Hybrid CSI-Based Hierarchical Beamforming for Flexible Duplex MIMO Systems

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***Abstract*—Flexible duplex (FD) techniques, including dynamic time-division duplex and dynamic frequency division duplex, are widely recognized as pivotal solutions to meeting the asymmetric and dynamic traffic demands in cellular systems. However, the coexistence of opposite transmission directions at the same time between neighboring cells inevitably results in cross-link interference (CLI), leading to significant degradation in network throughput. To tackle this issue, we exploit the trade-off between interference cancellation and spatial multiplexing capabilities in FD systems, and develop a hybrid channel state information (CSI) based hierarchical beamforming algorithm to mitigate the CLI. Compared with the existing state-of-the-art schemes, the proposed algorithm can substantially suppress the CLI with a much lower overhead of CSI acquisition between base stations. Simulation results demonstrate the superior performance of our proposed algorithm.**

***Index Terms*—Flexible duplex, cross-link interference, beamforming, MIMO, dynamic time-division duplex.**

I. Introduction

With asymmetric and dynamic traffic demands emerging in the 5G-advanced and the future 6G services, flexible duplex (FD) techniques, including the dynamic time-division duplex (D-TDD) and the dynamic frequency-division duplex (D-FDD), are recognized as promising solutions for achieving a more flexible transmission, since they allow the uplink (UL) and downlink (DL) transmission directions to be changed dynamically for adapting to the instantaneous traffic variation [1]. However, it is challenging to fully exploit the potential advantages of FD system due to the inevitable cross-link interference (CLI), which occurs when base stations (BSs) in neighboring cells simultaneously transmit/receive data in opposite directions on identical or partially-overlapping time-frequency resources.

Following the integration of both enhanced interference mitigation and improved traffic adaptation features into 3GPP Release 12 (LTE-Advanced), various approaches have been proposed to address this challenge, e.g., mitigating CLI through scheduling and resource allocation [2], [3], UL/DL configuration [4], advanced receiver design [5] and machine learning (ML) [6]. However, most of these approaches can achieve interference cancellation (IC) only when massive real-time information, e.g., traffic conditions, transmission direction and scheduling information, can be shared by neighboring cells. This information exchange will cost significant resource and must be carefully designed, which presents a considerable implementation challenge [7].

Beamforming-based CLI mitigation scheme provides a competitive alternative, which achieves IC by adjusting beam directions, without incurring extra massive information exchange. However, its effectiveness is significantly constrained by both the overhead and accuracy of channel state information (CSI) acquisition. In response to this issue, the work [8] proposed a decentralized beamforming solution, which only utilizes local CSI to mitigate the BS-to-BS CLI. However, massive signaling still needs to be shared between BSs and the amount of signaling overhead increases as the number of iterations grows. Furthermore, to eliminate the need for information exchange between BSs, the authors of [9] proposed a distributed interference alignment (IA) technique that operates with local CSI. Nevertheless, this method still relies on overhearing different types of reference signals in all BSs to obtain local CSI. Although these methods use local CSI to reduce the overhead of CSI acquisition, most existing beamforming-based CLI mitigation methods suppress BS-to-BS CLI by relying on instantaneous CSI between BSs, which results in considerable signaling overhead and leads to significant IC performance degradation due to various factors.[1]

Achieving the BS-to-BS CLI suppression through beamforming also leads to communication performance degradation [10], [11]. This is because beamforming-based CLI suppression consumes degrees of freedom (DoFs) that could otherwise be used for communication. The discrepancy in the corresponding optimal beamforming solutions makes it impossible to find a beamforming matrix that can perfectly satisfy both IC and communication requirements at the same time, leading to the trade-off between IC and communication when conducting beamforming design. No systematic guideline is found on characterizing such a trade-off between IC and communication performance in beamforming-based FD systems. Finding appropriate beamforming vectors to balance this trade-off remains a significant challenge.

Against this background, we develop a low-overhead beamforming-based IC scheme to suppress the BS-to-BS CLI. To achieve this challenging design, a new DoF-based perspective on the fundamental trade-off between IC and spatial multiplexing (SM) capabilities is first provided. Then, according to this perspective, we develop a hybrid CSI-based hierarchical beamforming algorithm, in which the BS-to-BS CLI is suppressed by utilizing the statistical CSI between adjacent BSs and the DL real-time CSI is needed for communication design. Specifically, our key contributions are summarized as follows.

- First, we provide a new DoF-based perspective on the fundamental trade-off between IC and SM capabilities in FD systems.
- Then, we propose a hybrid CSI-based hierarchical beamforming algorithm that reduces the overhead of real-time CSI acquisition and achieves effective IC performance.

Notations: The superscripts $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$ and $(\cdot)^H$ represent the transpose, conjugate and conjugate transpose operators, respectively. $\mathbb{E}\{\cdot\}$ denotes the expectation with respect to all random variables within the bracket, $\mathrm{rank}(\mathbf{A})$ represents the rank of matrix $\mathbf{A}$ and $\|\cdot\|^2$ denotes the Frobenius norm. $(\cdot)^{\frac{1}{2}}$ and $(\cdot)^{-1}$ represent the matrix principal square root and inverse operators, respectively. $\mathbf{I}_M$ and $\mathbf{0}_{M\times N}$ represent $(M\times M)$-dimension identity matrix and $(M\times N)$-dimension all-zero

[1]The instantaneous CSI is not always accurate due to various non-ideal factors, e.g., feedback delay, the channel estimation and quantization error.

Received 15 December 2024; revised 20 May 2025; accepted 3 July 2025. Date of publication 10 July 2025; date of current version 18 December 2025. This work was supported by the Beijing Municipal Natural Science Foundation under Grant L242013. The review of this article was coordinated by Prof. Haejoon Jung. *(Corresponding author: Shaoshi Yang.)*

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Digital Object Identifier 10.1109/TVT.2025.3587463

