# Revision of Lecture Twenty-Two

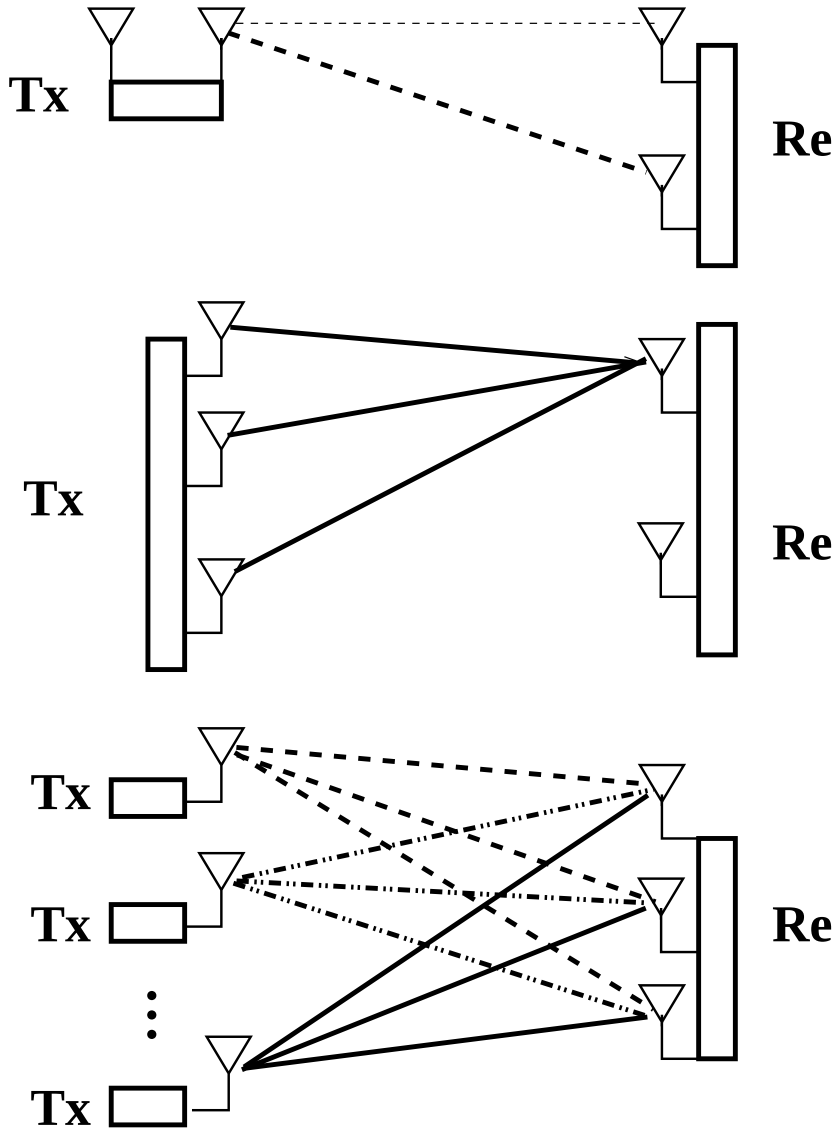
- OFDM fundamentals: what OFDM are "good" for? and any "bad"?
- OFDM implementation: IFFT / FFT

  Cyclic extension for removing inter-frame interference

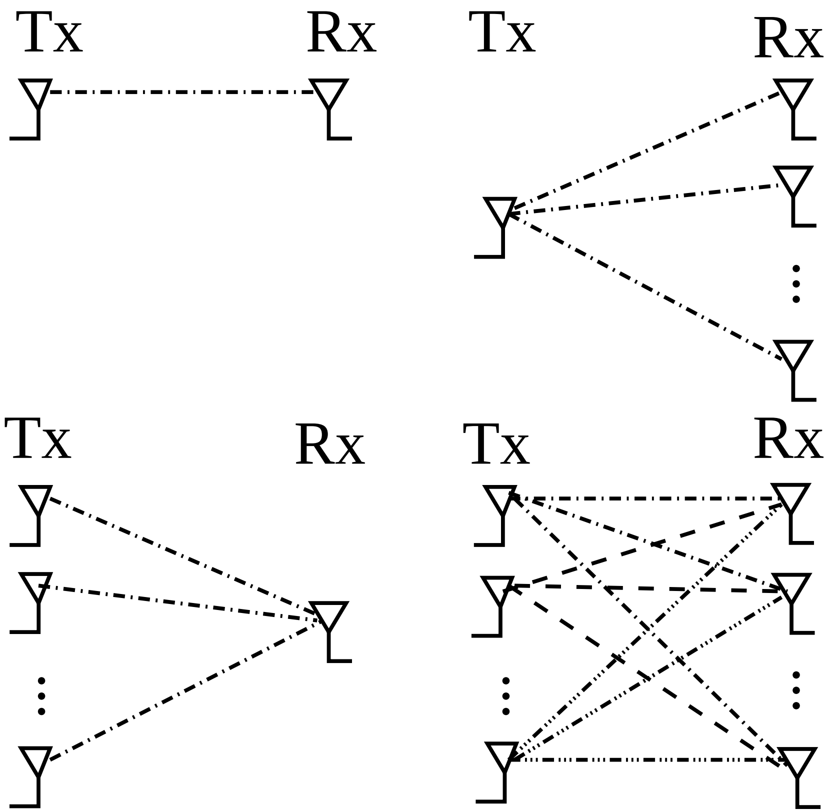
  Equalisation in OFDM becomes "automatic"
- The last two lectures we will look into some A, B, C of MIMO

