Adaptive Space-Time Equalisation for Multiple-Antenna Assisted Multiple-Input Multiple-Output Systems

A. Livingstone and S. Chen

School of Electronics and Computer Science
University of Southampton, Southampton SO17 1BJ, U.K.
E-mails: {al902, sqc}@ecs.soton.ac.uk

ABSTRACT

This paper investigates an adaptive space-time equalisation (STE) assisted multiuser detection scheme for multiple-antenna aided multiuser systems. A minimum bit error rate (MBER) design is compared with the standard minimum mean square error (MMSE) design. It is shown that the MBER design provides significant performance enhancement, in terms of achievable system bit error rate, over the MMSE design for the multiple-antenna assisted multiple-input multiple-output communication scenario. Adaptive implementation of the MBER STE is realised using a stochastic gradient based least bit error rate algorithm, which is demonstrated to consistently outperform the adaptive least mean square based STE.

I. INTRODUCTION

Mobile wireless system designers are increasingly faced with a number of challenges, including limited bandwidth, a hostile, time-varying propagation channel and ever increasing pressure to improve capacity and data rates. Smart antenna aided space-time processing is capable of substantially improving the achievable wireless system capacity, coverage and quality by suppressing the effects of both intersymbol interference and co-channel interference [1]-[12]. We consider the multiple-input multiple-output (MIMO) system based on a space division multiple access (SDMA) scheme, where each transmitter employs a single antenna, while the receiver has multiple antennas. To interpret the multiuser supporting capability of such an SDMA based MIMO system [13], it is useful to compare it with classic code division multiple access (CDMA) multiuser systems [10]. In a CDMA system, each user is separated by a unique user-specific spreading code. By contrast, an SDMA system differentiates each user by the unique user-specific channel impulse response (CIR) encountered at the receiver antennas. In this analogy, the unique user-specific CIR plays the role of a user-specific CDMA signature. However, owing to the non-orthogonal nature of the CIRs, an effective multiuser detection (MUD) is required for separating the users in an SDMA based MIMO system.

We investigate a space-time equalisation (STE) based multiuser detection (MUD) for SDMA based MIMO systems. The most popular MIMO-receiver design is constituted by the minimum mean square error (MMSE) MUD [9]-[11],[14]. We investigate a minimum bit error rate (MBER) design for the STE based MUD, and we show that the MBER STE design is superior in comparison to the MMSE design in terms of achievable bit error rate (BER). This is significant, since the MMSE design is often considered to be the state-of-the-art technique in multiple antenna assisted systems [9]-[11],[14]. Our study thus demonstrates that the system capacity can further be enhanced beyond that of the MMSE solution. We also study an adaptive implementation of the MBER STE based MUD, known as the least bit error rate (LBER). Our simulation results demonstrate that the LBER STE aided MUD consistently outperforms the adaptive least mean square (LMS) based one.

II. SYSTEM MODEL

The system model investigated is the multiple antenna aided SDMA based MIMO system supporting M users, as depicted in Fig. 1, where each of the M users is equipped with a single transmit antenna and the receiver is assisted by an L-element antenna array. The symbol-rate received signal samples $x_l(k)$ for $1 \le l \le L$ are given by [9],[15]

$$x_l(k) = \sum_{m=1}^{M} \sum_{i=0}^{n_C-1} c_{i,l,m} s_m(k-i) + n_l(k) = \bar{x}_l(k) + n_l(k), \quad (1)$$

where $n_l(k)$ is a complex-valued additive white Gaussian noise with $E[|n_l(k)|^2] = 2\sigma_n^2$, $\bar{x}_l(k)$ denotes the noise-free part of the lth receive antenna's output, $s_m(k)$ is the kth transmitted symbol of user m, $\mathbf{c}_{l,m} = [c_{0,l,m} \; c_{1,l,m} \cdots c_{n_C-1,l,m}]^T$ denotes the tap vector of the CIR connecting the user m and the lth receive antenna, and n_C is the length of the CIR. Binary phase shift keying modulation is employed and hence $s_m(k) \in \{\pm 1\}$.

The MUD consists of a bank of the M STEs, as shown in Fig. 2. The outputs of the M detectors are given by

$$y_m(k) = \sum_{l=1}^{L} \sum_{i=0}^{n_F-1} w_{i,l,m}^* x_l(k-i), \; 1 \le m \le M, \quad (2)$$

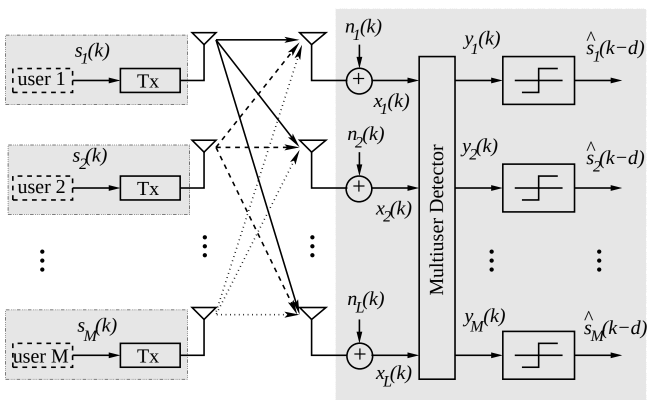

Fig. 1. An antenna array aided MIMO system, where each of the M users is equipped with a single transmit antenna and the receiver is assisted by an L-element antenna array.