TABLE I
DEFINITIONS OF SYMBOLS IN THIS PAPER

Symbol	Definition	Dimension/Type
N_{U}	Number of antennas at UL BS ($\mathrm{BS_U}$)	Positive integer
N_{D}	Number of antennas at DL BS ($\mathrm{BS_D}$)	Positive integer
M_d, M_u	Numbers of DL and UL UEs	Positive integers
$\mathbf{H}_{\mathrm{BS}}$	Inter-cell CLI channel ($\mathrm{BS_D} \rightarrow \mathrm{BS_U}$)	$\mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{D}}}$
$\mathbf{h}_{\mathrm{U}_j}$	Channel vector from jth UL UE to $\mathrm{BS_U}$	$\mathbb{C}^{N_{\mathrm{U}}\times 1}$
$\mathbf{H}_{\mathrm{DL}}$	DL channel matrix at $\mathrm{BS_{DL}}$	$\mathbb{C}^{M_d\times N_{\mathrm{D}}}$
$\mathbf{h}_{\mathrm{D}_k}$	Channel vector from $\mathrm{BS_D}$ to kth DL UE	$\mathbb{C}^{N_{\mathrm{D}}\times 1}$
$\mathbf{v}_j$	Receive beamforming vector for jth UL UE at $\mathrm{BS_U}$	$\mathbb{C}^{N_{\mathrm{U}}\times 1}$
$\mathbf{w}_k$	Transmit beamforming vector for kth DL UE at $\mathrm{BS_D}$	$\mathbb{C}^{N_{\mathrm{D}}\times 1}$
$\mathbf{W}$	Transmit beamforming matrix at $\mathrm{BS_D}$	$\mathbb{C}^{N_{\mathrm{D}}\times M_d}$
$\mathbf{W}_{\mathrm{IC}}$	Ideal beamforming solution for DL communication	$\mathbb{C}^{N_{\mathrm{D}}\times M_d}$
$\mathbf{W}_{\mathrm{SM}}$	Ideal beamforming solution for CLI suppression	$\mathbb{C}^{N_{\mathrm{D}}\times M_d}$
$\mathbf{W}_{\mathrm{opt}}$	Ideal beamforming solution meets need to suppress CLI while minimizing impact on DL communication	$\mathbb{C}^{N_{\mathrm{D}}\times M_d}$
$\mathbf{Q}_{\Omega}$	Null-space projection matrix for CLI suppression	$\mathbb{C}^{N_{\mathrm{D}}\times \Omega}$
Γ_{BS}	Threshold for determining CLI suppression rank r	Scalar ($0 \leq \Gamma_{\mathrm{BS}} \leq 1$)
$\mathrm{SINR}_j^{\mathrm{UL}}$	SINR of jth UL UE at $\mathrm{BS_U}$	Non-negative scalar
$\mathrm{SINR}_k^{\mathrm{DL}}$	SINR of kth DL UE at $\mathrm{BS_D}$	Non-negative scalar
$\mathbf{R}_{\mathrm{U}}, \mathbf{R}_{\mathrm{D}}$	Spatial correlation matrices of $\mathrm{BS_U}$ and $\mathrm{BS_D}$	$\mathbb{C}^{N_{\mathrm{U}}\times N_U}, \mathbb{C}^{N_{\mathrm{D}}\times N_{\mathrm{D}}}$
$\mathcal{D}, \mathcal{U}, \mathcal{K}$	Sets of DL, UL and all UE indices	$\mathcal{K} = \mathcal{D} \cup \mathcal{U}$

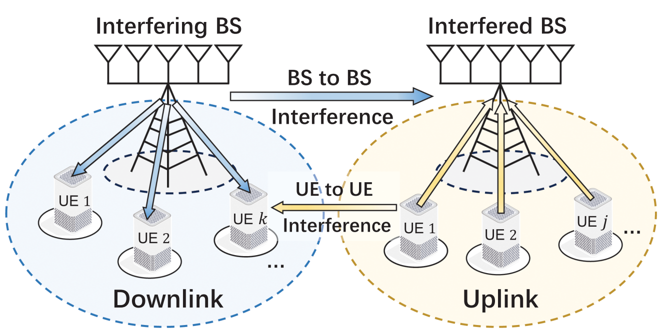

Fig. 1. System model of a two-cell FD MIMO including the BS-to-BS and the UE-to-UE CLI.

matrix, respectively. $\mathrm{diag}(\mathbf{a}_1,\ldots,\mathbf{a}_n)$ is a block diagonal matrix whose diagonal blocks are $\{\mathbf{a}_1,\ldots,\mathbf{a}_n\}$, and $\mathrm{Tr}\{\cdot\}$ denote the trace of a square matrix. Table I lists the symbols and their definitions used in this paper.

II. System Model and Problem Formulation

A two-cell multi-user MIMO system operating in FD (e.g., the D-TDD) mode[2] is illustrated in Fig. 1. In the system, an N_{U}-antenna BS in UL transmission, denoted as $\mathrm{BS_U}$, is interfered by a neighboring N_{D}-antenna BS in DL transmission, denoted as $\mathrm{BS_D}$. Moreover, the set of all user equipment (UEs) indices is represented as $\mathcal{K} = \mathcal{D} \cup \mathcal{U}$, where $\mathcal{D} \triangleq \{1, 2, \ldots, M_d\}$ and $\mathcal{U} \triangleq \{1, 2, \ldots, M_u\}$ are the sets of indices corresponding to the UEs in DL and UL transmissions, respectively. For simplicity, we assume that each UL/DL UE transmits/receives only one single spatial stream to/from $\mathrm{BS_U}$/$\mathrm{BS_D}$, and the both BSs are perfectly synchronized with their associated UEs [12].

The CLI channel from $\mathrm{BS_D}$ to $\mathrm{BS_U}$ can be formulated as

$$\mathbf{H}_{\mathrm{BS}} = \breve{\mathbf{H}}_{\mathrm{BS}} + \overline{\mathbf{H}}_{\mathrm{BS}} = \mathbf{R}_{\mathrm{U}}^{\frac{1}{2}}\mathbf{H}_w\mathbf{R}_{\mathrm{D}}^{\frac{1}{2}} + \overline{\mathbf{H}}_{\mathrm{BS}} \in \mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{D}}}, \quad (1)$$

where $\breve{\mathbf{H}}_{\mathrm{BS}} \in \mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{D}}}$ and $\overline{\mathbf{H}}_{\mathrm{BS}} \in \mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{D}}}$ denote the non-line-of-sight (NLoS) and the line-of-sight (LoS) components of the channel, respectively [13]. Moreover, $\mathbf{R}_{\mathrm{U}} \in \mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{U}}}$ and $\mathbf{R}_{\mathrm{D}} \in \mathbb{C}^{N_{\mathrm{D}}\times N_{\mathrm{D}}}$ represent the deterministic spatial correlation matrices of $\mathrm{BS_U}$ and $\mathrm{BS_D}$, respectively, while $\mathbf{H}_w \in \mathbb{C}^{N_{\mathrm{U}}\times N_{\mathrm{D}}}$ consists of the random components of the channel, whose elements are statistically independent and identically distributed (i.i.d.) with zero-mean and unit-variance. Due to the strong LoS paths between BSs as noted in [9], we assume that the CLI channel $\mathbf{H}_{\mathrm{BS}}$ between $\mathrm{BS_U}$ and $\mathrm{BS_D}$ is *rank-deficient* [14], i.e., $\mathrm{rank}(\mathbf{H}_{\mathrm{BS}}) \leq \min\{N_{\mathrm{U}}, N_{\mathrm{D}}\}$.

