Revision of Lecture Five

- Recall basic components of MODEM in slide 48

- A central point is associated with a digital modulation scheme, there is **channel capacity** (What is channel capacity?)

  This channel capacity is smaller than the channel capacity of ideal AWGN channel with Gaussian signal (recall capacity is maximised if signal PDF is Gaussian)

  Nevertheless, we may use the latter as upper limit for our practical digital modulated channel as first approximation

- As channel capacity is linked with bandwidth and signal to noise ratio, not surprisingly, performance measures of a digital modulation scheme are: **power efficiency** and **bandwidth efficiency**

- This lecture we continue on Modem, and look into **phase shift keying modulation**, in particular, BPSK and QPSK, with emphasis on operations of **carrier recovery** and **timing recovery**

Phase Shift Keying

- In PSK, the modulation signal set is:

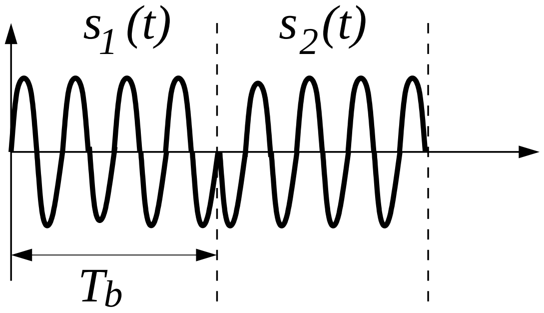
$$s_i(t) = A\cos(2\pi f_c t + \phi_i(t)),\ i = 1, \cdots, M,\ 0 \le t \le T_s$$

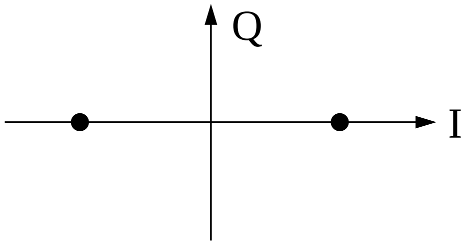
  - T_s is symbol period, A is carrier amplitude (constant), "**phase**" $\phi_i(t)$ carries symbol information, and $\log_2 M$ bits per symbol
  - BPSK, QPSK, 8-PSK, etc with 1 bit per symbol, 2 bits per symbol, 3 bits per symbol, etc, and minimum phase separation 180°, 90°, 45°, etc, respectively

- BPSK: $M = 2$. It is convention to use $m_1 = 1$ for bit 0, and $m_2 = -1$ for bit 1

$$s_i(t) = A\cos(2\pi f_c t + (i-1)\pi + \theta_c),\ 0 \le t \le T_b$$

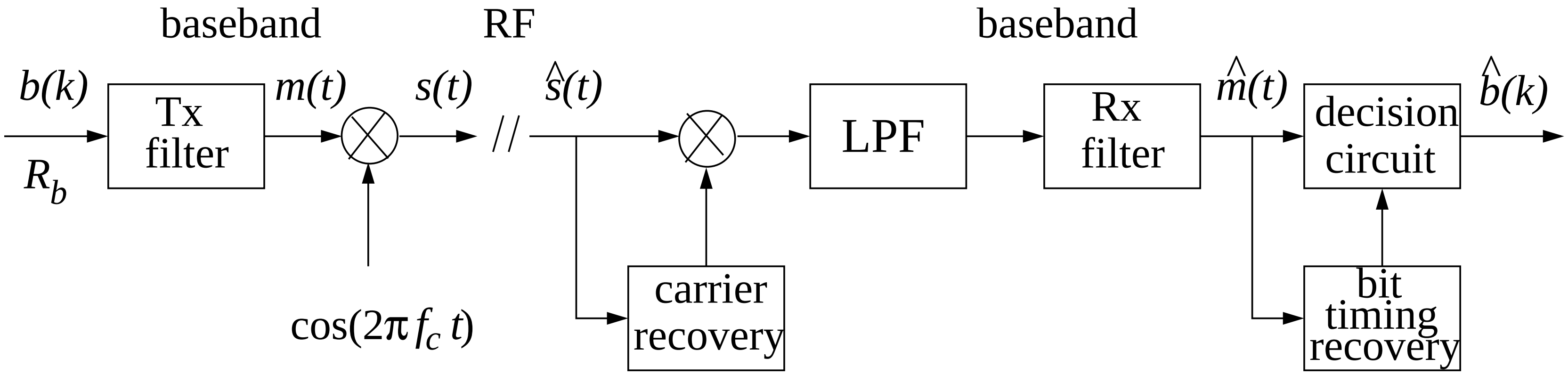
T_b: bit period
θ_c: an initial phase
Energy per bit for BPSK: $E_b = \frac{1}{2}A^2 T_b$
or $A = \sqrt{\frac{2E_b}{T_b}}$