# MIMO Introduction

- Create **diversity** for combating fading
  - With sufficient antenna spacing (10 wavelengths), each antenna experiences independent fading $\longrightarrow$ When one signal is in its deep fade, others are unlikely the same
- Increase **throughput**
  - Data stream is first S/P, each sub-sequence mapped to an antenna $\longrightarrow$ This creates many "digital pipes" to support higher rate
- Support multiple users
  - With multiple receive antennas, each spatially separated user has a unique set of CIRs seen at receiver $\longrightarrow$ This enables SDMA



# MIMO Channel Models



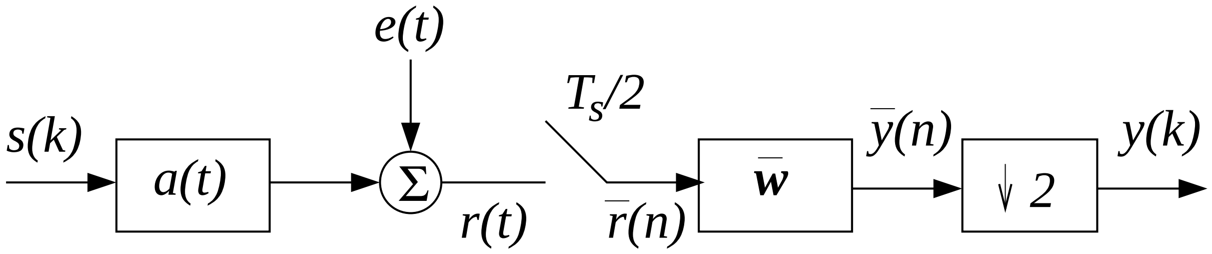

- Channel model classification
  - SISO: single-input single-output systems
  - SIMO: single-input multi-output systems
  - MISO: multi-input single-output systems
  - MIMO: **multi-input multi-output systems**
- Many ways to arrive MIMO, e.g. multi-user systems, multiple transmit and/or receive antennas, sampling faster than symbol rate, etc.
- Consider single-user system with **fractional-spaced sampling**, i.e. at each symbol period take more than one samples, which will result multiple (symbol-rate) subchannel models
- There are **advantages** of fractional-spaced sampling
  - More robust to carrier recovery error and timing recovery error
  - To achieve a perfect reconstruction (remove ISI completely), a finite equaliser can be sufficient (unlike symbol-rate sampling, which requires infinite length)

  A disadvantage: noise samples are no longer white

# Fractional-Spaced Sampling

- **Baseband** model with $T_s/2$-spaced receiver:

  Take two samples during each symbol period




- $T_s/2$-spaced equaliser

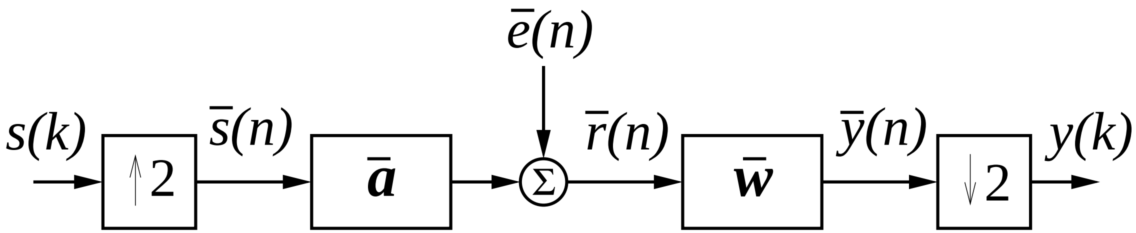
$$\bar{y}(n) = \bar{\mathbf{w}}^H \bar{\mathbf{r}}(n)$$

  with $\bar{\mathbf{r}}(n) = [\bar{r}(n)\ \bar{r}(n-1) \cdots \bar{r}(n-2m+1)]^T$ and $\bar{\mathbf{w}} = [\bar{w}_0\ \bar{w}_1 \cdots \bar{w}_{2m-1}]^T$