[2]While the BS-to-UE and the UE-to-BS interference exist in FD deployments, in this paper we mainly focus on CLI suppression.

The UL signal received by $\mathrm{BS_U}$ for the desired jth UE in the set $\mathcal{U}$ can be expressed as

$$y_j^{\mathrm{UL}} = \underbrace{\mathbf{v}_j^{\mathrm{H}}\mathbf{h}_{\mathrm{U}_j}x_j}_{\text{Desired Signal}} + \underbrace{\mathbf{v}_j^{\mathrm{H}}\sum\nolimits_{j'\neq j, j'\in\mathcal{U}}\mathbf{h}_{\mathrm{U}_{j'}}x_{j'}}_{\text{UL Intra-cell Interference}} + \underbrace{\mathbf{v}_j^{\mathrm{H}}\left(\mathbf{H}_{\mathrm{BS}}\sum\nolimits_{k\in\mathcal{D}}\mathbf{w}_k x_k\right)}_{\text{BS-to-BS Interference}} + \underbrace{\mathbf{v}_j^{\mathrm{H}}z_j}_{\text{Noise}}, \quad (2)$$

where $\mathbf{h}_{\mathrm{U}_j} \in \mathbb{C}^{N_{\mathrm{U}}\times 1}$ is the UL channel between the jth UL UE and $\mathrm{BS_U}$, $x_j \in \mathbb{C}$ is the data symbol transmitted by the jth UL UE with $\mathbb{E}\{x_j\} = 0$ and $\mathbb{E}\{|x_j|^2\} = 1$, and $\mathbf{v}_j \in \mathbb{C}^{N_{\mathrm{U}}\times 1}$ is the associated receive beamforming vector of $\mathrm{BS_U}$, while $x_k \in \mathbb{C}$ is the data symbol transmitted by $\mathrm{BS_D}$ to the targeted kth DL UE in the set $\mathcal{D}$, which has zero-mean and unit-variance, and $\mathbf{w}_k \in \mathbb{C}^{N_{\mathrm{D}}\times 1}$ is the associated transmit beamforming vector of $\mathrm{BS_D}$. Additionally, $z_j \sim \mathcal{CN}(0, \sigma_j^2)$ is the additive white Gaussian noise (AWGN) with zero-mean and variance σ_j^2.

Similarly, the received signal at the kth DL UE with $k \in \mathcal{D}$ in the interfering cell can be expressed as

$$y_k^{\mathrm{DL}} = \underbrace{\mathbf{h}_{\mathrm{D}_k}^{\mathrm{H}}\mathbf{w}_k x_k}_{\text{Desired Signal}} + \underbrace{\sum\nolimits_{k'\neq k, k'\in\mathcal{D}}\mathbf{h}_{\mathrm{D}_k}^{\mathrm{H}}\mathbf{w}_{k'}x_{k'}}_{\text{DL Intra-cell Interference}} + \underbrace{\sum\nolimits_{j\in\mathcal{U}}Q_{k,j}x_j}_{\text{UE-to-UE Interference}} + \underbrace{z_k}_{\text{Noise}}, \quad (3)$$

where $\mathbf{h}_{\mathrm{D}_k}^{\mathrm{H}} \in \mathbb{C}^{1\times N_{\mathrm{D}}}$ is the DL channel vector between $\mathrm{BS_D}$ and the kth DL UE, $Q_{k,j} \in \mathbb{C}$ denotes the channel between the jth UL UE and the kth DL UE, and $z_k \sim \mathcal{CN}(0, \sigma_k^2)$ is the AWGN with zero-mean and variance σ_k^2.

Then, the signal-to-interference-plus-noise ratio (SINR) for the communication link between $\mathrm{BS_D}$ and the kth DL UE can be expressed as

$$\mathrm{SINR}_k^{\mathrm{DL}} = \frac{\left|\mathbf{h}_{\mathrm{D}_k}^{\mathrm{H}}\mathbf{w}_k\right|^2}{\sum_{k'\neq k, k'\in\mathcal{D}}\left|\mathbf{h}_{\mathrm{D}_k}^{\mathrm{H}}\mathbf{w}_{k'}\right|^2 + \sum_{j\in\mathcal{U}}|Q_{k,j}|^2 + \sigma_k^2}, \quad (4)$$

and the SINR of the jth UL UE in the interfered cell is

$$\mathrm{SINR}_j^{\mathrm{UL}} = \frac{\left|\mathbf{v}_j^{\mathrm{H}}\mathbf{h}_{\mathrm{U}_j}\right|^2}{\sum_{j'\neq j, j'\in\mathcal{U}}\left|\mathbf{v}_j^{\mathrm{H}}\mathbf{h}_{\mathrm{U}_{j'}}\right|^2 + \sum_{k\in\mathcal{D}}\left|\mathbf{v}_j^{\mathrm{H}}\mathbf{H}_{\mathrm{BS}}\mathbf{w}_k\right|^2 + \tilde{\sigma}_j^2}, \quad (5)$$

where $\tilde{\sigma}_j^2 = \sigma_j^2\|\mathbf{v}_j^{\mathrm{H}}\|^2$. Furthermore, the sum rates of $\mathrm{BS_D}$ and $\mathrm{BS_U}$ can be calculated as $R_{\mathrm{sum}}^{\mathrm{DL}} = \sum_{k\in\mathcal{D}}\log_2(1 + \mathrm{SINR}_k^{\mathrm{DL}})$ and $R_{\mathrm{sum}}^{\mathrm{UL}} = \sum_{j\in\mathcal{U}}\log_2(1 + \mathrm{SINR}_j^{\mathrm{UL}})$, respectively.

Considering the low transmission power of UEs, the UE-to-UE interference (the third term in (3)) is typically smaller than the BS-to-BS CLI (the third term in (2)). Also there exist many works focusing on UE-to-UE CLI suppression. Here, we focus on BS-to-BS CLI suppression by designing the beamforming matrix $\mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_{M_d}] \in \mathbb{C}^{N_{\mathrm{D}}\times M_d}$ for $\mathrm{BS_D}$.

