Revision of Lecture Twenty-Three

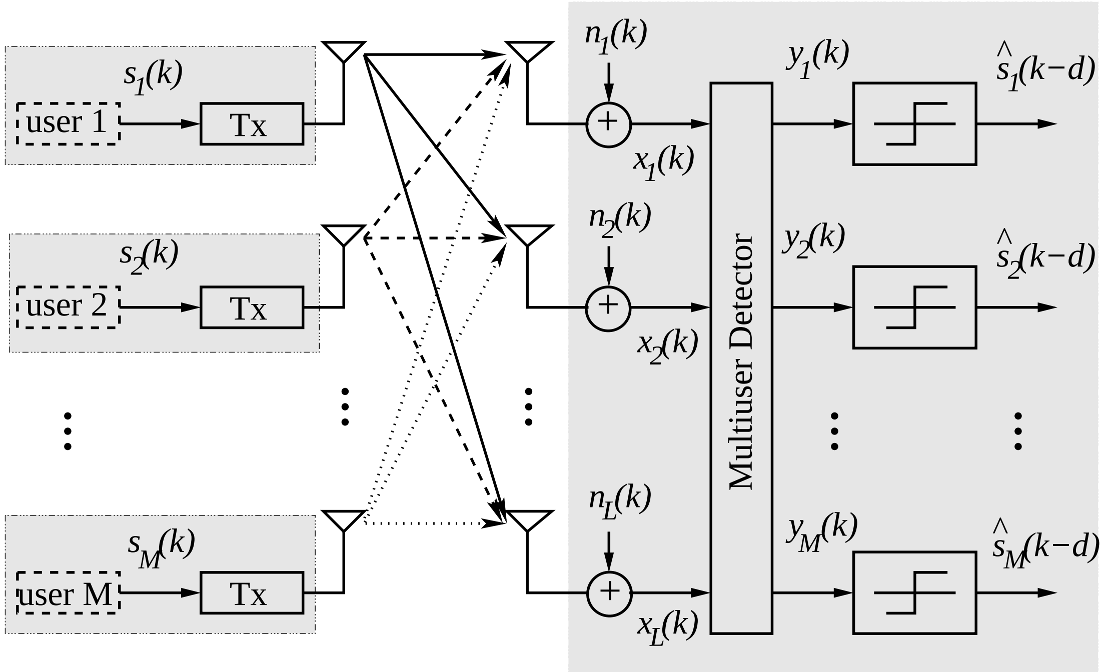
- MIMO classification: roughly three classes – create diversity, increase throughput, support multi-users
- Single-user fractional-spaced receiver

  Baseband continuous-time model, discrete-time multirate model, discrete-time multichannel model
- Adaptive beamforming assisted receiver
- Last lecture carries on MIMO A, B, C

SDMA Systems

- SDMA induced MIMO system:
  - Assume one transmit antenna and L receiver antennas supporting M users
  - No specific antenna array structure is assumed, so it is most generic
  - Channels are frequency selective, and CIR connecting user m and lth receiver antenna is