- $\bar{y}(n)$ is decimated by a factor of 2 to get $T_s$-spaced output $y(k)$

- Discrete-time baseband model:

  **multirate** model with $T_s/2$-spaced equaliser




  where $k$ indicates $T_s$-spaced quantities and $n$ indicates $T_s/2$-spaced quantities

# Multirate Model (continue)

- $T_s/2$-spaced sequence $\{\bar{s}(n)\}$ is zero-filled version of transmitted symbol sequence $\{s(k)\}$ defined by

$$\bar{s}(n) = \begin{cases} s(n/2), & \text{for even } n \\ 0, & \text{for odd } n \end{cases}$$

- Received $T_s/2$-spaced signal sample is

$$\bar{r}(n) = \sum_{i=0}^{2N_c-1} \bar{a}_i \bar{s}(n-i) + \bar{e}(n)$$

- $T_s/2$-spaced complex-valued channel impulse response (CIR) is given by

$$\bar{\mathbf{a}} = [\bar{a}_0 \ \bar{a}_1 \ \bar{a}_2 \ \bar{a}_3 \cdots \bar{a}_{2N_c-1}]^T$$

- Sampling at twice of symbol rate $\Rightarrow$ Two symbol-spaced models: Odd sample model and even sample model, i.e. **multichannel** model

# Multichannel Model

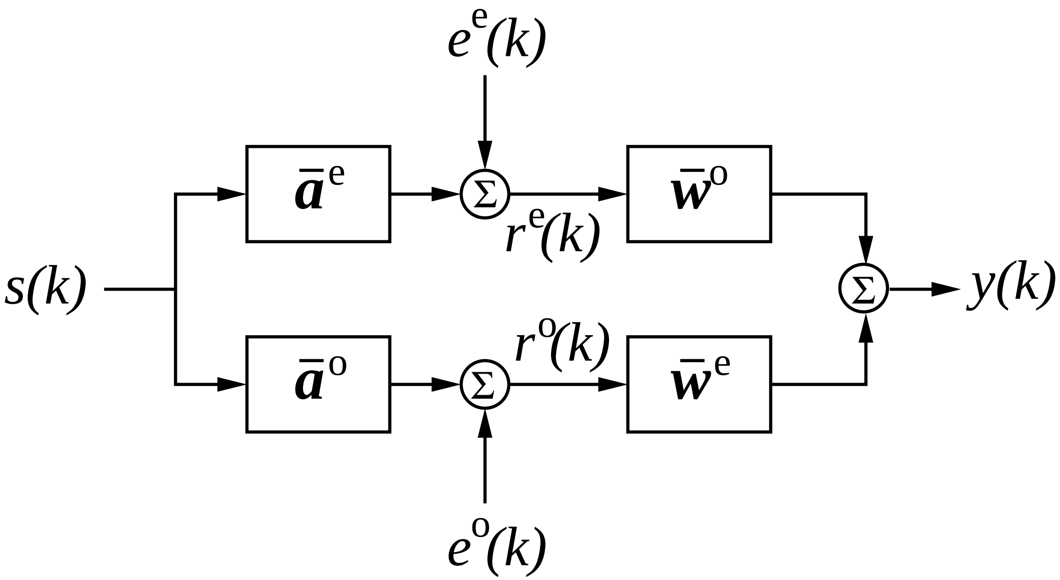
- Take even and odd samples of noise and Rx signal

$$e^{\mathrm{e}}(k) = \bar{e}(2n), \;\; e^{\mathrm{o}}(k) = \bar{e}(2n+1), \;\; r^{\mathrm{e}}(k) = \bar{r}(2n), \;\; r^{\mathrm{o}}(k) = \bar{r}(2n+1)$$