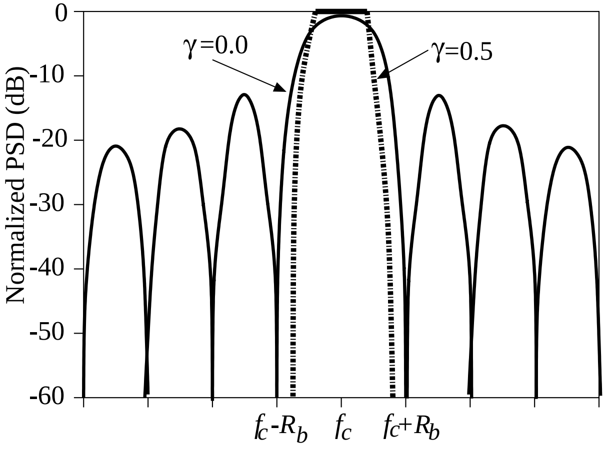


BPSK MODEM

- Simplified BPSK MODEM diagram



- **Transmitter**: Data bit stream with bit rate R_b are filtered by a lowpass filter (square root of raised cosine pulse shaping filter) to generate baseband signal $m(t)$, which is then modulated by carrier
  - PSD of BPSK RF signal with raised cosine pulse shaping:



  - Baseband complex envelope signal

$$g(t) = m(t) A \exp(j\theta_c)$$

    $m(t)$ being pulse shaped symbol m_1 or m_2
  - Transmitted BPSK signal

$$s(t) = \text{Re}[g(t) \exp(j2\pi f_c t)] = m(t) A \cos(2\pi f_c t + \theta_c)$$

BPSK MODEM (continue)

- **Receiver**: Assuming no channel distortion, omitting noise and dropping amplitude A, the received signal:

  $$\hat{s}(t) = m(t)\cos(2\pi f_c t + \theta)$$

  θ includes phase shift due to channel time delay

  - Carrier recovery obtains carrier $\cos(2\pi f_c t + \theta)$, and

    $$\hat{s}(t) \cdot \cos(\omega_c t + \theta) = \frac{m(t)}{2} \cdot (1 + \cos(2\omega_c t + 2\theta))$$