To meet different design objectives, specifically, to completely cancel the DL intra-cell interference (the second term in (3)) and to achieve the complete BS-to-BS CLI suppression (the third term in (2)), the beamforming matrix $\mathbf{W}$ to be designed for $\mathrm{BS_D}$ should respectively satisfy:

$$\mathbf{H}_{\mathrm{DL}} \cdot \mathbf{W}_{\mathrm{SM}} = \mathbf{I}_{M_d}, \tag{6}$$

$$\mathbf{H}_{\mathrm{BS}} \cdot \mathbf{W}_{\mathrm{IC}} = \mathbf{0}_{N_{\mathrm{U}} \times M_d}, \tag{7}$$

where $\mathbf{H}_{\mathrm{DL}} = [\mathbf{h}_{\mathrm{D}_1}, \mathbf{h}_{\mathrm{D}_2}, \ldots, \mathbf{h}_{\mathrm{D}_{M_d}}]^{\mathrm{H}} \in \mathbb{C}^{M_d \times N_{\mathrm{D}}}$, while $\mathbf{W}_{\mathrm{SM}} \in \mathbb{C}^{N_{\mathrm{D}} \times M_d}$ and $\mathbf{W}_{\mathrm{IC}} \in \mathbb{C}^{N_{\mathrm{D}} \times M_d}$ represent the ideal beamforming solutions for the interference-free DL communication and BS-to-BS CLI suppression designs, respectively. Specifically, the focus of (6) is to align multiple interfering signals in the same direction at unintended receivers at $\mathrm{BS_D}$, while in (7), all the signals from $\mathrm{BS_D}$ are nullified at $\mathrm{BS_U}$ to suppress the BS-to-BS CLI.

Simultaneously satisfying both (6) and (7) is challenging, since the different design objectives lead to different optimal solutions, namely, $\mathbf{W}_{\mathrm{SM}} \neq \mathbf{W}_{\mathrm{IC}}$ in general. In this paper, we aim to determine a beamforming matrix $\mathbf{W}_{\mathrm{opt}}$ that can suppress the BS-to-BS CLI, while minimizing its impact on DL communication performance.

III. Proposed Hierarchical Beamforming Design

We first present a new perspective on the trade-off between SM and IC capabilities in FD systems, and then develop a hybrid CSI-based hierarchical beamforming algorithm to suppress CLI according to this perspective.

A. Trade-Off Analysis Between SM and IC

We begin by considering the constraints (6) and (7) separately to obtain insights on their respective optimal solutions. Then, we analyze the differences between the two solutions and provide the fundamental trade-offs in design.

To meet (6), the desired $\mathbf{W}_{\mathrm{SM}}$ can be constructed based on the DL channel matrix under different design criteria, such as the zero-forcing (ZF) which yields $\mathbf{W}_{\mathrm{SM}} = \mathbf{H}_{\mathrm{DL}}^{\mathrm{H}}(\mathbf{H}_{\mathrm{DL}}\mathbf{H}_{\mathrm{DL}}^{\mathrm{H}})^{-1}$, according to [15].

In (7), the IC constraint can be rewritten as

$$\mathbf{W}_{\mathrm{IC}}^{\mathrm{H}} \cdot \mathbf{H}_{\mathrm{BS}}^{\mathrm{H}} \cdot \mathbf{H}_{\mathrm{BS}} \cdot \mathbf{W}_{\mathrm{IC}} = \mathbf{0}_{M_d \times M_d}. \tag{8}$$

$\mathbf{H}_{\mathrm{BS}}^{\mathrm{H}}\mathbf{H}_{\mathrm{BS}}$ can be decomposed as $\mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{\mathrm{H}}$, where $\mathbf{Q} \in \mathbb{C}^{N_{\mathrm{D}} \times N_{\mathrm{D}}}$ comprises the eigenvectors associated with the eigenvalues in the diagonal matrix $\mathbf{\Lambda} = \mathrm{diag}\{\lambda_1, \ldots, \lambda_{N_{\mathrm{D}}}\} \in \mathbb{C}^{N_{\mathrm{D}} \times N_{\mathrm{D}}}$, and the eigenvalues $\{\lambda_i\}_{i=1}^{N_{\mathrm{D}}}$ in $\mathbf{\Lambda}$ are arranged in descending order. Besides, the eigenvectors correspond to Ω zero eigenvalues in $\mathbf{\Lambda}$ span a $(N_{\mathrm{D}} \times \Omega)$-dimension null-space $\mathbf{Q}_\Omega$, where $\Omega = N_{\mathrm{D}} - r_{\mathrm{BS}}$ and $r_{\mathrm{BS}} = \mathrm{rank}(\mathbf{H}_{\mathrm{BS}})$. Moreover, any beamforming vector selected from $\mathbf{Q}_\Omega$ satisfies (7).

The optimal beamforming matrix $\mathbf{W}_{\mathrm{opt}}$ should meet the need to suppress BS-to-BS CLI while minimizing the impact on DL communication. Let the optimal beamforming matrix that meets the IC condition (7) be $\mathbf{W}_{\mathrm{opt}} = \mathbf{Q}_\Omega\mathbf{P}$, where $\mathbf{P} \in \mathbb{C}^{\Omega \times M_d}$ is the projection coordinate matrix of the M_d beamforming vectors in $\mathbf{Q}_\Omega$. To minimize the impact on DL transmission, the following design criterion of $\mathbf{P}$ is considered

$$\widetilde{\mathbf{P}} = \arg\min_{\mathbf{P}} \|\mathbf{W}_{\mathrm{SM}} - \mathbf{Q}_\Omega\mathbf{P}\|^2, \tag{9}$$

where the minimization is achieved when $\mathbf{Q}_\Omega^{\mathrm{H}}(\mathbf{W}_{\mathrm{SM}} - \mathbf{Q}_\Omega\mathbf{P}) = \mathbf{0}_{\Omega \times M_d}$. By using the fact $\mathbf{Q}_\Omega^{\mathrm{H}}\mathbf{Q}_\Omega = \mathbf{I}_\Omega$, the optimal solution can be derived as $\widetilde{\mathbf{P}} = (\mathbf{Q}_\Omega^{\mathrm{H}}\mathbf{Q}_\Omega)^{-1}\mathbf{Q}_\Omega^{\mathrm{H}}\mathbf{W}_{\mathrm{SM}} = \mathbf{Q}_\Omega^{\mathrm{H}}\mathbf{W}_{\mathrm{SM}}$. Finally, we obtain $\mathbf{W}_{\mathrm{opt}} = \mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}}\mathbf{W}_{\mathrm{SM}}$, which is an approximation to the given $\mathbf{W}_{\mathrm{SM}}$.