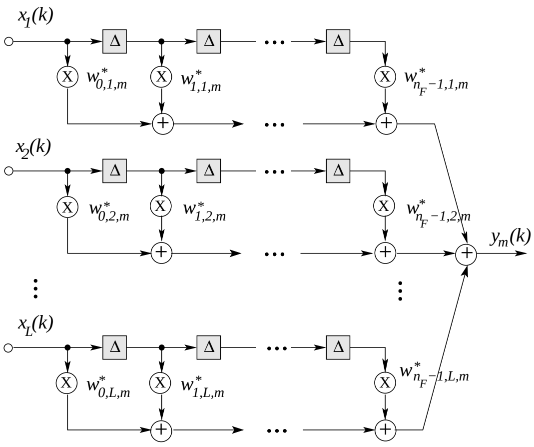

Fig. 2. STE assisted MUD for user m, where Δ denotes the symbol-spaced delay, L is the number of receive antennas, $1 \le m \le M$, and M is the number of users.

where $\mathbf{w}_{l,m} = [w_{0,l,m}\ w_{1,l,m} \cdots w_{n_F-1,l,m}]^T$ denotes the mth user detector's equaliser weight vector for the lth receive antenna, and n_F is the length of temporal equaliser filter. The M user detectors' decisions are defined by

$$\hat{s}_m(k-d) = \text{sgn}\left(y_{R_m}(k)\right),\ 1 \le m \le M, \tag{3}$$

where $\hat{s}_m(k-d)$ is the estimate of $s_m(k-d)$, $y_{R_m}(k) = \Re[y_m(k)]$, and d is the detector decision delay.

Let us define the $n_F \times (n_F + n_C - 1)$ CIR convolution matrix associated with the user m and lth receive antenna as

$$\mathbf{C}_{l,m} = \left[\begin{array}{cccccc} c_{0,l,m} & \cdots & c_{n_C-1,l,m} & 0 & \cdots & 0 \\ 0 & c_{0,l,m} & \cdots & c_{n_C-1,l,m} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & c_{0,l,m} & \cdots & c_{n_C-1,l,m} \end{array}\right] \tag{4}$$

and introduce the overall system CIR convolution matrix as

$$\mathbf{C} = \left[\begin{array}{cccc} \mathbf{C}_{1,1} & \mathbf{C}_{1,2} & \cdots & \mathbf{C}_{1,M} \\ \mathbf{C}_{2,1} & \mathbf{C}_{2,2} & \cdots & \mathbf{C}_{2,M} \\ \vdots & \vdots & \dots & \vdots \\ \mathbf{C}_{L,1} & \mathbf{C}_{L,2} & \cdots & \mathbf{C}_{L,M} \end{array}\right]. \tag{5}$$

Then the received signal vector $\mathbf{x}(k)$ can be expressed by

$$\mathbf{x}(k) = \mathbf{C}\,\mathbf{s}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k), \tag{6}$$

where

$$\mathbf{x}(k) = [\mathbf{x}_1^T(k)\ \mathbf{x}_2^T(k) \cdots \mathbf{x}_L^T(k)]^T \tag{7}$$

with $\mathbf{x}_l(k) = [x_l(k)\ x_l(k-1) \cdots x_l(k-n_F+1)]^T$,

$$\mathbf{n}(k) = [\mathbf{n}_1^T(k)\ \mathbf{n}_2^T(k) \cdots \mathbf{n}_L^T(k)]^T \tag{8}$$

with $\mathbf{n}_l(k) = [n_l(k)\ n_l(k-1) \cdots n_l(k-n_F+1)]^T$, and

$$\mathbf{s}(k) = [\mathbf{s}_1^T(k)\ \mathbf{s}_2^T(k) \cdots \mathbf{s}_M^T(k)]^T \tag{9}$$

with $\mathbf{s}_m(k) = [s_m(k)\ s_m(k-1) \cdots s_m(k-n_F-n_C+2)]^T$. Let us further define

$$\mathbf{w}_m = [\mathbf{w}_{1,m}^T\ \mathbf{w}_{2,m}^T \cdots \mathbf{w}_{L,m}^T]^T. \tag{10}$$

Then the output of the mth detector can be written as

$$\begin{aligned} y_m(k) &= \sum_{l=1}^{L} \mathbf{w}_{l,m}^H \mathbf{x}_l(k) = \mathbf{w}_m^H \mathbf{x}(k) \\ &= \mathbf{w}_m^H(\bar{\mathbf{x}}(k) + \mathbf{n}(k)) = \bar{y}_m(k) + e_m(k), \end{aligned} \tag{11}$$

where $e_m(k)$ is Gaussian distributed, having a zero mean and $E[|e_m(k)|^2] = 2\mathbf{w}_m^H \mathbf{w}_m \sigma_n^2$.