    $\mathbf{c}_{l,m} = [c_{0,l,m} \, c_{1,l,m} \cdots c_{n_C-1,l,m}]^T$



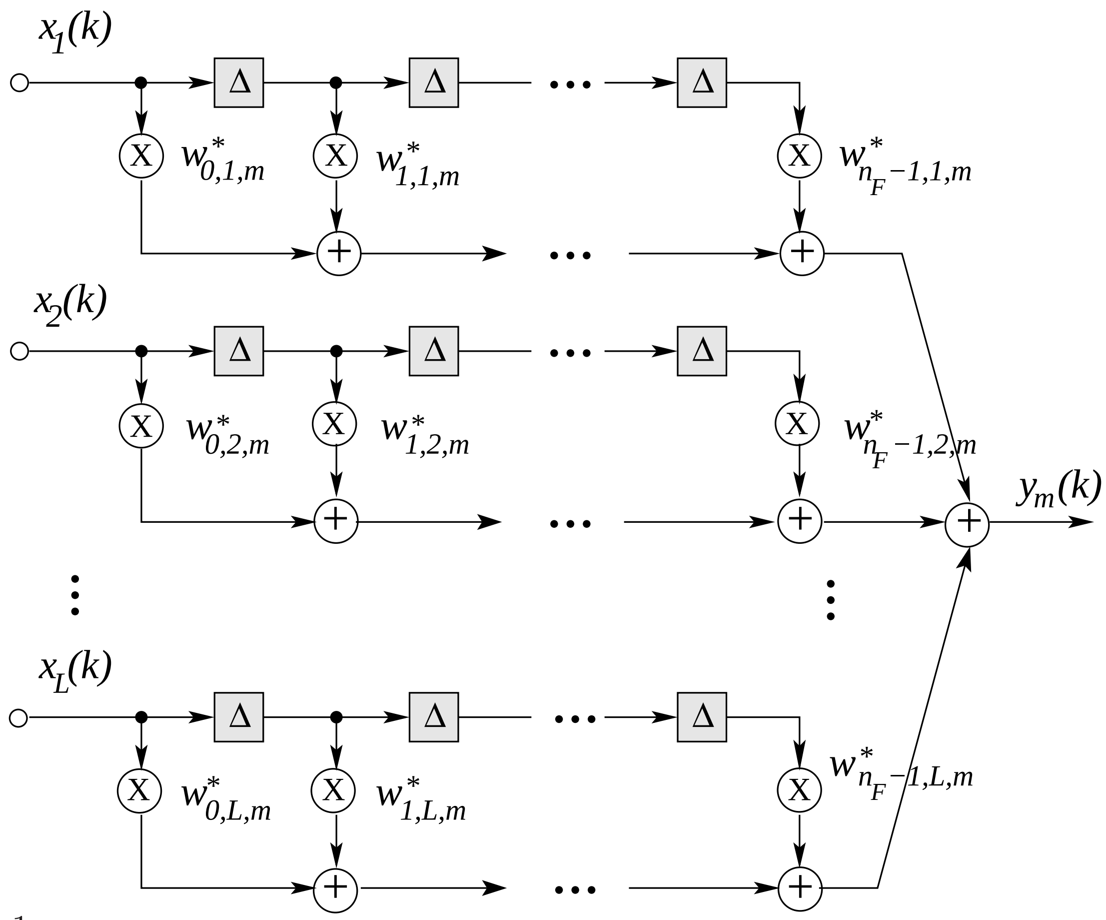
- Symbol-rate received signal samples $x_l(k)$ for $1 \le l \le L$ are given by

$$x_l(k) = \sum_{m=1}^{M} \sum_{i=0}^{n_C-1} c_{i,l,m} s_m(k-i) + n_l(k) = \bar{x}_l(k) + n_l(k)$$

$n_l(k)$ is complex-valued AWGN with $E[|n_l(k)|^2] = 2\sigma_n^2$, $\bar{x}_l(k)$ is noise-free part of lth receive antenna's output, $s_m(k)$ is kth transmitted symbol of user m (assuming BPSK for simplicity)

Multiuser Detection in SDMA Systems

- Multiuser supporting capability
  - CDMA: each user is separated by a unique user-specific spreading code
  - SDMA: each user is associated with a unique user-specific CIR encountered at receiver antennas
  - Unique user-specific CIR plays role of user-specific CDMA signature
  - Owing to non-orthogonal nature of CIRs, effective multiuser detection is required for separating users
- A bank of M space-time equalisers forms MUD, whose soft outputs are



$$y_m(k) = \sum_{l=1}^{L} \sum_{i=0}^{n_F-1} w^*_{i,l,m} x_l(k-i), \;\; 1 \le m \le M$$