The normalized average beamforming error $\bar{\varepsilon}_{\mathrm{SM}}^2$ between $\mathbf{W}_{\mathrm{opt}}$ and $\mathbf{W}_{\mathrm{SM}}$ can be evaluated as

$$\begin{aligned}\bar{\varepsilon}_{\mathrm{SM}}^2 &= \frac{\mathbb{E}\{\|\mathbf{W}_{\mathrm{opt}} - \mathbf{W}_{\mathrm{SM}}\|^2\}}{\|\mathbf{W}_{\mathrm{SM}}\|^2} = \frac{\mathbb{E}\{\|(\mathbf{I}_{N_{\mathrm{D}}} - \mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}})\mathbf{W}_{\mathrm{SM}}\|^2\}}{\|\mathbf{W}_{\mathrm{SM}}\|^2} \\ &= \frac{\mathbb{E}\{\mathrm{Tr}\{\mathbf{W}_{\mathrm{SM}}^{\mathrm{H}}(\mathbf{I}_{N_{\mathrm{D}}} - \mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}})(\mathbf{I}_{N_{\mathrm{D}}} - \mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}})\mathbf{W}_{\mathrm{SM}}\}\}}{\mathrm{Tr}\{\mathbf{W}_{\mathrm{SM}}^{\mathrm{H}}\mathbf{W}_{\mathrm{SM}}\}} \\ &= \frac{\mathrm{Tr}\{\mathbf{W}_{\mathrm{SM}}^{\mathrm{H}}(\mathbf{I}_{N_{\mathrm{D}}} - \mathbb{E}\{\mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}}\})\mathbf{W}_{\mathrm{SM}}\}}{\mathrm{Tr}\{\mathbf{W}_{\mathrm{SM}}^{\mathrm{H}}\mathbf{W}_{\mathrm{SM}}\}} = \frac{r_{\mathrm{BS}}}{N_{\mathrm{D}}},\end{aligned} \tag{10}$$

where we use the fact that $\mathbb{E}\{\mathbf{Q}_\Omega\mathbf{Q}_\Omega^{\mathrm{H}}\} = \frac{\Omega}{N_{\mathrm{D}}}\mathbf{I}_{N_{\mathrm{D}}}$. By utilizing the DoF-based model outlined in [16], it is easy and tractable to analyze the IC and SM capabilities from a DoF-based perspective. Some basic insights provided by [16] are summarized as follows.

- Considering $M_d \leq N_{\mathrm{D}}$, the total available DoFs in $\mathrm{BS_D}$ is determined by the maximum number of dimensions spanned by $\{\mathbf{w}_k\}_{k=1}^{M_d}$, namely, N_{D}.
- As the consumed DoFs are determined by the number of linearly independent constraints imposed on $\mathbf{W}$, the number of DoFs required for IC equals the rank of the interfering channel $\mathbf{H}_{\mathrm{BS}}$, namely, r_{BS}.

Thus, the beamforming error $\bar{\varepsilon}_{\mathrm{SM}}^2$ in (10) reflects the fundamental trade-off between IC and SM capabilities in FD systems. Specifically, $\bar{\varepsilon}_{\mathrm{SM}}^2$ equals the ratio of the DoFs consumed for IC to the total available DoFs. When $r_{\mathrm{BS}} = 0$, there is no beamforming error for SM and all DoFs can be allocated to SM without IC consideration. As r_{BS} increases, the DoFs consumed for IC also increases, reducing the remaining DoFs available for SM and consequently limiting communication performance.

The insight gained from this perspective is that a flexible selection of r_{BS} can offer DoF savings, which can ensure a certain level of CLI suppression while reducing the impact on DL transmission.

B. Hybrid CSI-Based Hierarchical Beamforming Design

This subsection proposes a hybrid CSI-based hierarchical two-stage beamforming algorithm. Specifically, in *Stage I* we mitigate the BS-to-BS CLI, using only the statistical CSI of the CLI channel $\mathbf{H}_{\mathrm{BS}}$. In *Stage II*, we further utilize the instantaneous CSI of DL UEs to eliminate the mutual interference in the DL cell.

1) Stage I. Statistical Beamforming for BS-to-BS IC: To suppress the average BS-to-BS CLI, the statistical properties of $\mathbf{H}_{\mathrm{BS}}$ are leveraged. By defining $\mathbb{E}\{\mathbf{H}_{\mathrm{BS}}^{\mathrm{H}}\mathbf{H}_{\mathrm{BS}}\} = \mathbf{\Phi}_{\mathrm{BS}}$, we first rewrite (8) as

$$\mathbf{W}_{\mathrm{opt}}^{\mathrm{H}}\mathbf{\Phi}_{\mathrm{BS}}\mathbf{W}_{\mathrm{opt}} = \mathbf{W}_{\mathrm{opt}}^{\mathrm{H}}\widetilde{\mathbf{Q}}\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{Q}}^{\mathrm{H}}\mathbf{W}_{\mathrm{opt}} = \mathbf{0}_{M_d \times M_d}, \tag{11}$$

where $\mathbf{\Phi}_{\mathrm{BS}} = \widetilde{\mathbf{Q}}\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{Q}}^{\mathrm{H}}$ is the eigen-decomposition of $\mathbf{\Phi}_{\mathrm{BS}}$. The eigenvalues $\{\lambda_i(\mathbf{\Phi}_{\mathrm{BS}})\}_{i=1}^{N_{\mathrm{D}}}$ in the diagonal matrix $\widetilde{\mathbf{\Lambda}} = \mathrm{diag}\{\lambda_1(\mathbf{\Phi}_{\mathrm{BS}}), \lambda_2(\mathbf{\Phi}_{\mathrm{BS}}), \ldots, \lambda_{N_{\mathrm{D}}}(\mathbf{\Phi}_{\mathrm{BS}})\}$, arranged in descending order, represent the intensity of CLI with larger eigenvalues indicating stronger CLI in the corresponding directions. Given the number of DoFs to be consumed for IC, suppressing CLI along the eigenvectors associated with the larger eigenvalues can ensure optimal IC performance.