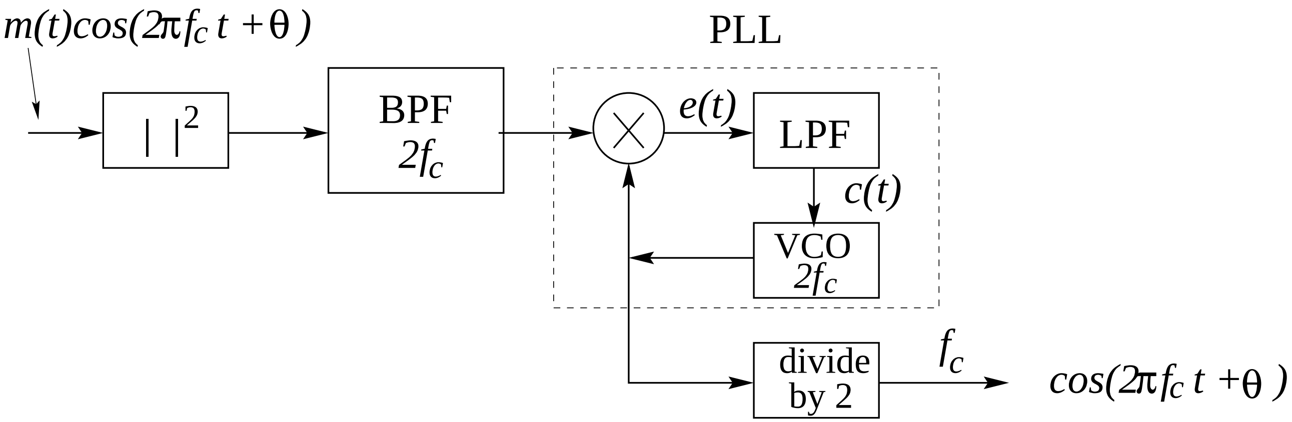
  - LPF at receiver filters this to obtain $m(t)$
  - Bit (symbol) timing recovery recovers clock pulses to obtain samples at appropriate instances for decision circuit, which detects transmitted bits (symbols)

- Carrier recovery: operate at RF to try to align receiver local oscillator with transmitted carrier frequency (and phase), which is only required for coherent or synchronous demodulation
- Clock recovery: operate at baseband to try to synchronise receiver clock with baseband symbol rate transmitter clock, which is needed for any receiver (coherent or non-coherent demodulation)
- Coherent receiver has better performance but higher complexity than non-coherent receiver

Carrier Recovery for BPSK

- Let received RF signal be $\hat{s}(t) = m(t)\cos(\omega_c t + \theta)$
  - If receiver knows carrier $\cos(\omega_c t + \theta)$, it can use this information to demodulate the $\hat{s}(t)$ so as to obtain baseband signal $m(t)$
  - Recover the carrier (phase): we discuss **time-2 carrier recovery** scheme, which works well for BPSK signals, but not quadrature signals with equal average power in each quadrature branch

- Time-2 carrier recovery:
  A nonlinear square device, a BPF, a PLL – which produces $\cos(2\omega_c t + 2\theta)$, a frequency divider finally generates $\cos(\omega_c t + \theta)$



- A square device generates $m^2(t)\cos^2(\omega_c t + \theta) = \frac{1}{2} + \frac{1}{2}\cos(2\omega_c t + 2\theta)$. The BPF centred at $2f_c$ gets $\cos(2\omega_c t + 2\theta)$ and uses it to drive a phase locked loop

Time-2 Carrier Recovery (continue)

- The phase locked loop consists of a lowpass filter, a multiplier and a voltage controlled oscillator. The frequency divider produces $\cos(\omega_c t + \theta)$, the carrier
- The VCO oscillates at $2f_c$ with an initial phase $\hat{\theta}$. Its output, $\sin(2(\omega_c t + \hat{\theta}))$, is multiplied by $\cos(2(\omega_c t + \theta))$ to obtain

$$e(t) = \frac{1}{2}\sin(4\omega_c t + 2(\theta + \hat{\theta})) + \frac{1}{2}\sin(2(\theta - \hat{\theta}))$$