$\mathbf{w}_{l,m} = [w_{0,l,m} \; w_{1,l,m} \cdots w_{n_F-1,l,m}]^T$ is mth user detector's equaliser weight vector associated with lth receive antenna, STE has order n_F and decision delay d

System Model

- Define $n_F \times (n_F + n_C - 1)$ CIR matrix associated with user m and lth receive antenna

$$\mathbf{C}_{l,m} = \begin{bmatrix} c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & 0 & \cdots & 0 \\ 0 & c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c_{0,l,m} & c_{1,l,m} & \cdots & c_{n_C-1,l,m} \end{bmatrix}$$

- Introduce overall system CIR convolution matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \cdots & \mathbf{C}_{1,M} \\ \mathbf{C}_{2,1} & \mathbf{C}_{2,2} & \cdots & \mathbf{C}_{2,M} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{C}_{L,1} & \mathbf{C}_{L,2} & \cdots & \mathbf{C}_{L,M} \end{bmatrix}$$

- Then received signal vector $\mathbf{x}(k) = [\mathbf{x}_1(k)\ \mathbf{x}_2(k) \cdots \mathbf{x}_L(k)]^T$ can be expressed by

$$\mathbf{x}(k) = \mathbf{C}\,\mathbf{s}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

where $\mathbf{x}_l(k) = [x_l(k)\ x_l(k-1) \cdots x_l(k-n_F+1)]^T$ for $1 \le l \le L$, $\mathbf{n}(k) = [\mathbf{n}_1(k)\ \mathbf{n}_2(k) \cdots \mathbf{n}_L(k)]^T$ with $\mathbf{n}_l(k) = [n_l(k)\ n_l(k-1) \cdots n_l(k-n_F+1)]^T$, and $\mathbf{s}(k) = [\mathbf{s}_1^T(k)\ \mathbf{s}_2^T(k) \cdots \mathbf{s}_M^T(k)]^T$ with $\mathbf{s}_m(k) = [s_m(k)\ s_m(k-1) \cdots s_m(k-n_F-n_C+2)]^T$

Space-Time Equalisation

- Output of mth STE detector can be written as

$$y_m(k) = \sum_{l=1}^{L} \mathbf{w}_{l,m}^H \mathbf{x}_l(k) = \mathbf{w}_m^H \mathbf{x}(k)$$

where $\mathbf{w}_m = [\mathbf{w}_{1,m}^T \ \mathbf{w}_{2,m}^T \cdots \mathbf{w}_{L,m}^T]^T$

- With $y_{R_m}(k) = \text{Re}[y_m(k)]$, M user detectors' decisions are defined by

$$\hat{s}_m(k-d) = \text{sgn}\left(y_{R_m}(k)\right), \ 1 \le m \le M$$

- Minimum mean square error solution is defined by closed-form

$$\mathbf{w}_{(\text{MMSE})m} = \left(\mathbf{C}\,\mathbf{C}^H + 2\sigma_n^2 \mathbf{I}\right)^{-1} \mathbf{C}_{|(m-1)(n_F+n_C-1)+(d+1)}$$

for $1 \le m \le M$, where $\mathbf{I}$ denotes $Ln_F \times Ln_F$ identity matrix and $\mathbf{C}_{|i}$ the ith column of $\mathbf{C}$

- Adaptive implementation using LMS algorithm

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu \mathbf{x}(k) \epsilon^*(k)$$