- Define even and odd channels and equalisers as:

$$\begin{aligned}\bar{\mathbf{a}}^{\mathrm{e}} &= [\bar{a}_0 \; \bar{a}_2 \cdots \bar{a}_{2N_c-2}]^T \\ \bar{\mathbf{a}}^{\mathrm{o}} &= [\bar{a}_1 \; \bar{a}_3 \cdots \bar{a}_{2N_c-1}]^T \\ \bar{\mathbf{w}}^{\mathrm{e}} &= [\bar{w}_0 \; \bar{w}_2 \cdots \bar{w}_{2m-2}]^T \\ \bar{\mathbf{w}}^{\mathrm{o}} &= [\bar{w}_1 \; \bar{w}_3 \cdots \bar{w}_{2m-1}]^T\end{aligned}$$

  All are symbol-rate sequences




- Symbol-rate output

$$y(k) = \sum_{i=0}^{2m-1} w_i^* r(k-i) = \mathbf{w}^H \mathbf{r}(k)$$

with $\mathbf{w} = \left[ (\bar{\mathbf{w}}^{\mathrm{o}})^T \;\; (\bar{\mathbf{w}}^{\mathrm{e}})^T \right]^T$ and $\mathbf{r}(k) = \left[ (\mathbf{r}^{\mathrm{e}}(k))^T \;\; (\mathbf{r}^{\mathrm{o}}(k))^T \right]^T$

# Multichannel Model (continue)

- In general, $T_s/K$-spaced sampling will result in $K$ channel models
- As we have symbol-rate model $y(k) = \mathbf{w}^H \mathbf{r}(k)$, all equalisation results apply
- **Blind equalisation example**: consists of a 22-tap $T_s/2$ channel and a 26-tap $T_s/2$ equaliser with 256-QAM and SNR$= 60$ dB
- In simulation, an estimated MSE based on a separate block of data and the maximum distortion measure defined by

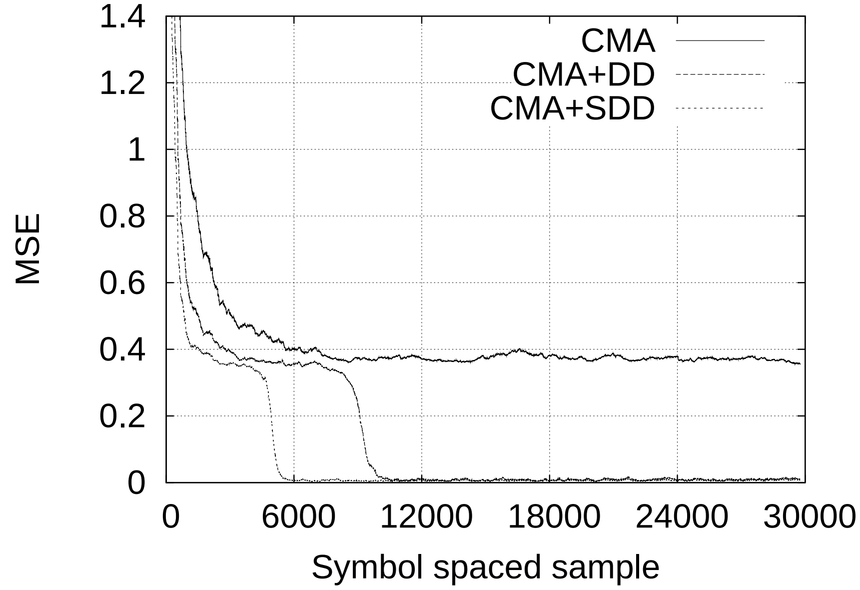
$$\mathrm{MD} = \frac{\sum_{i=0}^{n_{\mathrm{tot}}} |f_i| - |f_{i_{\max}}|}{|f_{i_{\max}}|}$$

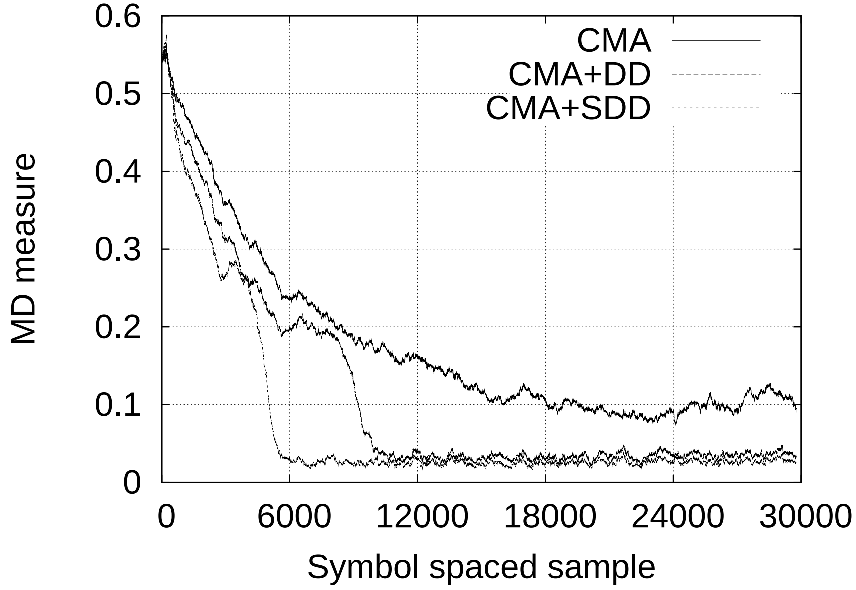
are used to assess convergence rate, where