III. Space-Time Equalisation

The task of designing the STE (11) is to choose an optimal weight vector $\mathbf{w}_m$ according to some design criterion.

A. Minimum Mean Square Error Design

Classically, the mth STE detector's weight vector $\mathbf{w}_m$ is determined by minimising the mean square error metric of $E[|s_m(k-d) - y_m(k)|^2]$, which leads to the following MMSE solution [9]-[11],[14]

$$\mathbf{w}_{(\text{MMSE})m} = \left(\mathbf{C}\,\mathbf{C}^H + 2\sigma_n^2 \mathbf{I}\right)^{-1} \mathbf{C}_{|(m-1)(n_F+n_C-1)+(d+1)}, \tag{12}$$

for $1 \le m \le M$, where $\mathbf{I}$ denotes the $Ln_F \times Ln_F$ identity matrix and $\mathbf{C}_{|i}$ the ith column of $\mathbf{C}$. An adaptive implementation of the MMSE solution can readily be realised using the LMS algorithm [16]-[18]

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu\left(s_m(k-d) - y_m(k)\right)^* \mathbf{x}(k), \tag{13}$$

where μ is the step size.

B. Minimum Bit Error Rate Design

As recognized by [19] in a CDMA context and by [20] in a beamforming-based MUD scenario, a better strategy is to choose the detector's coefficients by directly minimising the system's BER. A main objective of this study is to investigate the MBER solution for the STE based MUD (11). Following the notations used in [19],[20], let us denote the $N_s = 2^{M(n_F+n_C-1)}$ number of possible transmitted symbol sequences of $\mathbf{s}(k)$ as $\mathbf{s}^{(q)}$, $1 \le q \le N_s$. Denote furthermore the $((m-1)(n_F+n_C-1)+(d+1))$th element of $\mathbf{s}^{(q)}$, corresponding to the desired symbol $s_m(k-d)$, as $s_{m,d}^{(q)}$. The noise-free part of the mth detector input signal $\bar{\mathbf{x}}(k)$ assumes values from the vector signal set defined as

$$\mathcal{X}_m = \{\bar{\mathbf{x}}^{(q)} = \mathbf{C}\,\mathbf{s}^{(q)}, 1 \le q \le N_s\}. \tag{14}$$

Similarly, the noise-free part of the mth detector's output $\bar{y}_m(k)$ assumes values from the scalar set $\mathcal{Y}_m = \{\bar{y}_m^{(q)} = \mathbf{w}_m^H \bar{\mathbf{x}}^{(q)}, 1 \le q \le N_s\}$. Thus $\bar{y}_{R_m}(k) = \Re[\bar{y}_m(k)]$ can only take the values from the set

$$\mathcal{Y}_{R_m} = \{\bar{y}_{R_m}^{(q)} = \Re[\bar{y}_m^{(q)}], 1 \le q \le N_s\}, \tag{15}$$

and $\mathcal{Y}_{R_m}$ can be divided into the two subsets conditioned on the value of $s_m(k-d)$

$$\mathcal{Y}_{R_m}^{(\pm)} = \{\bar{y}_{R_m}^{(q,\pm)} \in \mathcal{Y}_{R_m} : s_m(k-d) = \pm 1\}. \tag{16}$$

The probability density function (PDF) of $y_{R_m}(k)$ is a Gaussian mixture defined by

$$p_m(y_R) = \frac{1}{N_s} \sum_{q=1}^{N_s} \frac{1}{\sqrt{2\pi\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}} e^{-\frac{\left(y_R - \bar{y}_{R_m}^{(q)}\right)^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}}, \tag{17}$$

where $\bar{y}_{R_m}^{(q)} \in \mathcal{Y}_{R_m}$. Thus the BER of the mth detector associated with the detector's weight vector $\mathbf{w}_m$ can be shown to be [19],[20]

$$P_E(\mathbf{w}_m) = \frac{1}{N_s} \sum_{q=1}^{N_s} Q\left(g^{(q)}(\mathbf{w}_m)\right), \tag{18}$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} e^{-\frac{v^2}{2}}\, dv \tag{19}$$

and

$$g^{(q)}(\mathbf{w}_m) = \frac{\mathrm{sgn}(s_{m,d}^{(q)}) \bar{y}_{R_m}^{(q)}}{\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}}. \tag{20}$$

Note that the BER is invariant to a positive scaling of $\mathbf{w}_m$.