Furthermore, inspired by the fundamental trade-off between IC and SM capabilities, we here propose a flexible selection to determine the value of r as[3]:

$$r = \arg\min_k \left\{ \frac{\sum_{l=1}^{k} \lambda_l \left(\mathbf{\Phi}_{\mathrm{BS}}\right)}{\sum_{l=1}^{N_{\mathrm{D}}} \lambda_l \left(\mathbf{\Phi}_{\mathrm{BS}}\right)} \geq \Gamma_{\mathrm{BS}} \right\}, \tag{12}$$

where $\Gamma_{\mathrm{BS}} \in [0, 1]$ is defined as the suppression threshold on the CLI link and r DoFs are allocated for CLI suppression.

With a determined value of r, we reformulate $\mathbf{\Phi}_{\mathrm{BS}} = \widetilde{\mathbf{Q}}\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{Q}}^{\mathrm{H}}$ as $\mathbf{\Phi}_{\mathrm{BS}} = [\widetilde{\mathbf{Q}}_r, \widetilde{\mathbf{Q}}_n]\mathbf{\Lambda}[\widetilde{\mathbf{Q}}_r, \widetilde{\mathbf{Q}}_n]^{\mathrm{H}}$, where $\widetilde{\mathbf{Q}}_r \in \mathbb{C}^{N_{\mathrm{D}} \times r}$ and $\widetilde{\mathbf{Q}}_n \in \mathbb{C}^{N_{\mathrm{D}} \times (N_{\mathrm{D}} - r)}$ are the subspaces correspond to the first r and last $(N_{\mathrm{D}} - r)$ eigenvalues, respectively. With r DoFs determined to be consumed for IC, we aim to suppress the CLI along the first r dominating eigenvectors, namely, $\widetilde{\mathbf{Q}}_r$. In the next section, we will validate the impact of this selected r on CLI suppression performance through simulations.

To meet (11), we consider to set the first r rows of $\widetilde{\mathbf{Q}}^{\mathrm{H}}\mathbf{W}_{\mathrm{opt}}$ as a $(r \times M_d)$-dimension all zero matrix to satisfy $\widetilde{\mathbf{\Lambda}}\widetilde{\mathbf{Q}}^{\mathrm{H}}\mathbf{W}_{\mathrm{opt}} = \mathbf{0}_{r \times M_d}$. By defining $\overline{\mathbf{W}}_{\mathrm{opt}} = \widetilde{\mathbf{Q}}^{\mathrm{H}}\mathbf{W}_{\mathrm{opt}}$, we have

$$\overline{\mathbf{W}}_{\mathrm{opt}} = \left[\mathbf{0}_{r \times M_d}^{\mathrm{T}}, \overline{\mathbf{W}}^{\mathrm{T}}\right]^{\mathrm{T}}, \tag{13}$$

where $\overline{\mathbf{W}} \in \mathbb{C}^{(N_{\mathrm{D}} - r) \times M_d}$ denotes the beamforming matrix to be designed. Then, using the fact that $\widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^{\mathrm{H}} = \mathbf{I}_{N_{\mathrm{D}}}$, we have

$$\mathbf{W}_{\mathrm{opt}} = \widetilde{\mathbf{Q}}\overline{\mathbf{W}}_{\mathrm{opt}} = \left[\widetilde{\mathbf{Q}}_r, \widetilde{\mathbf{Q}}_n\right] \left[\mathbf{0}_{r \times M_d}^{\mathrm{T}}, \overline{\mathbf{W}}^{\mathrm{T}}\right]^{\mathrm{T}}. \tag{14}$$

By utilizing the statistical CSI of $\mathbf{H}_{\mathrm{BS}}$ to suppress the CLI, frequent channel estimation between $\mathrm{BS_U}$ and $\mathrm{BS_D}$ is avoided and this reduces the overhead of real-time CSI acquisition. It is noteworthy that without low-overhead consideration, the instantaneous CSI of $\mathbf{H}_{\mathrm{BS}}$ can also be used here to further improve CLI suppression performance and we validate this in next section.

2) Stage II. Instantaneous Beamforming for Intra-Cell IC: To mitigate the intra-cell interference among M_d UEs in the DL cell, we consider to align the intra-cell interference into the null-space of the desired signal. Considering $\widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^{\mathrm{H}} = \mathbf{I}_{N_{\mathrm{D}}}$, the condition (6) can be reformulated as

$$\mathbf{H}_{\mathrm{DL}}\widetilde{\mathbf{Q}}\widetilde{\mathbf{Q}}^{\mathrm{H}}\mathbf{W}_{\mathrm{opt}} = \overline{\mathbf{H}}_{\mathrm{DL}}\overline{\mathbf{W}}_{\mathrm{opt}} = \mathbf{I}_{N_{\mathrm{D}} - r}, \tag{15}$$

where $\overline{\mathbf{H}}_{\mathrm{DL}} = \mathbf{H}_{\mathrm{DL}}\widetilde{\mathbf{Q}}$. Note that the first r rows of $\overline{\mathbf{W}}_{\mathrm{opt}}$ contain all zero elements, resulting the identity matrix in (15) has a maximum $(N_{\mathrm{D}} - r)$-dimension.[4]

With regard to (13), we partition $\overline{\mathbf{H}}_{\mathrm{DL}}$ into $\overline{\mathbf{H}}_{\mathrm{DL}} = [\overline{\mathbf{H}}_1\, \overline{\mathbf{H}}_2]$, with $\overline{\mathbf{H}}_1 \in \mathbb{C}^{M_d \times r}$ and $\overline{\mathbf{H}}_2 \in \mathbb{C}^{M_d \times (N_{\mathrm{D}} - r)}$. Then, (15) can be reformulated as $\overline{\mathbf{H}}_2\overline{\mathbf{W}} = \mathbf{I}_{M_d}$, which has the similar form to the ZF criterion [17] and consequently the solution can be derived as $\overline{\mathbf{W}} = \overline{\mathbf{H}}_2^{\mathrm{H}}(\overline{\mathbf{H}}_2\overline{\mathbf{H}}_2^{\mathrm{H}})^{-1}$.

Thus, the final beamforming matrix is

$$\mathbf{W}_{\mathrm{opt}} = \widetilde{\mathbf{Q}} \begin{bmatrix} \mathbf{0}_{r \times M_d} \\ \overline{\mathbf{W}} \end{bmatrix} = \widetilde{\mathbf{Q}} \begin{bmatrix} \mathbf{0} \\ \overline{\mathbf{H}}_2^{\mathrm{H}} \left(\overline{\mathbf{H}}_2\overline{\mathbf{H}}_2^{\mathrm{H}}\right)^{-1} \end{bmatrix}. \tag{16}$$

[3]In fact, $\mathbf{\Phi}_{\mathrm{BS}}$ may exhibit a full or almost full rank. Determine the DoFs consumed for IC only by the non-zero condition is not practical.