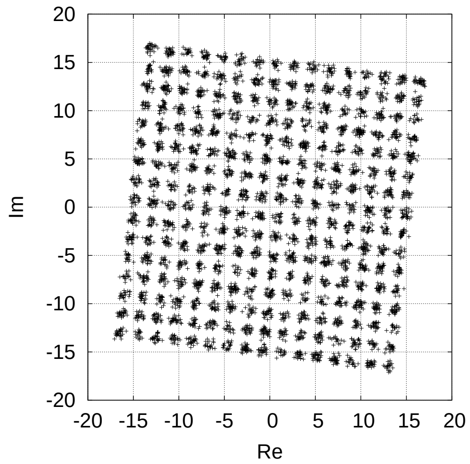
$$\mathbf{f} = [f_0 \ f_1 \cdots f_{n_{\mathrm{tot}}}]^T = (\bar{\mathbf{w}}^{\mathrm{o}})^* \star \bar{\mathbf{a}}^{\mathrm{e}} + (\bar{\mathbf{w}}^{\mathrm{e}})^* \star \bar{\mathbf{a}}^{\mathrm{o}}$$

is the combined impulse response of the channel and equaliser, $f_{i_{\max}} = \max\{f_i, \ 0 \le i \le n_{\mathrm{tot}}\}$

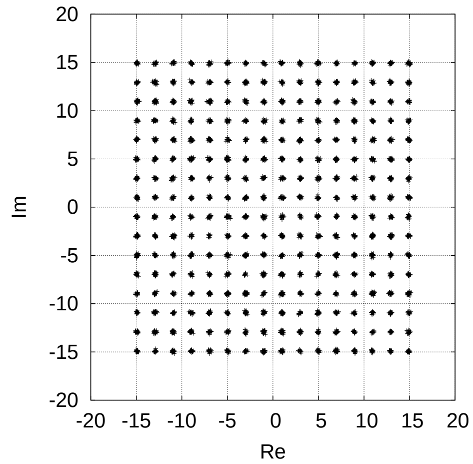
# Simulation Results



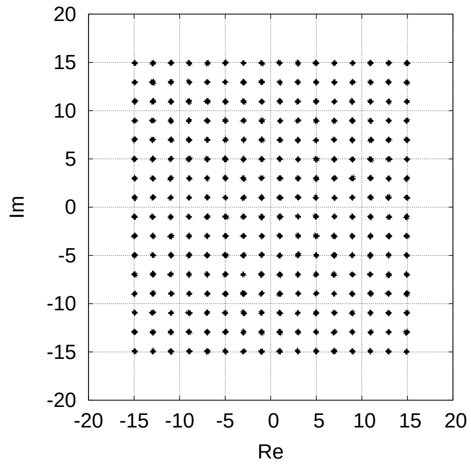




(a) CMA




(b) CMA+DD




(c) CMA+SDD

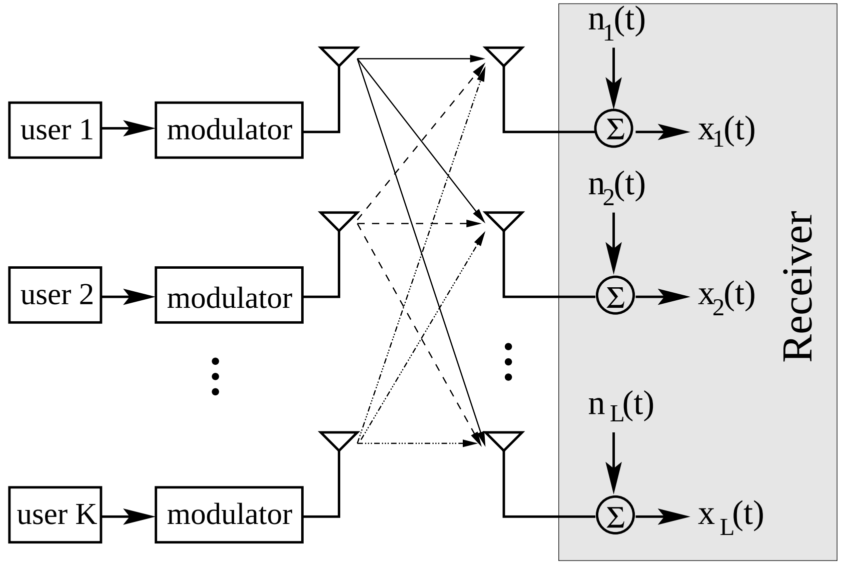
# Adaptive Beamforming Assisted Receiver