where $\epsilon(k) = s_m(k-d) - y_m(k)$

Bit Error Rate of Space-Time Equaliser

- Note transmitted symbol sequence $\mathbf{s}(k) \in \{\mathbf{s}^{(q)}, 1 \le q \le N_s\}$, where $N_s = 2^{M(n_F+n_C-1)}$
- Let the element of $\mathbf{s}^{(q)}$ corresponding to desired symbol $s_m(k-d)$ be $s_{m,d}^{(q)}$
- Noise-free part of mth detector input signal $\bar{\mathbf{x}}(k)$ assumes values from signal set $\mathcal{X}_m = \{\bar{\mathbf{x}}^{(q)} = \mathbf{C}\,\mathbf{s}^{(q)}, 1 \le q \le N_s\}$
- $\mathcal{X}_m$ can be partitioned into two subsets, depending on the value of $s_m(k-d)$, as follows $\mathcal{X}_m^{(\pm)} = \{\bar{\mathbf{x}}^{(q,\pm)} \in \mathcal{X}_m : s_m(k-d) = \pm 1\}$
- Similarly, noise-free part of mth detector's output $\bar{y}_m(k)$ assumes values from the scalar set

$$\mathcal{Y}_m = \{\bar{y}_m^{(q)} = \mathbf{w}_m^H \bar{\mathbf{x}}^{(q)}, 1 \le q \le N_s\}$$

- Thus $\bar{y}_{Rm}(k) = \text{Re}[\bar{y}_m(k)]$ can only take the values from the set

$$\mathcal{Y}_{Rm} = \{\bar{y}_{Rm}^{(q)} = \text{Re}[\bar{y}_m^{(q)}], 1 \le q \le N_s\}$$

- $\mathcal{Y}_{Rm}$ can be divided into the two subsets conditioned on the value of $s_m(k-d)$

$$\mathcal{Y}_{Rm}^{(\pm)} = \{\bar{y}_{Rm}^{(q,\pm)} \in \mathcal{Y}_{Rm} : s_m(k-d) = \pm 1\}$$

Bit Error Rate of STE (continue)

- Conditional PDF of $y_{Rm}(k)$ given $s_m(k-d) = +1$ is a Gaussian mixture

$$p_m(y_R| + 1) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}} e^{-\frac{\left(y_R - \bar{y}_{Rm}^{(q,+)}\right)^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}}$$

  where $\bar{y}_{Rm}^{(q,+)} \in \mathcal{Y}_{Rm}^{(+)}$ and $N_{sb} = N_s/2$ is the number of points in $\mathcal{Y}_{Rm}^{(+)}$

- Thus BER of the mth detector associated with the detector's weight vector $\mathbf{w}_m$ is given by

$$P_E(\mathbf{w}_m) = \frac{1}{N_{sb}} \sum_{q=1}^{N_{sb}} Q\left(g^{(q,+)}(\mathbf{w}_m)\right)$$

  where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{v^2}{2}} \, d\, v \quad \text{and} \quad g^{(q,+)}(\mathbf{w}_m) = \frac{\text{sgn}(s_{m,d}^{(q)}) \bar{y}_{Rm}^{(q,+)}}{\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}}$$

- Note that BER is invariant to a positive scaling of $\mathbf{w}_m$
- Alternatively, the BER may be calculated based on the other subset $\mathcal{Y}_{Rm}^{(-)}$.

Minimum Bit Error Rate Solution

- MBER solution for the mth STE detector is defined as

$$\mathbf{w}_{(\mathrm{MBER})m} = \arg \min_{\mathbf{w}_m} P_E(\mathbf{w}_m)$$

- No closed-form solution, but gradient of $P_E(\mathbf{w}_m)$ is

$$\nabla P_E(\mathbf{w}_m) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}_m^H\mathbf{w}_m}} \sum_{q=1}^{N_{sb}} e^{-\frac{\left(\bar{y}_{R_m}^{(q,+)}\right)^2}{2\sigma_n^2\mathbf{w}_m^H\mathbf{w}_m}} \mathrm{sgn}\left(s_{m,d}^{(q)}\right)\left(\frac{\bar{y}_{R_m}^{(q,+)}\mathbf{w}_m}{\mathbf{w}_m^H\mathbf{w}_m} - \bar{\mathbf{x}}^{(q,+)}\right)$$