The MBER solution for the mth STE detector is then defined as the weight vector that minimises the error probability (18)

$$\mathbf{w}_{(\mathrm{MBER})m} = \arg\min_{\mathbf{w}_m} P_E(\mathbf{w}_m). \tag{21}$$

The gradient of $P_E(\mathbf{w}_m)$ with respect to $\mathbf{w}_m$ is given by

$$\nabla P_E(\mathbf{w}_m) = \frac{1}{2N_s\sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}_m^H \mathbf{w}_m}} \sum_{q=1}^{N_s} e^{-\frac{\left(\bar{y}_{R_m}^{(q)}\right)^2}{2\sigma_n^2 \mathbf{w}_m^H \mathbf{w}_m}}$$

$$\times \mathrm{sgn}\left(s_{m,d}^{(q)}\right) \left(\frac{\bar{y}_{R_m}^{(q)} \mathbf{w}_m}{\mathbf{w}_m^H \mathbf{w}_m} - \bar{\mathbf{x}}^{(q)} \right). \tag{22}$$

Given the gradient (22), the optimisation problem (21) can be solved iteratively by commencing the iterations from an appropriate initial point using a gradient-based optimisation algorithm, such as the simplified conjugate gradient algorithm [19],[20],[21].

Following the derivations presented in [19],[20], an adaptive implementation of the MBER STE based MUD can be realised using the LBER algorithm which takes the form of

$$\mathbf{w}_m(k+1) = \mathbf{w}_m(k) + \mu \frac{\mathrm{sgn}(s_m(k-d))}{2\sqrt{2\pi}\rho_n} e^{-\frac{y_{R_m}^2(k)}{2\rho_n^2}} \mathbf{x}(k). \tag{23}$$

The adaptive gain μ and the kernel width ρ_n are the two algorithmic parameters that have to be set appropriately to ensure a fast convergence rate and small steady-state BER misadjustment.

IV. Simulation Study

The simulations was carried out using MATLAB.

A. Stationary System

The systems used in the simulation supported $M = 3$ users with $L = 2$ and $L = 4$ receiver antennas. All the three users had an equal transmit power. The $M \times L = 6$ (for $L = 2$) and $M \times L = 12$ (for $L = 4$) CIRs are listed in Table I, each CIR having $n_C = 2$ taps. In the actual simulation, all the CIRs were normalised to provide unit channel energy, i.e. $\|\mathbf{c}_{l,m}\|^2 = 1$ for all l and m. Each equaliser temporal filter had a length of $n_F = 3$ and the detector decision delay was chosen to be $d = 1$. Fig. 3 compares the BER performance of the MMSE and MBER STE based MUDs. The BER of a STE based MUD was computed using the theoretic BER formula (18), the MMSE STE weight vector was calculated using the formula (12), and the MBER STE solution was computed numerically using the simplified conjugate gradient algorithm. It can be seen that for both $L = 2$ and $L = 4$ as well as for all three users the MBER STE detectors had better BER performance than the corresponding MMSE detectors. By comparing Fig. 3 (a)-(c) with Fig. 3 (d)-(f), it confirms that the system with $L = 4$ antennas achieved better performance than the system with $L = 2$ antennas.

The LMS and LBER adaptive STE based MUDs were investigated in simulation, and the BERs of the both LMS and LBER MUDs, after training convergence and averaging over 30 runs, are compared in Fig. 4, where better performance of the adaptive LBER detectors over the corresponding LMS ones is self evident. The step size for the LMS algorithm was chosen as $\mu = 0.01$, while for the LBER algorithm the step size $\mu = 0.1$ and the kernel width $\rho_n = 4\sigma_n$.

B. Slow Fading System

The system again supported $M = 3$ users with $L = 2$ and $L = 4$ antennas. However, fading channels were simulated and moreover each CIR had $n_C = 3$ taps. The CIR coefficients were generated using a Doppler fading process with Rayleigh distributed magnitudes, normalised to unit power. The coefficients were varied every symbol period. The process used was based on the Clarke and Gans fading model, as described in [15],[22],[23]. Each equaliser temporal filter had a length of $n_F = 5$ and the detector decision delay was set to $d = 2$. The transmission frame structure consisted of 50 training symbols followed by 450 data symbols. In the simulations, the normalised Doppler frequency for the simulated system was 10^{-5}, which for a carrier of 900 MHz and a symbol rate of 3 Msymbols/s corresponded to a user velocity of 10 m/s (36 km/h). The step size for the LMS algorithm was chosen as $\mu = 0.005$, while for the LBER algorithm the step size $\mu = 0.1$ and the kernel width $\rho_n = 4\sigma_n$. The BER of an adaptive STE based MUD was calculated using Monte Carlo simulation. Fig. 5 compares the BERs of the LBER STE based MUDs with those of the LMS based ones. It can be seen from Fig. 5 that the LBER STE based MUD consistently outperformed the LMS STE based MUD for all three users and in both the cases of $L = 2$ and $L = 4$ antennas.