  - The first term is removed by the LPF, while the second term

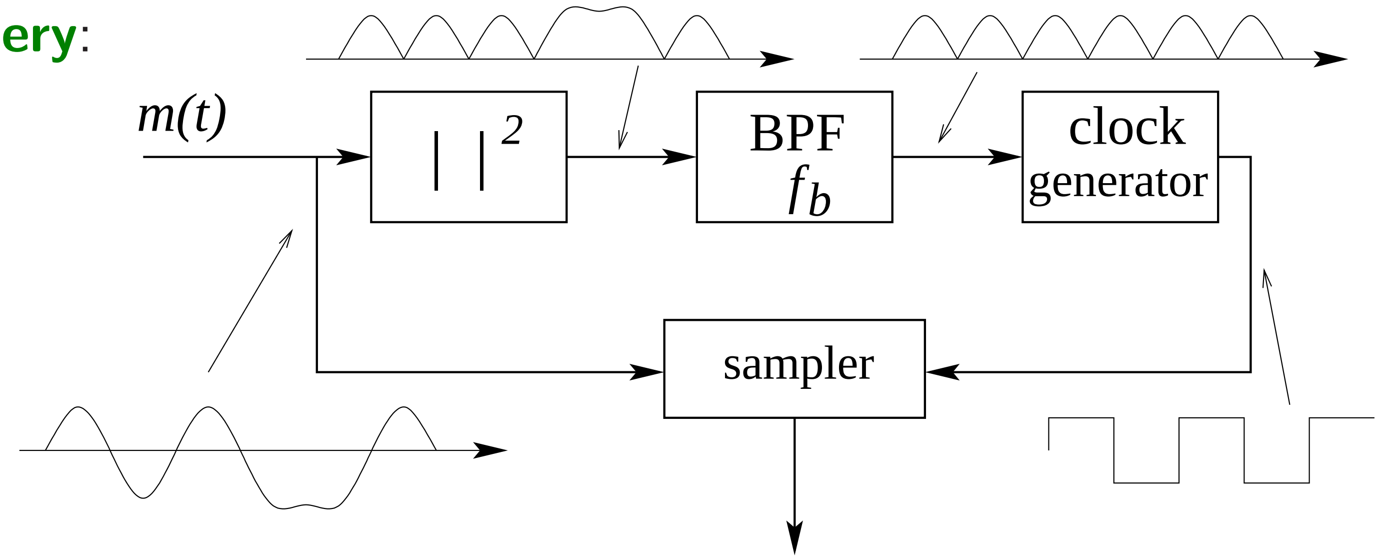
$$c(t) = \frac{1}{2}\sin(2(\theta - \hat{\theta})) \approx \Delta\theta \text{ if } \Delta\theta \ll 1$$

    is used to drive the VCO, so that its phase locks to θ
  - In order for $c(t) \rightarrow 0$, initial phase $\hat{\theta}$ of VCO should not be far away from true carrier phase θ

Alternative PLL directly produces carrier $\cos(\omega_c t + \theta)$ with VCO operates at f_c – What does circuit diagram look like?

Time-2 Clock Recovery

- **Time-2 clock recovery**:



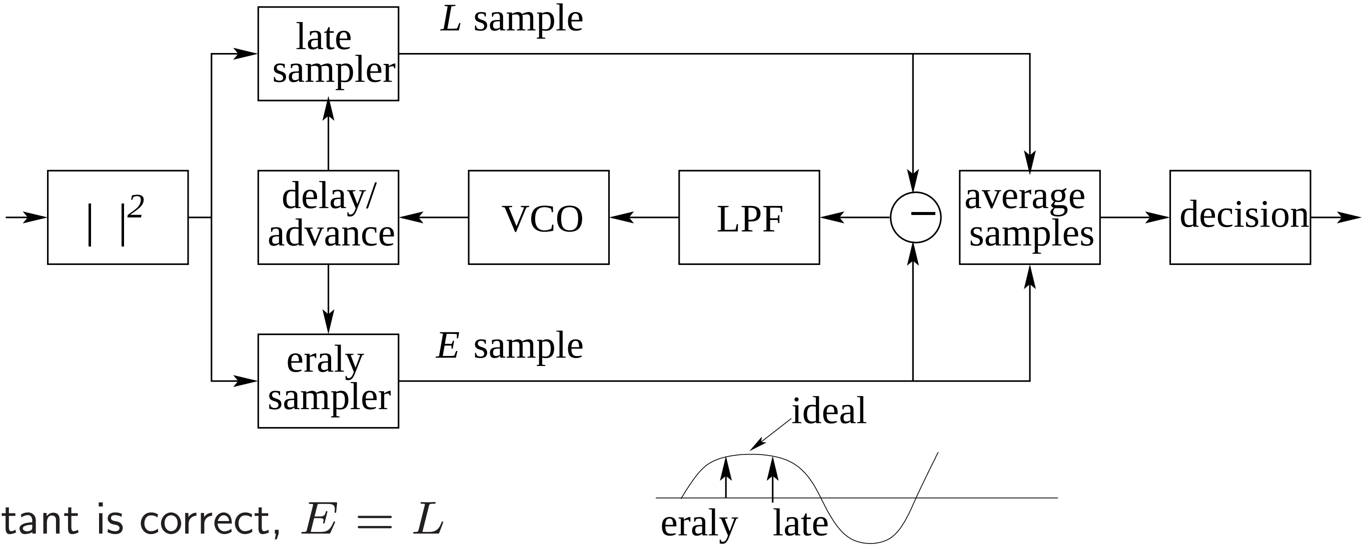
- When the received demodulated signal is squared, it possesses a periodic frequency domain component at the symbol rate. A BPF tuned close to the symbol rate extracts this periodic signal. A clock pulse regenerator (a saturating amplifier) produces a rectangular pulse shape with the required timing information