  Gradient optimisation can be applied to obtain a $\mathbf{w}_{(\mathrm{MBER})m}$

- Adaptive implementation using LBER algorithm

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu \frac{\mathrm{sgn}(s_m(k-d))}{2\sqrt{2\pi}\rho_n} e^{-\frac{y_{R_m}^2(k)}{2\rho_n^2}} \mathbf{x}(k)$$

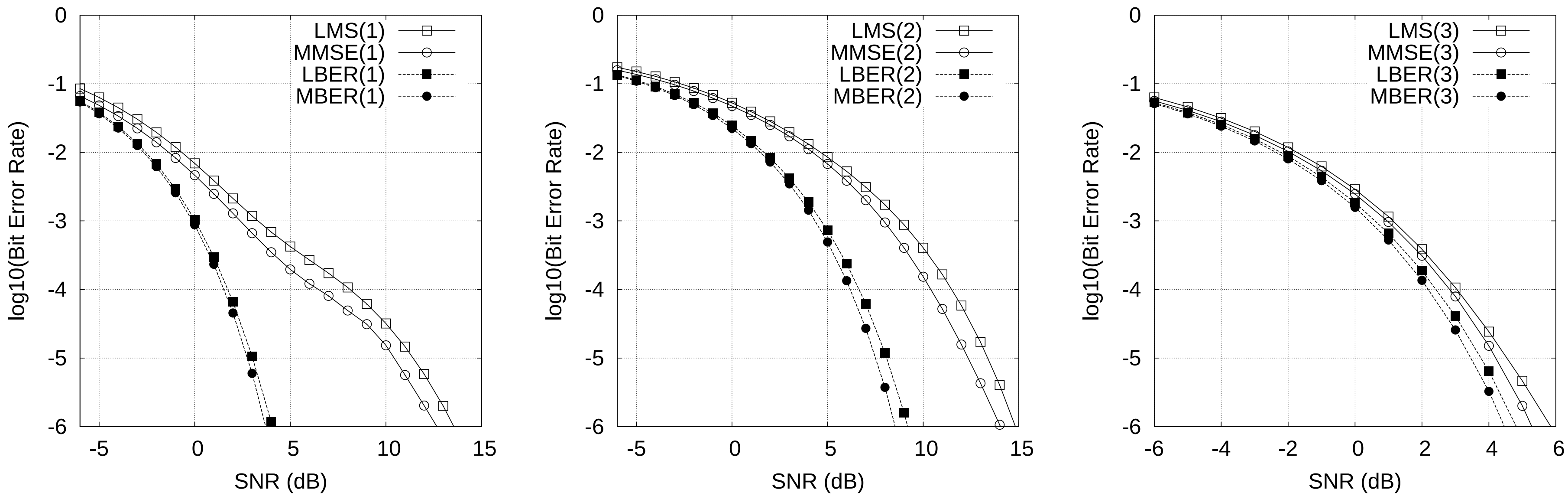
  where μ is adaptive gain, and ρ_n kernel width