V. Conclusions

Multiuser detection based on the space-time equalisation has been investigated for multiple antenna aided space division multiple access based MIMO systems. An MBER design has been investigated for the STE based MUD. It has been shown that the MBER

$\mathbf{c}_{l,m}$	$l=1$	$l=2$	$l=3$	$l=4$
$m=1$	-0.5+0.4j, 0.7+0.6j	0.5-0.4j, -0.8-0.3j	0.4-0.4j, -0.7-0.8j	0.5+0.5j, 0.6-0.9j
$m=2$	-0.1-0.2j, 0.7+0.6j	-0.3+0.5j, -0.7-0.9j	-0.1-0.2j, 0.7+0.6j	-0.6-0.4j, 0.9-0.4j
$m=3$	-0.7+0.9j, 0.6+0.4j	-0.6+0.8j, -0.6-0.7j	0.3-0.5j, 0.9+0.1j	-0.6-0.6j, 0.8+0.0j

TABLE I

CIRs USED IN THE STATIONARY SYSTEM SIMULATION.

STE assisted MUD can obtain significant performance gains over the standard MMSE design, in terms of achievable system BER. Adaptive implementation of the MBER STE assisted MUD has been considered using the LBER algorithm. The simulation results have demonstrated that the adaptive LBER STE assisted MUD consistently achieves better BER performance over the classical LMS STE assisted MUD.

REFERENCES

[1] J.H. Winters, J. Salz and R.D. Gitlin, "The impact of antenna diversity on the capacity of wireless communication systems," *IEEE Trans. Communications*, Vol.42, No.2, pp.1740–1751, 1994.

[2] A.J. Paulraj and C.B. Papadias, "Space-time processing for wireless communications," *IEEE Signal Processing Magazine*, Vol.14, No.6, pp.49–83, 1997.

[3] L.C. Godara,"Applications of antenna arrays to mobile communications, Part I: Performance improvement, feasibility, and system considerations," *Proc. IEEE*, Vol.85, No.7, pp.1031–1060, 1997.

[4] J.H. Winters, "Smart antennas for wireless systems," *IEEE Personal Communications*, Vol.5, No.1, pp.23–27, 1998.

[5] S.N. Diggavi, "On achievable performance of spatial diversity fading channels," *IEEE Trans. Information Theory*, Vol.47, No.1, pp.308–325, 2001.

[6] P. Vandenameele, L. van Der Perre and M. Engels, *Space Division Multiple Access for Wireless Local Area Networks*. Boston: Kluwer Academic Publishers, 2001.

[7] S. Bellofiore, C.A. Balanis, J. Foutz and A.S. Spanias, "Smart-antenna systems for mobile communication networks. Part 1. Overview and antenna design," *IEEE Antennas and Propagation Magazine*, Vol.44, No.3, pp.145–154, 2002.

[8] J.S. Blogh and L. Hanzo, *Third Generation Systems and Intelligent Wireless Networking – Smart Antenna and Adaptive Modulation*. Chichester: John Wiley, 2002.

[9] A. Paulraj, R. Nabar and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge: Cambridge University Press, 2003.

[10] L. Hanzo, L-L. Yang, E-L. Kuan and K. Yen, *Single- and Multi-Carrier DS-CDMA: Multi-User Detection, Space-Time Spreading, Synchronisation, Standards and Networking*. IEEE Press - John Wiley, 2003.

[11] A.J. Paulraj, D.A. Gore, R.U. Nabar and H. Bölcskei, "An overview of MIMO communications – A key to gigabit wireless," *Proc. IEEE*, Vol.92, No.2, pp.198–218, 2004.

[12] S.N. Diggavi, N. Al-Dhahir, A. Stamoulis and A.R. Calderbank, "Great expectations: the value of spatial diversity in wireless networks," *Proc. IEEE*, Vol.92, No.2, pp.219–270, 2004.

[13] L. Hanzo, M. Münster, B.J. Choi and T. Keller, *OFDM and MC-CDMA*. West Sussex, England: John Wiley and IEEE Press, 2003.

[14] D.N.C. Tse and S.V. Hanly, "Linear multiuser receivers: effective interference, effective bandwidth and user capacity," *IEEE Trans. Information Theory*, Vol.45, No.2, pp.641–657, 1999.