[4]An insight present here is that the maximum number of the remaining DoFs can be allocated to SM is $N_{\mathrm{D}} - r$, after consuming r DoFs on implementing CLI suppression in *Stage I*.

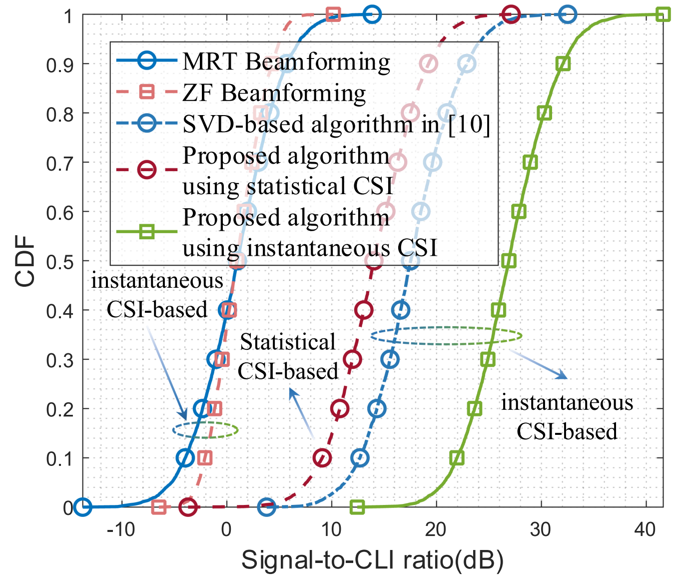

Fig. 2. CDFs of the UL signal-to-CLI ratio for different IC methods.

IV. Numerical Results

In this section, Monte Carlo simulations are conducted to evaluate the performance of the proposed algorithm. In the simulations, the numbers of antennas at $\mathrm{BS_D}$ and $\mathrm{BS_U}$ are set to $N_{\mathrm{D}} = 128$ and $N_{\mathrm{U}} = 4$, respectively. Moreover, $M_d = 4$ DL UEs in the DL cell and $M_u = 3$ UL UEs in the UL cell are randomly deployed within the circular regions with a radius ranging from 20 m to 100 m.[5]

Firstly, we evaluate the CLI suppression performance in Fig. 2. Specifically, the cumulative distribution function (CDF) of the signal-to-CLI ratio is illustrated. The CLI suppression performance of the proposed algorithm is validated by using both instantaneous and statistical CSI of $\mathbf{H}_{\mathrm{BS}}$. When instantaneous CSI is used, our proposed algorithm demonstrates superior CLI suppression performance over the singular value decomposition (SVD)-based CLI mitigation algorithm [10]. At 90% CDF, our algorithm offers above 9 dB gain in signal-to-CLI ratio over the SVD-based algorithm. Moreover, the performance gap between the SVD-based algorithm using instantaneous CSI and our algorithm using the statistical CSI is small, only around 4 dB in signal-to-CLI ratio at 90% CDF. By using the statistical CSI, our algorithm avoids frequent channel estimation between BSs and reduces the overhead of CSI acquisition dramatically. It can also be seen from Fig. 2 that both the maximum ratio transmission (MRT) and ZF beamforming designs based on the consideration of canceling the DL intra-cell interference only perform poorly, because they cannot suppress BS-to-BS CLI.

Next, Fig. 3 evaluates the impact of the threshold parameter Γ_{BS} on CLI suppression performance for our proposed algorithm using both instantaneous and statistical CSI. As expected, the average signal-to-CLI ratio increases with the increase of Γ_{BS}. A larger threshold Γ_{BS} selects a larger number of DoFs r for CLI suppression, leading to an enhanced performance. Also as expected, using instantaneous CSI significantly improves CLI suppression performance at the cost of imposing considerable CSI acquisition overhead.

In Figs. 4 and 5, we evaluate the average bit error rate (BER) performance of different IC methods for the DL and UL cells, respectively. In

[5]Here, the spatial correlation channel $\mathbf{H}_{\mathrm{BS}}$ between $\mathrm{BS_D}$ and $\mathrm{BS_U}$ is modeled as a Toeplitz matrix following the approach in [13], which is widely used to characterize the channel correlation in massive MIMO systems.

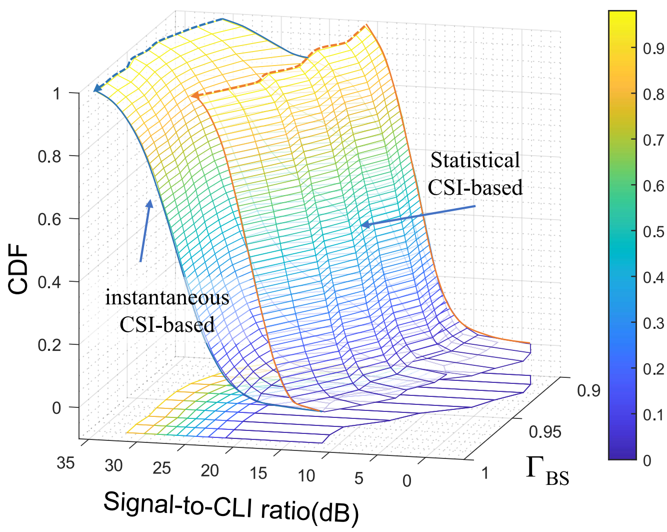

Fig. 3. CDFs of the UL signal-to-CLI ratio under increasing threshold Γ_{BS} for our proposed algorithm using both instantaneous and statistical CSI.

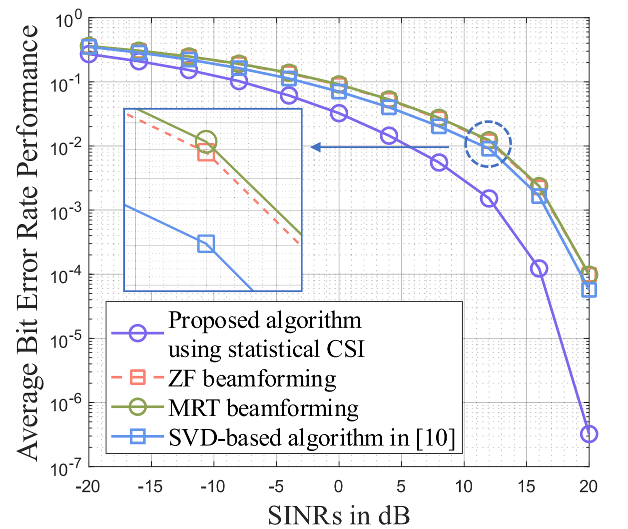

Fig. 5. The average BER performance of UL UEs, where the SVD-based algorithm, MRT beamforming and ZF beamforming use instantaneous CSI, while our proposed algorithm uses statistical CSI.