Simulation Results: Stationary System

- CIRs of 3-user 4-antenna stationary system

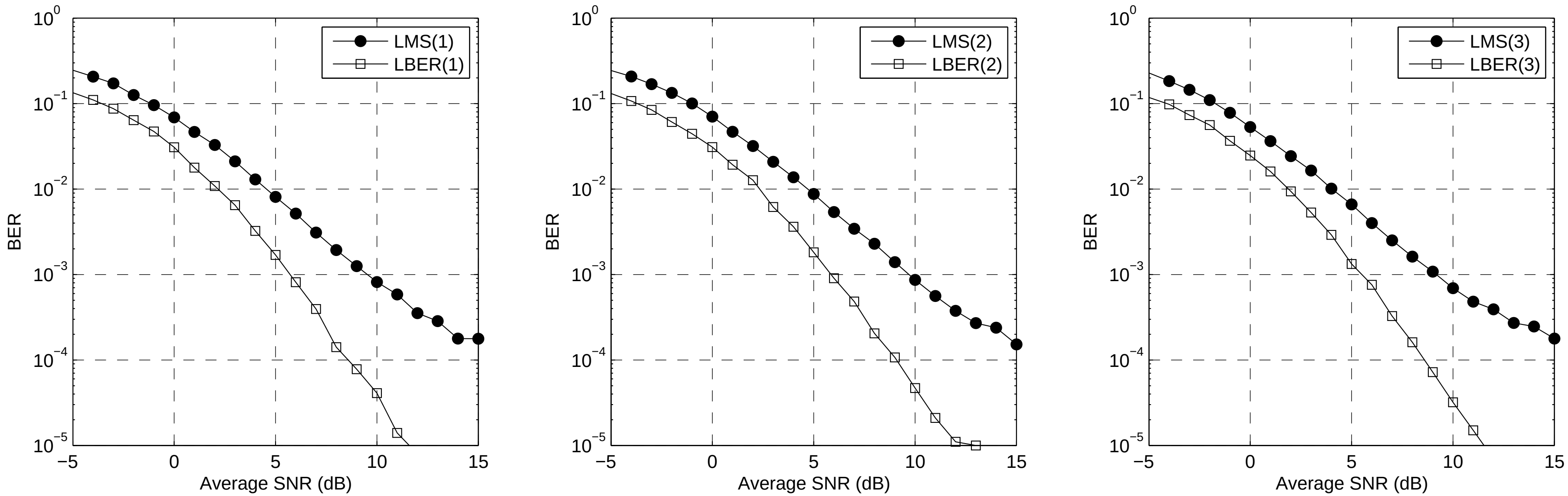
$C_{l,m}(z)$	$m=1$	$m=2$	$m=3$
$l=1$	$(-0.5+j0.4)+(0.7+j0.6)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(-0.7+j0.9)+(0.6+j0.4)z^{-1}$
$l=2$	$(0.5-j0.4)+(-0.8-j0.3)z^{-1}$	$(-0.3+j0.5)+(-0.7-j0.9)z^{-1}$	$(-0.6+j0.8)+(-0.6-j0.7)z^{-1}$
$l=3$	$(0.4-j0.4)+(-0.7-j0.8)z^{-1}$	$(-0.1-j0.2)+(0.7+j0.6)z^{-1}$	$(0.3-j0.5)+(0.9+j0.1)z^{-1}$
$l=4$	$(0.5+j0.5)+(0.6-j0.9)z^{-1}$	$(-0.6-j0.4)+(0.9-j0.4)z^{-1}$	$(-0.6-j0.6)+(0.8+j0.0)z^{-1}$

- CIR order $n_C = 2$, STE order $n_F = 3$ and decision delay $d = 1$
- BER comparison of MMSE/MBER and LMS/LBER for three users



Simulation Results: Fading System

- 3 users, 4 receive antennas, and Rayleigh fading channels with each of 12 CIRs having $n_C = 3$ taps
- Each channel tap has root mean power of $\sqrt{0.5} + j\sqrt{0.5}$
- Normalised Doppler frequency for simulated system was 10^{-5}, which for a carrier of 900 MHz and a symbol rate of 3 Msymbols/s corresponded to a user velocity of 10 m/s (36 km/h)
- STE order $n_F = 5$ and decision delay $d = 2$
- Frame structure: 50 training symbols followed by 450 data symbols
- BER comparison of LMS/LBER for three users