- It works well for binary modulation schemes, but not so for multilevel signalling schemes, as the symbol rate component is less clear in the squared signal

Early-Late Clock Recovery

- **Early-late clock recovery**: It takes two samples E, L, both equi-spaced around predicted sampling instant

  Assumption is peaks in squared waveform are correct sampling points



  - If predicted sampling instant is correct, $E = L$
  - If $E > L$, recovered clock is sampling too late
  - If $E < L$, recovered clock is sampling too early
  - With LPF used to reduce noise, filtered difference signal adjusts frequency of VCO to delay or advance the arrival of next clock impulse, and results are averaged over several samples

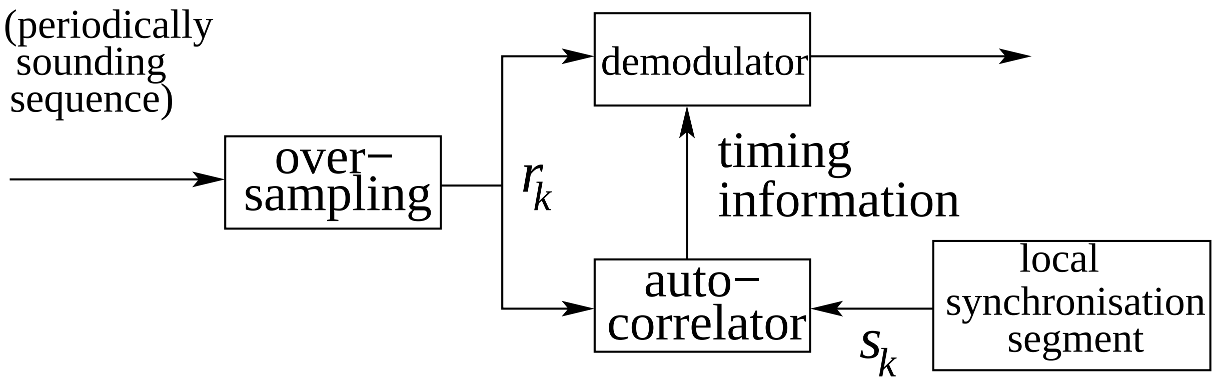
- It works well for binary modulation schemes but less so with multilevel ones, as there are fewer distinctive peaks for a (squared) multilevel baseband signal

Zero Crossing Clock Recovery

- **Zero crossing clock recovery**: it looks for zero crossings at the incoming waveform, instead of peaks of incoming squared waveform as in the early-late scheme
  - For binary signalling, the circuit is similar to that of the early-late
  - This works as for symmetrical signalling the received waveform will pass through zero midway between the sampling points
- The time-2, early-late, and zero crossing clock recovery schemes all work well for binary signalling but less successful for multilevel signalling
  - The time-2 clock recovery performs poorly for multilevel signalling
  - As zero crossing is not always at middle of sampling period for multilevel signalling, a control logic block is needed for zero crossing scheme to enable or disable adjustments,
    - If a transition occurs between two symbols of equal magnitude but opposite polarity, the zero crossing associated with this transition is in the middle of a symbol period, and only these zero crossings are used to update the timing
  - Later we will discuss modified early-late clock recovery for multilevel signalling