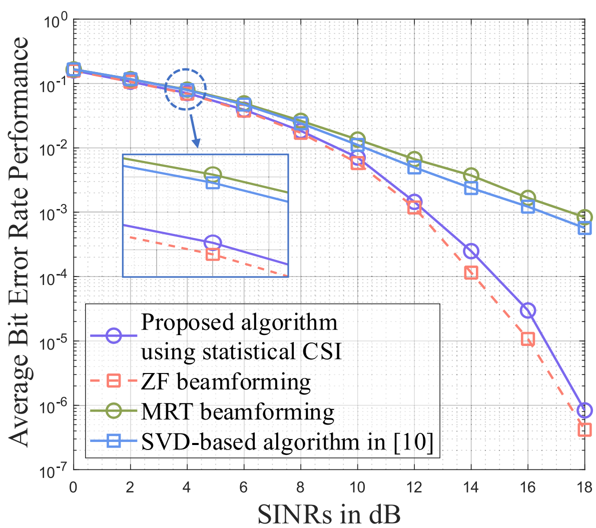

Fig. 4. The average BER performance of DL UEs, where the SVD-based algorithm, MRT beamforming and ZF beamforming use instantaneous CSI, while our proposed algorithm uses statistical CSI.

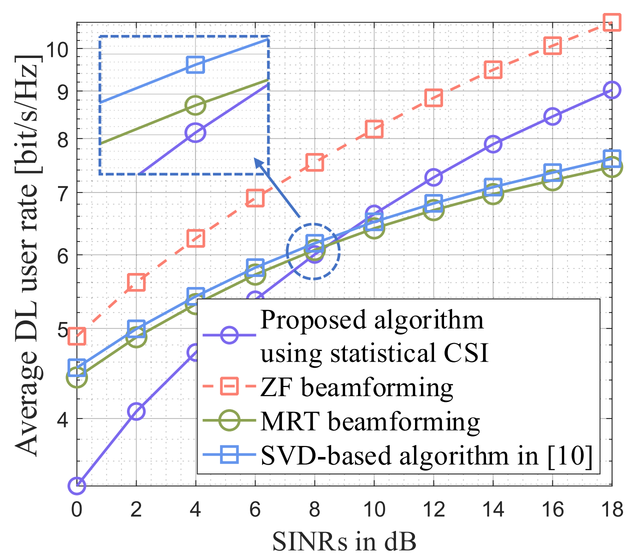

Fig. 6. The average user rate performance of DL UEs, where the SVD-based algorithm, MRT beamforming and ZF beamforming use instantaneous CSI, while our proposed algorithm uses statistical CSI.

DL transmission, when the SINR is larger than 10 dB, the proposed algorithm using statistical CSI considerably outperforms the SVD-based algorithm and MRT beamforming, and its BER performance is close to that of the ZF beamforming. Note that the other three methods use instantaneous CSI. In UL transmission, our algorithm outperforms all the other algorithms, providing approximately a 5 dB gain in SINR at the BER level of 10^{-3} over the other three methods. It can also be seen that similar to the simulation results of [10], the SVD-based algorithm only performs slightly better than the MRT and ZF methods which do not consider CLI suppression. The results of Figs. 4 and 5 thus show that the SVD-based algorithm utilizes part of DoFs to suppress CLI, at the expense of considerably decreased DL performance. In contrast, the proposed algorithm effectively balances the requirements for IC and SM. The pre-beamforming design in *Stage I* mitigates the average CLI, leading to improved UL BER performance in Fig. 5. By using a similar ZF criterion in *Stage II*, the proposed algorithm effectively mitigates the DL intra-cell interference, and considerably enhances the communication in the DL cell compared to the SVD-based algorithm.

Finally, Figs. 6 and 7 evaluate the average user rate performance of different IC methods for the DL and UL cells, respectively. As depicted, in Fig. 6, the ZF beamforming achieves the best DL user rate, and the proposed algorithm outperforms the MRT beamforming and the SVD-based algorithm in the high SINR region. In Fig. 7, for UL average user rate, the MRT and ZF methods have the similar poor performance since neither is designed for CLI mitigation. The SVD-based algorithm only performs slightly better than the MRT and ZF methods. By contrast, the proposed algorithm demonstrates significant improvement, benefiting from its IC design in *Stage I*.

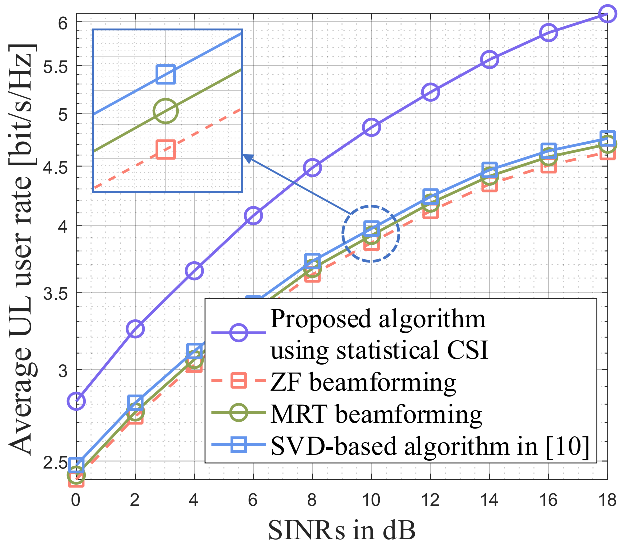

Fig. 7. The average user rate performance of UL UEs, where the SVD-based algorithm, MRT beamforming and ZF beamforming use instantaneous CSI, while our proposed algorithm uses statistical CSI.

V. Conclusion

In this paper, we have investigated the beamforming-based CLI suppression scheme in FD systems. Specifically, we have presented a new perspective on the fundamental trade-off between IC and SM in FD systems. Subsequently, we have proposed a hybrid CSI-based hierarchical beamforming algorithm, in which the BS-to-BS CLI is suppressed by utilizing the statistical CSI of the channel between BSs. Frequent channel estimation of the CLI link can thus be avoided and the overhead of CSI acquisition can be reduced. Monte Carlo simulation results have confirmed that the proposed beamforming algorithm can appropriately balance IC and SM requirements to effectively enhance both UL cell and DL cell communication performance.

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