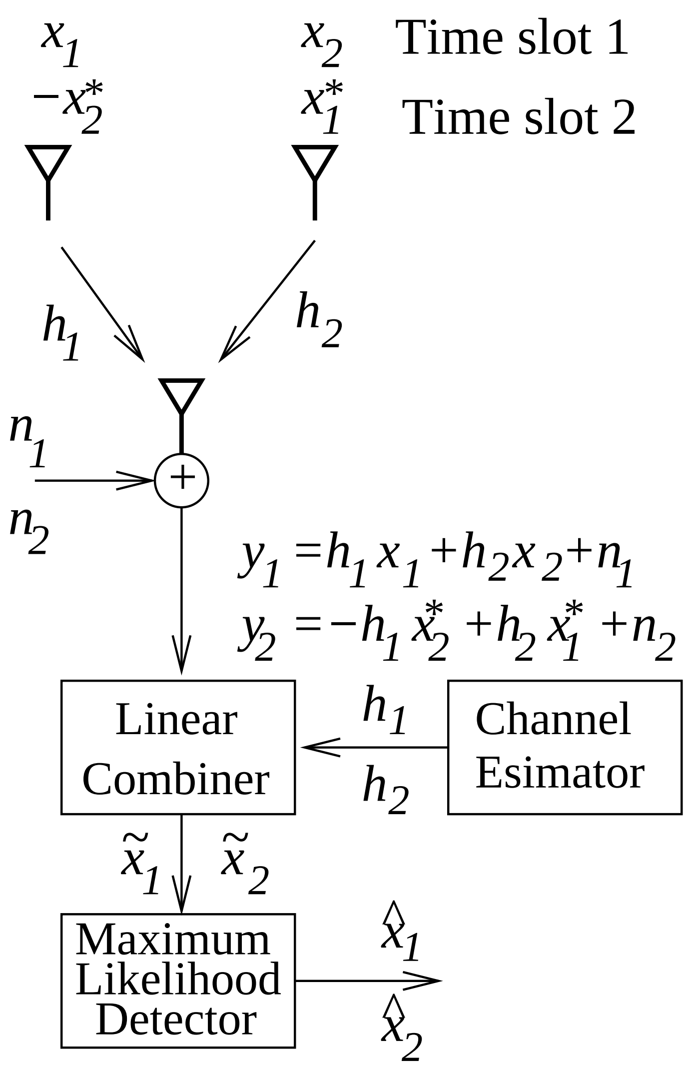
$\mathbf{G}_2$ Space-Time Block Code

- Alamouti's G_2 space-time block code uses two transmitter antennas and one receiver antenna
  - In time slot 1 (one symbol period), two symbols (x_1, x_2) are transmitted
  - While in time slot 2, transformed (x_1, x_2), i.e. $(-x_2^*, x_1^*)$, are transmitted

- Assume narrowband channels with channel 1, $h_1 = |h_1|e^{j\alpha_1}$ and channel 2, $h_2 = |h_2|e^{j\alpha_2}$

- Antenna spacing is sufficiently large, e.g. 10 wavelengths, so two channels are independently faded

- Fading is sufficiently slow so during two time slots channels h_1, h_2 are unchanged



G_2 STBC (continue)

- Received signals at two time slots are respectively

$$\begin{aligned} y_1 &= h_1 x_1 + h_2 x_2 + n_1 \\ y_2 &= -h_1 x_2^* + h_2 x_1^* + n_2 \end{aligned}$$

- Assume perfect channel estimate h_1, h_2, linear combiner's outputs are

$$\begin{aligned} \tilde{x}_1 = h_1^* y_1 + h_2 y_2^* = (|h_1|^2 + |h_2|^2) x_1 + h_1^* n_1 + h_2 n_2^* \\ \tilde{x}_2 = h_2^* y_1 - h_1 y_2^* = (|h_1|^2 + |h_2|^2) x_2 + h_2^* n_1 + h_1 n_2^* \end{aligned}$$

- Maximum likelihood decoding involves minimising decision metric

$$|\tilde{x}_1 - (|h_1|^2 + |h_2|^2) x_1|^2$$

for decoding x_1 and minimising decision metric

$$|\tilde{x}_2 - (|h_1|^2 + |h_2|^2) x_2|^2$$

for decoding x_2

Space-Time Block Codes

- Encoding: generic STBC is defined by $n \times p$ transmission matrix

$$G = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1p} \\ g_{21} & g_{22} & \cdots & g_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ g_{n1} & g_{n2} & \cdots & g_{np} \end{bmatrix} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & \vdots & \cdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{bmatrix}$$