Synchroniser Clock Recovery

- **Synchroniser** clock recovery:



- Tx periodically sends a **sounding sequence**. Rx searches for it by performing an auto-correlation at a rate faster than the symbol rate (typically 4 times)

  Let r_k be the oversampled Rx signal and s_k the locally generated sounding signal

$$\tau_{\text{sam}} = \arg\max_{\tau} \sum_{k} r_{k+\tau} \cdot s_k$$

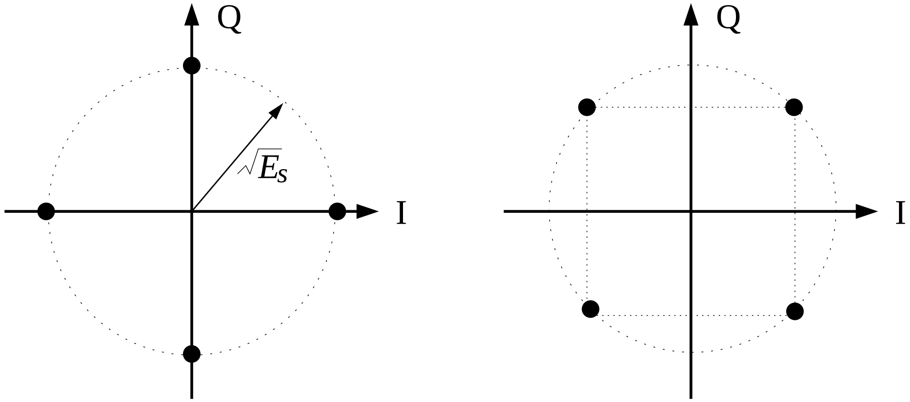
  τ_{sam} at which the **maximum correlation** occurs is the correct sampling point

- This method works well for both binary and multilevel modulation schemes but requires extra bandwidth overhead for sounding sequence

Quadrature Phase Shift Keying

- QPSK: $M=4$, 2 BPS, symbol period $T_s=2T_b$, energy per symbol $E_s=2E_b$
- QPSK signal constellation:

  Minmum phase separation is 90°



- QPSK signal set: $i=1,2,3,4$,

$$s_i(t)=\sqrt{\frac{2E_s}{T_s}}\cos\left(2\pi f_c t+(i-1)\frac{\pi}{2}\right) \quad 0\le t\le T_s$$

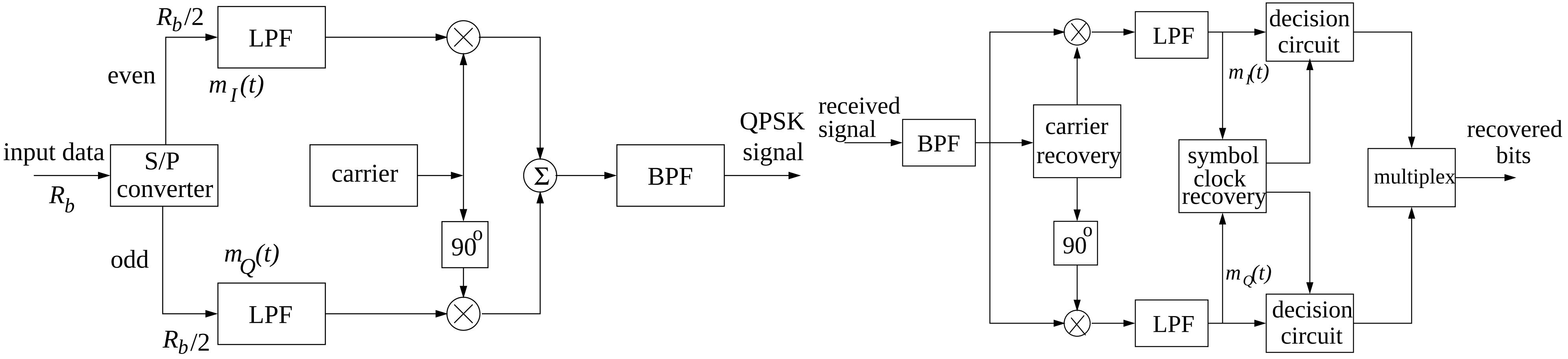
- The transmitted QPSK RF signal can also be written as:

$$s_i(t)=\sqrt{\frac{2E_s}{T_s}}\cos\left((i-1)\frac{\pi}{2}\right)\cos(\omega_c t)-\sqrt{\frac{2E_s}{T_s}}\sin\left((i-1)\frac{\pi}{2}\right)\sin(\omega_c t)$$

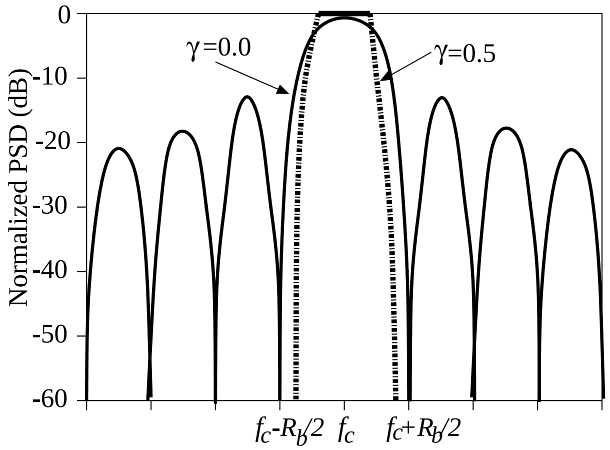
Note both inphase and quadrature branches (two orthogonal carriers) are utilised

QPSK (continue)

- QPSK transmitter/receiver: S/P splits bit stream into inphase and quadrature streams, LPFs are for pulse shaping, two orthogonal carriers are used for inphase and quadrature carrier modulations



- QPSK RF signal PSD (raised cosine):
- BPF at transmitter limits signal power spectrum within allocated band, as raised cosine pulse shaping is truncated
- BPF at receiver filters out out of band noise
- Time-2 carrier recovery **does not work** for QPSK
- As I and Q are BPSK, all clock recovery schemes work
- P/S after decision circuits recovers original data streams



Differential Phase Shift Keying

- **Coherent receiver** requires to know the carrier phase, which is a difficult job, while **non-coherent receiver** does not

  Coherent receiver has better performance but non-coherent receiver is easy and cheap to build

- DPSK: for non-coherent receiver
  - Let m_k be input bit sequence
  - Differentially encoded bit sequence d_k is given by $d_k = \overline{m_k \oplus d_{k-1}}$

m_k		1	0	0	1	0	1	1	0
d_{k-1}		1	1	0	1	1	0	0	0
d_k	1	1	0	1	1	0	0	0	1

  - **Differentially encoded** sequence $\{d_k\}$ is then used for BPSK modulation etc
  - At receiver, detected $\{d_k\}$ are used for recovering original $\{m_k\}$

- As m_k is determined by two bits d_k and d_{k-1}, an error in detection will cause two bit errors, leading to worst case performance penalty of 3 dB in SNR

Summary

- Phase shift keying: general form, phase carries symbol information
  - Bits per symbol: define system throughput or bandwidth efficiency
  - Coherent receiver: require carrier phase reference
  - BPSK: signal waveform and constellation, 1 BPS, transmitter/receiver
  - QPSK: signal waveform and constellation, 2 BPS, transmitter/receiver
  - DPSK: for non-coherent receiver, differential encoding
- Time-2 carrier recovery: suitable for binary modulation scheme
- Clock recovery
  - Time-2, early-late, zero crossing clock recovery schemes: suitable for binary signalling
  - Synchroniser clock recovery