency samples are obtained via FFT:

$$R_{i,k} = \sum_{n=0}^{N-1} r_{i,n} e^{-j2\pi \frac{n}{N} k}, \; 0 \le k \le N-1$$

- Using

$$e^{-j2\pi \frac{0}{N} k} = e^{-j2\pi \frac{N-1+1}{N} k} = e^{-j2\pi \frac{N-2+2}{N} k} = \cdots, \; \cdots$$

  we have

$$R_{i,k} = \sum_{n=0}^{N_h} h_n e^{-j2\pi \frac{n}{N} k} \sum_{n=0}^{N-1} s_{i,n} e^{-j2\pi \frac{n}{N} k} = H_k S_{i,k}$$

  where $\{H_k\}$ are the DFTs of the CIR $\{h_k\}$, call frequency domain channel transfer functions (FDCTFs)

Equalisation in OFDM

- In general, $R_{i,k} = H_k S_{i,k} + n_{i,k}$ with $n_{i,k}$ being a channel noise component

- The transmitted symbols are determined by passing $R_{i,k}$ through a decision device:

$$S_{i,k} = \text{Detector}\left(\frac{R_{i,k}}{H_k}\right), \; 0 \le k \le N-1$$

- Note that the intersymbol interference occurs in the received time samples $r_{i,k}$, but this does not matter, as the DFT removes this ISI $\rightarrow$ This is a beauty of OFDM: equalisation becomes very simple

- Equalisation in OFDM involves to estimate the FDCTFs $\{H_i\}_{i=0}^{N-1}$

- With the estimated channel $\{\hat{H}_i\}_{i=0}^{N-1}$, the estimated transmitted symbols are given by

$$\hat{S}_{i,k} = \text{Detector}\left(\frac{R_{i,k}}{\hat{H}_k}\right), \; 0 \le k \le N-1$$

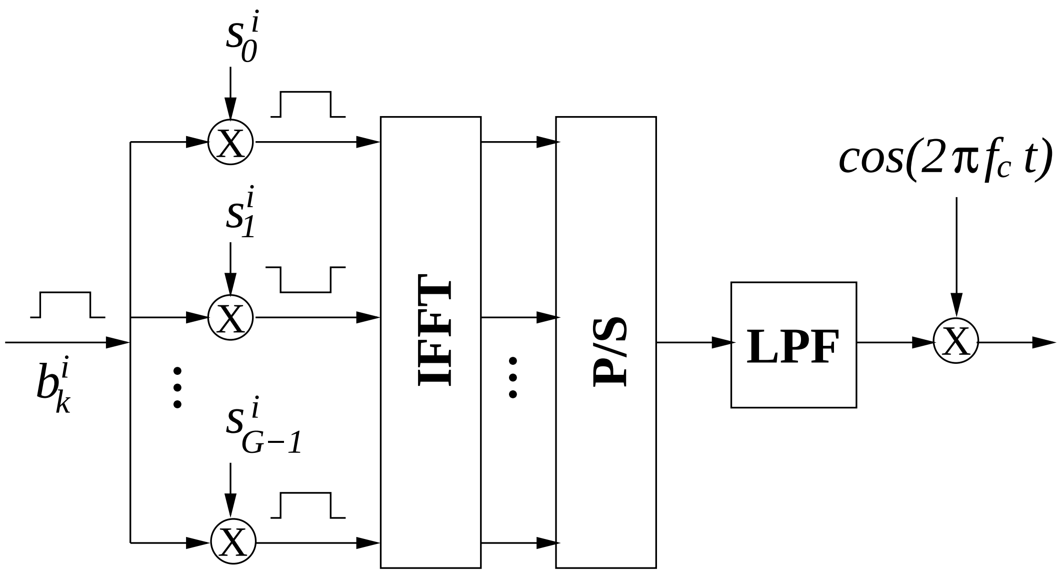
Multi-Carrier CDMA

- Map a different chip of a spreading sequence to an individual OFDM subcarrier
- Each OFDM subcarrier has a data rate identical to original input data rate
- Multicarrier absorbs increased rate due to spreading in a wider frequency band
- MC-CDMA transmitter:

  b_k^i: ith user's kth bit

  $\mathbf{s}^i = [s_0^i \ s_1^i \cdots s_{G-1}^i]$: ith user's spreading code

  Processing gain is G (subcarrier number is also G)



Multi-Carrier CDMA (continue)

- MC-CDMA receiver: Let number of users be I, and $i \in \{0, 1, \cdots I-1\}$; kth received symbol (sample) for subcarrier l is