  Each entry $g_{ij} = x_{i,j}$ is a linear combination of k input symbols $x_1, x_2, \cdots x_k$ and their conjugates

- Number of rows n is equal to number of time slots, and number of columns is equal to number of transmit antennas

- During time slot i, encoded symbols $x_{i,1}, x_{i,2}, \cdots, x_{i,p}$ are transmitted simultaneously from transmit antennas $1, 2, \cdots, p$, respectively

- Code rate is obviously $R = k/n$

- Assume L receiver antennas, and channel connecting jth transmit antenna and lth receiver antenna is $h_{j,l}$, then received signal arriving at receiver l during time slot i is

$$y_{i,l} = \sum_{j=1}^{p} h_{j,l} x_{i,j} + n_{j,l}$$

  where $n_{j,l}$ is AWGN for j, l-th channel

Space-Time Block Codes (continue)

- Decoding: assuming perfect channel estimate, maximum likelihood decoding decides in favour of specific entry $x_{i,j}$, $1 \le i \le n$, $1 \le j \le p$, that minimises the decision metric

$$\sum_{i=1}^{n}\sum_{l=1}^{L}\left| y_{i,l} - \sum_{j=1}^{p} h_{j,l} x_{i,j} \right|^2$$

- An alternative is maximum a posteriori probability decoding, for details see relevant reference
- STBC examples (transmit antennas $p = 2, 3, 4$)

$$G_2 = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}, \quad G_3 = \begin{bmatrix} x_1 & x_2 & x_3 \\ -x_2 & x_1 & -x_4 \\ -x_3 & x_4 & x_1 \\ -x_4 & -x_3 & x_2 \\ x_1^* & x_2^* & x_3^* \\ -x_2^* & x_1^* & -x_4^* \\ -x_3^* & x_4^* & x_1^* \\ -x_4^* & -x_3^* & x_2^* \end{bmatrix}, \quad G_4 = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}$$

G_2 has time slots $n = 2$, G_3 and G_4 have time slots $n = 8$

STBC Examples (continue)

- STBC examples (transmit antennas $p = 3, 4$)

$$H_3 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1-x_1^*+x_2-x_2^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2+x_2^*+x_1-x_1^*}{2} \end{bmatrix}, H_4 = \begin{bmatrix} x_1 & x_2 & \frac{x_3}{\sqrt{2}} & \frac{x_3}{\sqrt{2}} \\ -x_2^* & x_1^* & \frac{x_3}{\sqrt{2}} & -\frac{x_3}{\sqrt{2}} \\ \frac{x_3^*}{\sqrt{2}} & \frac{x_3^*}{\sqrt{2}} & \frac{-x_1-x_1^*+x_2-x_2^*}{2} & \frac{-x_2-x_2^*+x_1-x_1^*}{2} \\ \frac{x_3^*}{\sqrt{2}} & -\frac{x_3^*}{\sqrt{2}} & \frac{x_2+x_2^*+x_1-x_1^*}{2} & \frac{-x_1-x_1^*-x_2+x_2^*}{2} \end{bmatrix}$$

H_3 and H_4 have time slots $n = 4$

- Parameters of space-time block codes

space-time block code	code rate R	number of transmitters p	number of input symbol k	number of time slots n
G_2	1	2	2	2
G_3	1/2	3	4	8
G_4	1/2	4	4	8
H_3	3/4	3	3	4
H_4	3/4	4	3	4

Summary

- Multiuser capacity of SDMA systems
- Space-time equalisation assisted multiuser detection for SDMA systems

  MMSE design and MBER design, adaptive implementation
- Space-time block codes