[15] T.S. Rappaport, *Wireless Communications: Principles and Practice*. Upper Saddle River, N.J.: Prentice Hall, 1996.

[16] B. Widrow, J.M. McCool and M. Ball, "The complex lms algorithm," *Proc. IEEE*, Vol.63, No.4, pp.719–720, 1975.

[17] B. Widrow and S.D. Strearns, *Adaptive Signal Processing*. Englewood Cliffs, N.J.: Prentice Hall, 1985.

[18] E. Soria, J. Calpe, J. Chambers, M. Martinez, G. Camps and J.D.M. Guerrero, "A novel approach to introducing adaptive filters based on the LMS algorithm and its variants," *IEEE Trans. Education*, Vol.47, No.1. pp.127–133, 2004.

[19] S. Chen, A.K. Samingan, B. Mulgrew and L. Hanzo, "Adaptive minimum-BER linear multiuser detection for DS-CDMA signals in multipath channels," *IEEE Trans. Signal Processing*, Vol.49, No.6, pp.1240–1247, 2001.

[20] S. Chen, N.N. Ahmad and L. Hanzo, "Adaptive minimum bit error rate beamforming," *IEEE Trans. Wireless Communications*, Vol.4, No.2, pp.341–348, 2005.

[21] M.S. Bazaraa, H.D. Sherali and C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*. New York: John Wiley, 1993.

[22] W.C. Jakes, *Microwave Mobile Communications*. John Wiley and Sons, 1974.

[23] Y.X. Li and X.J. Huang, "The generation of independent Rayleigh faders," in *Proc. ICC*, 2000, Vol.1, pp.41–45.