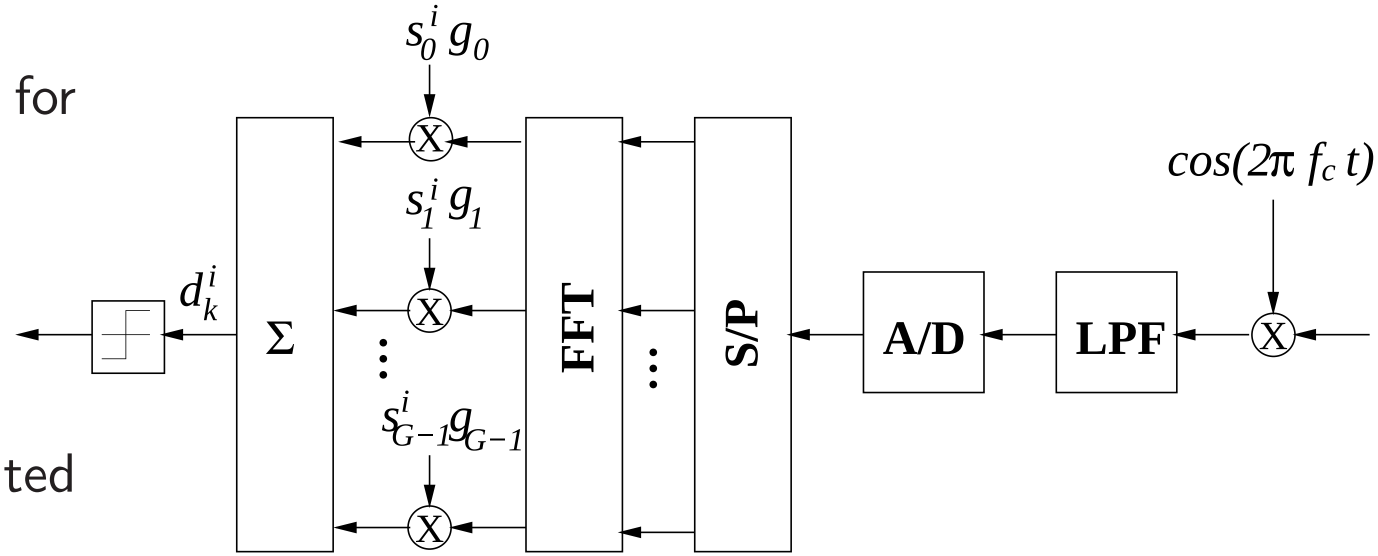
$$r_{k,l} = \sum_{i=1}^{I-1} H_l b_k^i s_l^i + n_{k,l}$$

H_l: frequency response of lth subcarrier (subchannel), $n_{k,l}$: noise sample

- Decision variable d_k^i for estimating b_k^i is

$$d_k^i = \sum_{l=0}^{G-1} s_l^i g_l r_{k,l}$$

g_l: reciprocal of estimated H_l

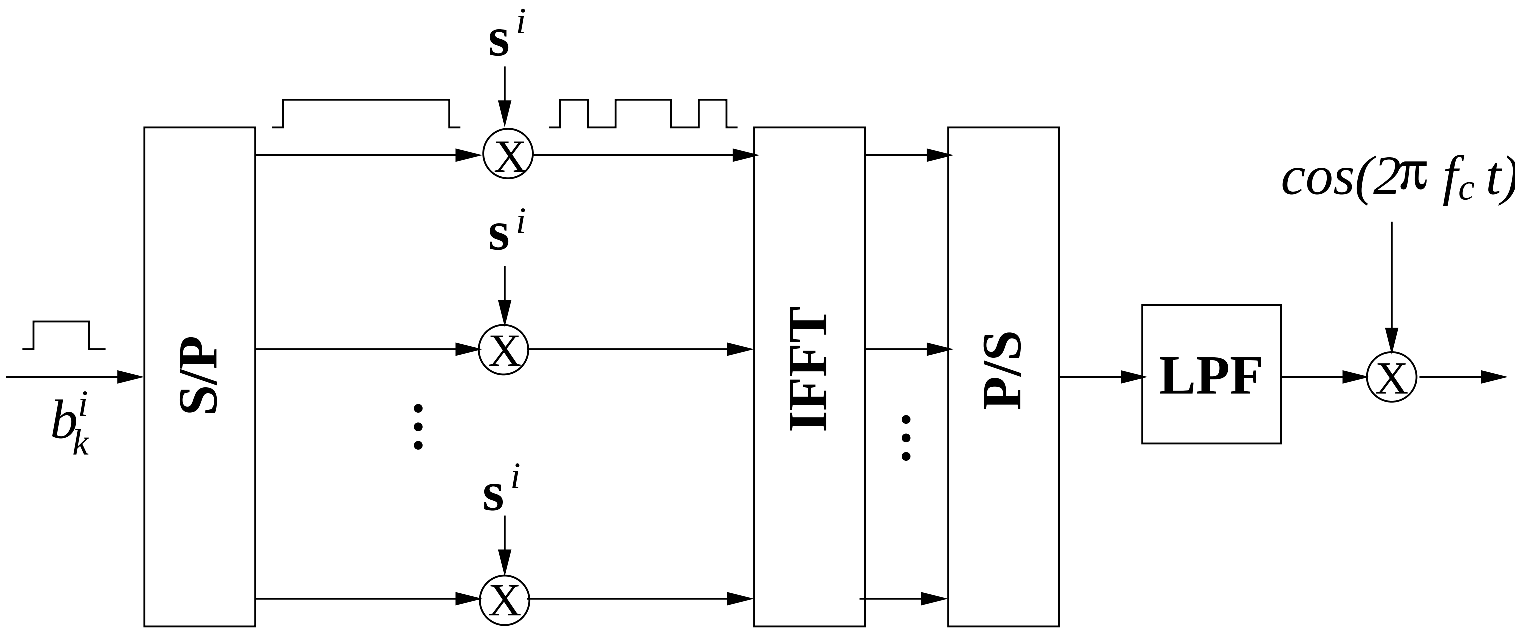


MC-DS-CDMA

- Parallel transmission of DS-CDMA signals using OFDM structure

  R_b: input bit rate, N: number of subcarriers, G: processing gain

- MC-DS-CDMA transmitter for user i: information data rate is R_b bps, after S/P rate is $\frac{R_b}{N}$ bps, after spreading rate is $\frac{R_b}{N}G$ bps



Summary

- OFDM implementation with FFT: transmitted complex symbols S_k are frequency samples, and transmitted time signal samples s_m are the IDFT of S_k

- Cyclic extension: the channel with CIR length N_h will spread the transmitted frame from length N to $N + N_h$

  By employing cyclic extension, the inter-frame interference can be removed by simply discard the first N_h received time samples

- Equalisation in OFDM becomes "automatic": DFT simply removes ISI in the received signal samples, all required are estimating frequency domain channel transfer functions

- MC-CDMA and MC-DS-CDMA