- SDMA induced MIMO system:

  Assume one transmit antenna and $L$ receiver antennas supporting $K$ users

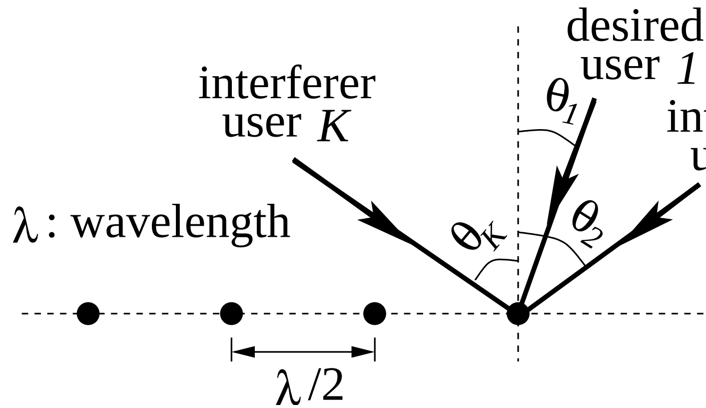
  Narrowband channels with $m_i(k) = A_i b_i(k)$, $A_i$: channel coefficient for user $i$ and $b_i(k)$: $k$th symbol of user $i$

  Assume symbol-rate sampling, user $1$ is desired user and rest are interferers




- Uniformly spaced linear antenna array:

  Let $t_l(\theta_i)$ be relative time delay at array element $l$ for user $i$, $\theta_i$ angle of arrival for user $i$ and carrier $\omega = 2\pi f_c$




$$x_l(k) = \sum_{i=1}^{K} m_i(k) \exp\left(j\omega t_l(\theta_i)\right) + n_l(k) = \bar{x}_l(k) + n_l(k), \;\; 1 \leq l \leq L$$

# Beamforming Assisted Receiver (continue)

- System model: Antenna array output $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is expressed as

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k)$$

where $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ has a covariance matrix of $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$ with $\mathbf{I}_L$ representing the $L \times L$ identity matrix, system matrix $\mathbf{P}$ is given by

$$\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_K\mathbf{s}_K]$$

steering vector for source $i$ is

$$\mathbf{s}_i = [\exp(j\omega t_1(\theta_i))\ \exp(j\omega t_2(\theta_i)) \cdots \exp(j\omega t_L(\theta_i))]^T$$

and transmitted symbol vector $\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_K(k)]^T$

- Beamformer's output is given by

$$y(k) = \mathbf{w}^H\mathbf{x}(k) = \mathbf{w}^H\bar{\mathbf{x}}(k) + \mathbf{w}^H\mathbf{n}(k) = \bar{y}(k) + e(k)$$

where $\mathbf{w} = [w_1\ w_2 \cdots w_L]^T$ is complex-valued beamformer weight vector and $e(k)$ Gaussian distributed having a zero mean and a variance of $E[|e(k;\mathbf{w})|^2] = 2\sigma_n^2\mathbf{w}^H\mathbf{w}$

# Beamforming Assisted QAM Receiver

- Assume $M$-QAM modulation and define combined impulse response of beamformer and system as

$$\mathbf{w}^H \mathbf{P} = \mathbf{w}^H [\mathbf{p}_1 \ \mathbf{p}_2 \cdots \mathbf{p}_K] = [c_1 \ c_2 \cdots c_K]$$