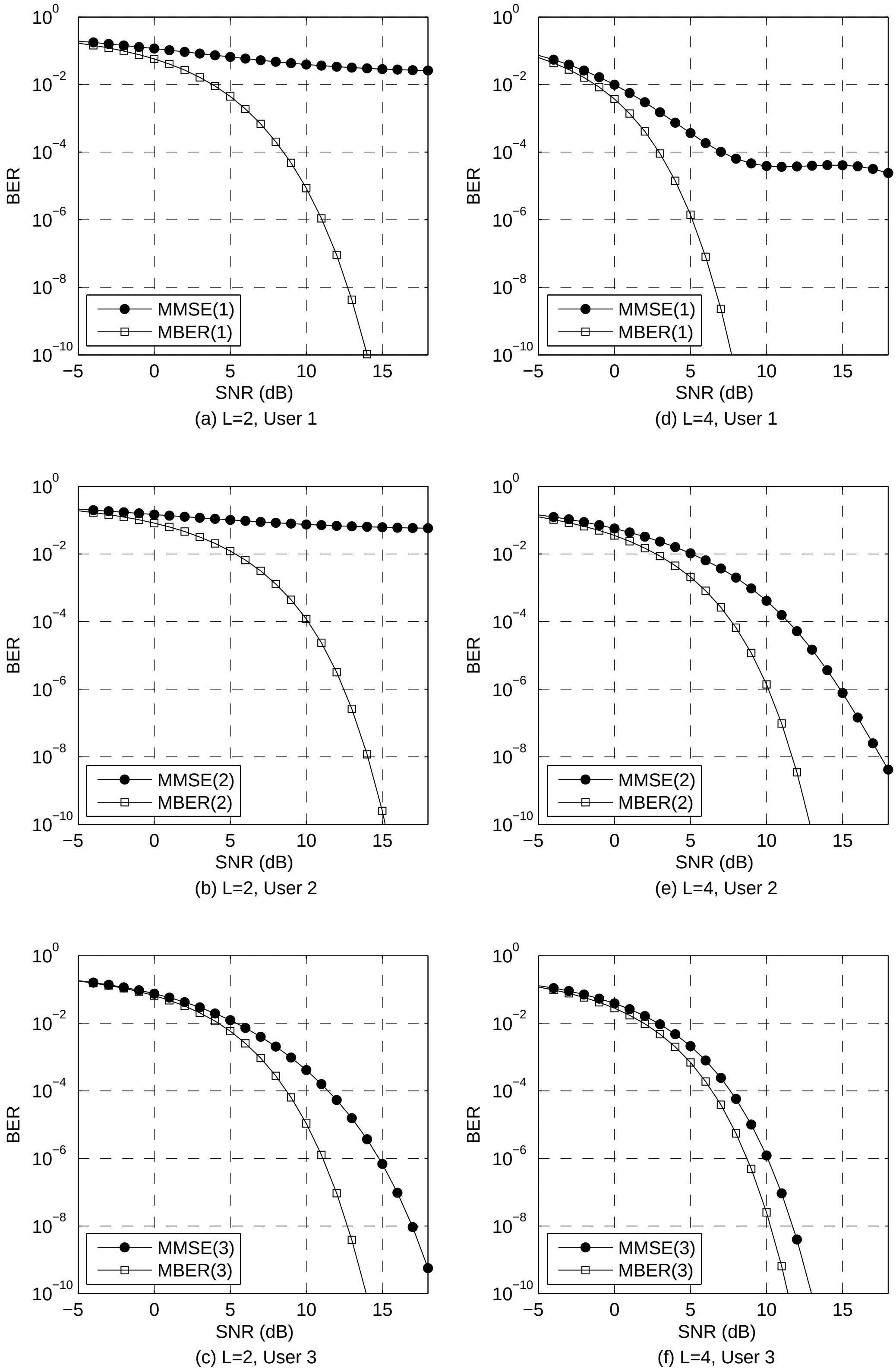

Fig. 3. BER comparison of the theoretical MMSE and MBER STE based MUDs for a 3-user L-antenna system, with the stationary CIRs listed in Table. I.

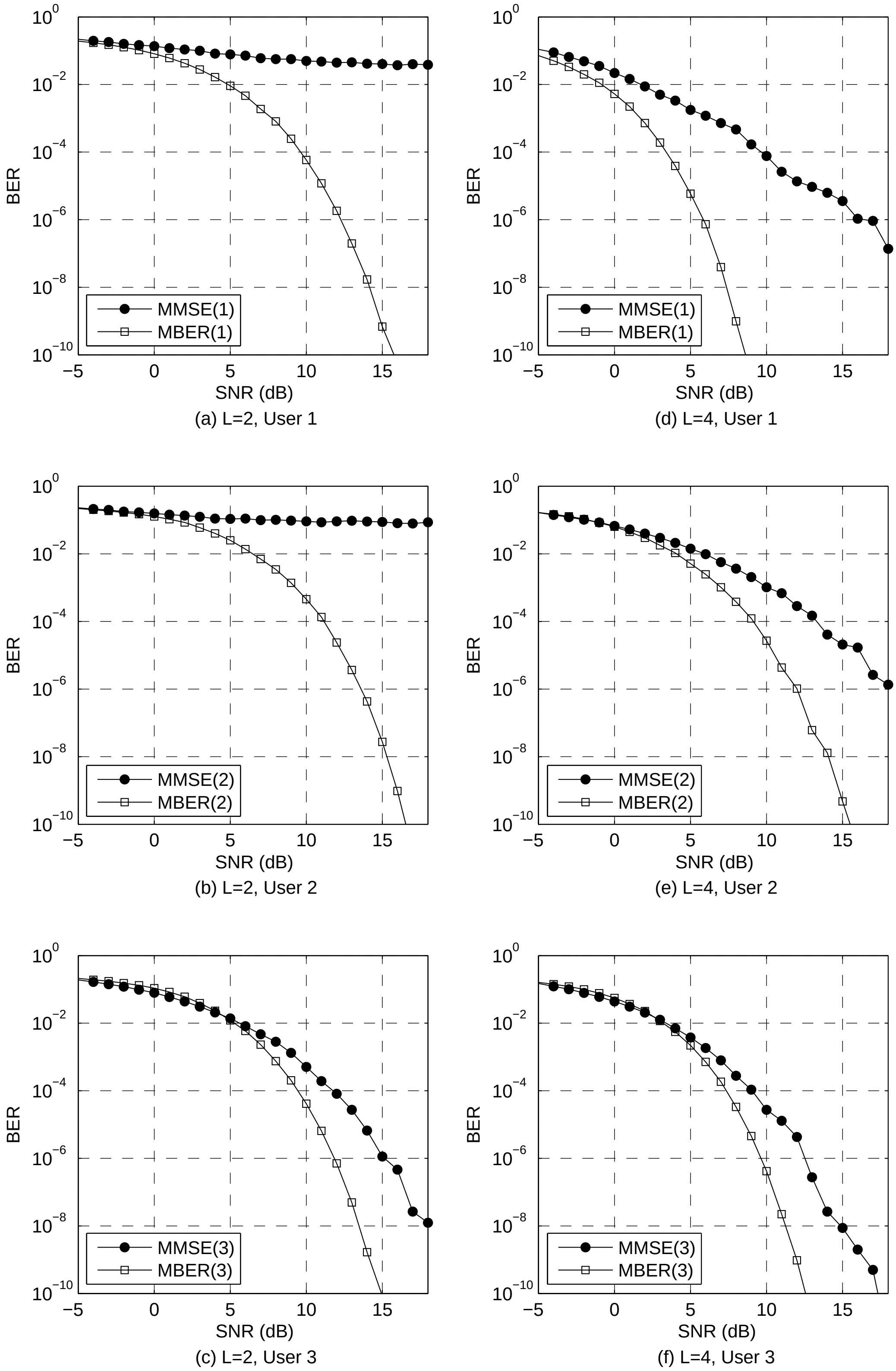

Fig. 4. BER comparison of the adaptive LMS and LBER STE based MUDs for a 3-user L-antenna system, with the stationary CIRs listed in Table. I. The training was done over $K = 500$ symbols and the performance was averaged over 30 runs. The LMS step size $\mu = 0.01$, the LBER step size $\mu = 0.1$ and its kernel width $\rho_n = 4\sigma_n$.

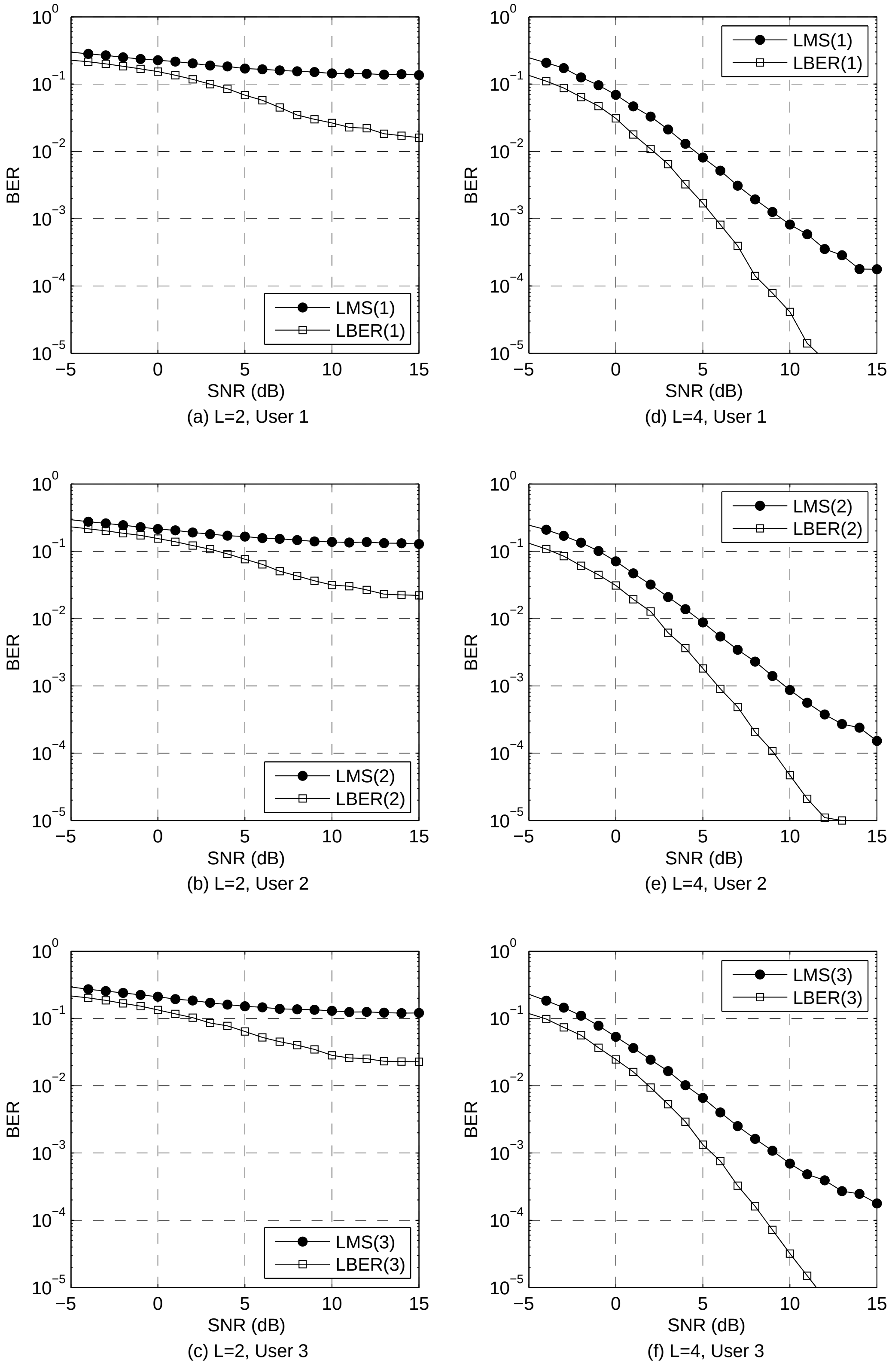

Fig. 5. BER comparison of the adaptive LMS and LBER STE based MUDs for a 3-user L-antenna system, with Doppler-faded time-varying CIRs. Normalised Doppler frequency 10^{-5}, the LMS step size $\mu = 0.005$, the LBER step size $\mu = 0.1$ and its kernel width $\rho_n = 4\sigma_n$.