The beamformer's output can also be expressed as

$$y(k) = c_1 b_1(k) + \sum_{i=2}^{K} c_i b_i(k) + e(k)$$

- Define decision variable as $d(k) = d_R(k) + j d_I(k) = \frac{y(k)}{c_1}$, then symbol decision $\hat{b}_1(k) = \hat{b}_{R_1}(k) + j\hat{b}_{I_1}(k)$ is given by

$$\hat{b}_{R_1}(k) = \begin{cases} u_1, & \text{if } \ d_R(k) \le u_1 + 1 \\ u_l, & \text{if } \ u_l - 1 < d_R(k) \le u_l + 1 \text{ for } 2 \le l \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } \ d_R(k) > u_{\sqrt{M}} - 1 \end{cases}$$

$$\hat{b}_{I_1}(k) = \begin{cases} u_1, & \text{if } \ d_I(k) \le u_1 + 1 \\ u_q, & \text{if } \ u_q - 1 < d_I(k) \le u_q + 1 \text{ for } 2 \le q \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } \ d_I(k) > u_{\sqrt{M}} - 1 \end{cases}$$

where $M$-QAM symbol set is defined as $\{u_l + j u_q, \ 1 \le l, q \le \sqrt{M}\}$

# Adaptive Beamforming Solutions

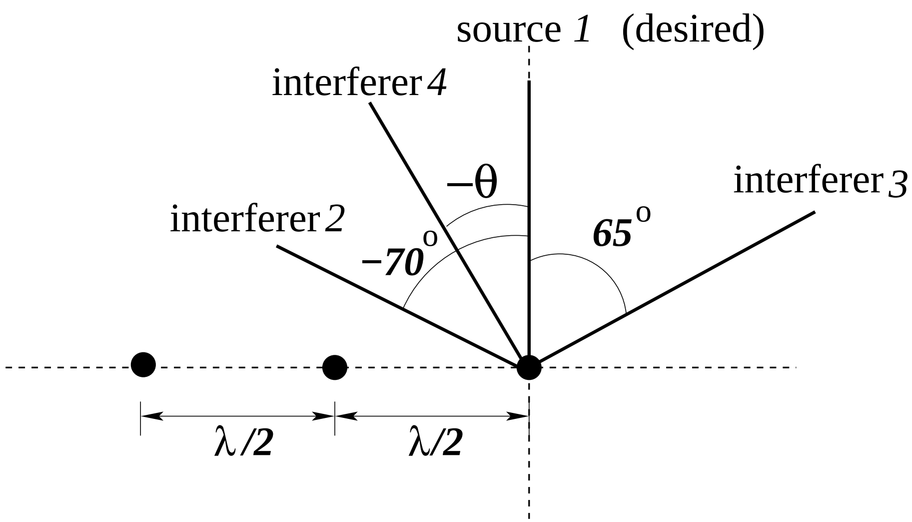
- Let $E[|b_1(k)|^2] = \sigma_s^2$. Then minimum mean square error solution:

$$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{P}\mathbf{P}^H + \frac{2\sigma_n^2}{\sigma_s^2}\mathbf{I}_L\right)^{-1} \mathbf{p}_1$$

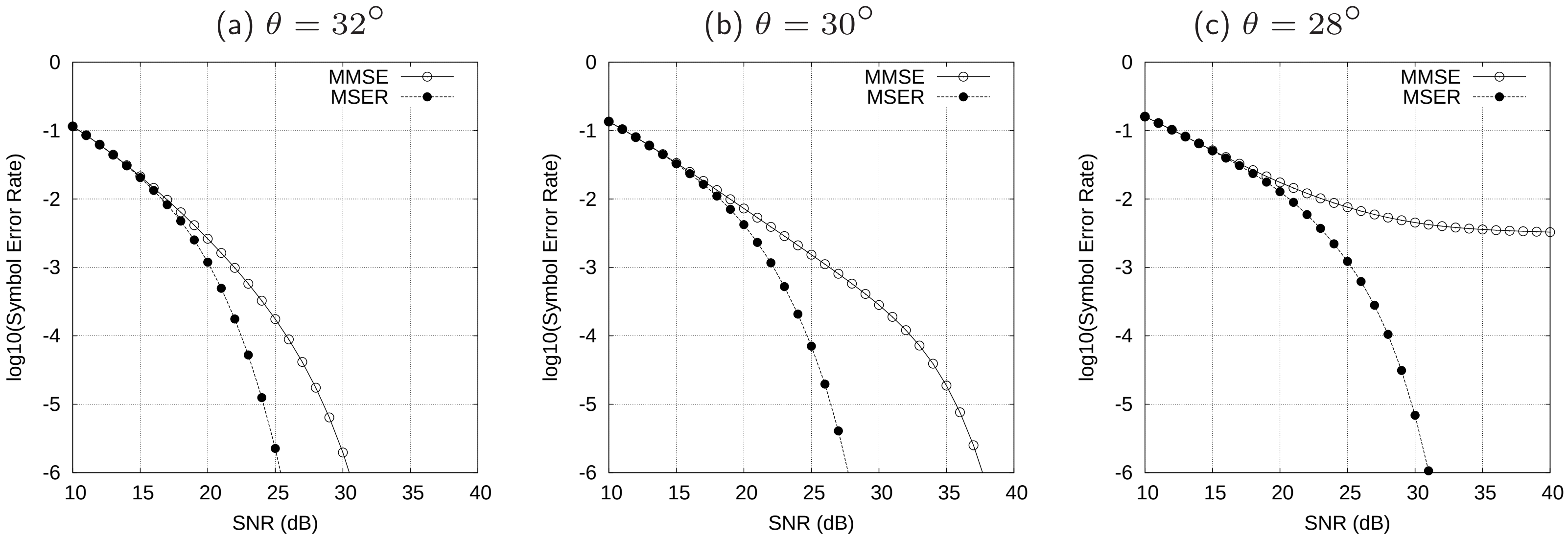
- MMSE solution can be implemented adaptively using the LMS or RLS algorithm
- We can derive minimum symbol error rate solution $\mathbf{w}_{\mathrm{MSER}}$ for general QAM, but unlike the MMSE solution, there is no closed-form solution for $\mathbf{w}_{\mathrm{MSER}}$ and gradient optimisation must be used
- MSER solution can be implemented adaptively using the LSER algorithm
- For details see:

  S. Chen, H.-Q. Du and L. Hanzo, "Adaptive minimum symbol error rate beamforming assisted receiver for quadrature amplitude modulation systems," in *VTC2006-Spring* (Melbourne, Australia), May 7-10, 2006

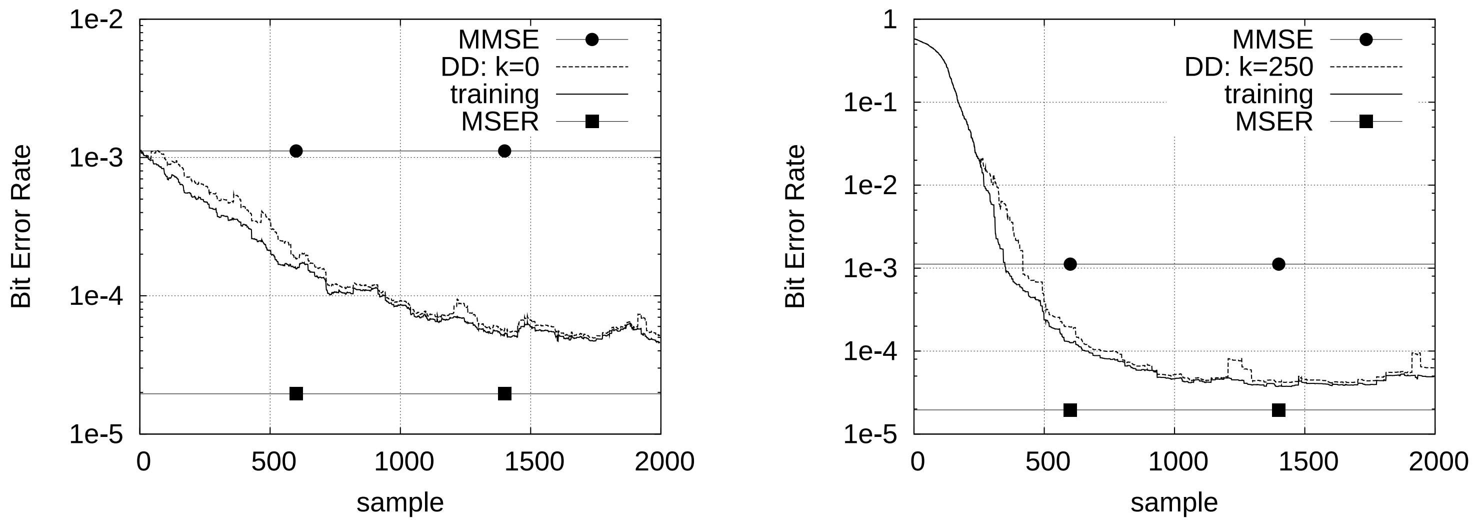
  PDF copy can be download from:
  http://www.ecs.soton.ac.uk/~sqc/EZ412-612/
- Example: 16QAM, 4 users, 3-element antenna array



# Beamforming Results




$\theta = 30^{\circ}$ and SNR$=26$ dB, LSER



# Summary

- MIMO system introduction
- Single-user fractional-spaced receiver

  Baseband continuous-time model, discrete-time multirate model, discrete-time multichannel model
- Adaptive beamforming assisted receiver for QAM modulation

  System model, beamformer model, MMSE solution and MSER solution, adaptive implementation