Blind Equalisation of High-Order QAM Channels Using a Fuzzy-Logic Tuned Constant Modulus Algorithm and Soft Decision-Directed Scheme

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Abstract— **We consider blind equalisation for high-order quadrature amplitude modulation channels using a low-complexity high-performance concurrent constant modulus algorithm (CMA) and soft decision-directed (SDD) scheme. Instead of using a constant step size, we design a fuzzy-logic (FL) tuning unit to adjust the step size of the CMA. Simulation investigation confirms that faster convergence can be achieved with this FL assisted CMA and SDD scheme, compared with the constant step-size CMA and SDD scheme. Specifically, the former requires several thousands fewer samples to converge to the same steady-state solution achieved by the latter.**

Keywords – Blind equalisation, constant modulus algorithm, soft decision-directed adaptation, adaptive step size, fuzzy logic

I. Introduction

For high-throughput quadrature amplitude modulation (QAM) systems [1], the constant modulus algorithm (CMA) based blind equaliser [2]–[5] offers a low-complexity scheme. The CMA is known to be very robust to imperfect carrier recovery. Many studies have investigated the performance and convergence behaviour of the CMA [6]. A serious problem associated with the CMA is that its steady-state mean square error (MSE) may not be sufficiently low for the system to achieve an adequate symbol error rate (SER) performance. A possible solution to this problem is to switch to a decision-directed (DD) adaptation after the convergence of the CMA, so to minimise the residual CMA steady-state MSE [7] and therefore to achieve a performance close to the minimum MSE (MMSE) solution. However, a successful switch to the DD adaptation requires that the CMA's steady-state MSE is sufficiently small, and in practice such a low level of MSE may not be achievable by the CMA scheme [8].

An interesting solution was suggested in [8] to overcome the above-mentioned problem of the CMA blind equaliser. The scheme of [8] operates a DD equaliser in parallel with a CMA equaliser. The weight adaptation of the DD equaliser follows that of the CMA equaliser and, to avoid error propagation due to incorrect decisions, the DD adjustment only takes place if the CMA adaptation is deemed to have achieved a successful adjustment of the equaliser weight vector with a high probability. At a cost of slightly more than doubling the complexity of the simple CMA, this combined CMA and DD equaliser is capable of achieve a significant improvement in equalisation performance over the CMA [8]. Later, a combined CMA and soft DD (SDD) blind equaliser was proposed [9]–[11], which achieves a faster convergence and has simpler implementation than the combined CMA and DD scheme of [8]. This combined CMA and SDD scheme is capable of achieving an equalisation performance that is close to the MMSE solution based on the perfect channel information.

For the training-based least mean square (LMS) algorithm, the step size must be sufficiently small to avoid divergence. Within the range of stable step size values, a smaller step size achieves better steady-state performance at the expense of slower convergence speed, while a larger step size improves convergence speed with poorer steady-state performance [12]. A constant step-size LMS algorithm thus has to trade off between the steady-state performance and convergence speed. In attempts to optimise both the steady-state performance and convergence speed, techniques based on fuzzy logic (FL) tuning of LMS's step size were developed [13]–[17]. An application of FL tuned step size algorithm to blind source separation is given in [18]. For the CMA, the step size has to be chosen with extreme care, much more so than the LMS algorithm. While there exist some works on variable step-size CMA techniques [19], [20], to our best knowledge, no published work considers FL tuning of CMA's step size for blind equalisation of high-order QAM systems. We investigate the fuzzy step size CMA in the context of high-order QAM blind equalisation. An FL tuning unit is designed to adjust the step size of the CMA, and this fuzzy step size CMA is combined with the SDD scheme to obtain the concurrent FL assisted CMA and SDD blind equaliser. We show that the FL assisted CMA and SDD scheme achieves faster convergence over the constant step-size CMA and SDD scheme.

II. Equalisation Signal Model

Let $\mathbf{c}_{\rm CIR} = [c_0 \ c_1 \cdots c_{n_{\rm ch}-1}]^T$ be the the symbol-rate channel impulse response (CIR), where $n_{\rm ch}$ is the CIR length and c_i, $0 \leq i \leq n_{\rm ch} - 1$, are complex-valued CIR taps. The symbol-rate received signal sample $x(k)$ is then given by [21]

$$x(k) = \sum_{i=0}^{n_{\rm ch}-1} c_i s(k-i) + e(k), \tag{1}$$

where $e(k)$ is a complex-valued additive white Gaussian noise with $E[|e(k)|^2] = 2\sigma_e^2$, and $s(k)$ is the k-th transmitted symbol with $E[|s(k)|^2] = \sigma_s^2$, taking the value from the symbol set

$$\mathcal{S} \triangleq \{s_{i,l} = u_i + ju_l,\ 1 \le i, l \le \sqrt{M}\}, \quad (2)$$

where $j \triangleq \sqrt{-1}$, the real-part symbol $\Re[s_{i,l}] = u_i = 2i - \sqrt{M} - 1$ and the imaginary part $\Im[s_{i,l}] = u_l = 2l - \sqrt{M} - 1$. The channel signal-to-noise ratio (SNR) is defined as

$$\text{SNR} \triangleq \left(\sum_{i=0}^{n_c - 1} |c_i|^2 \sigma_s^2 \right) / 2\sigma_e^2. \quad (3)$$

The equaliser has a length n_{eq}, and its output is given by

$$y(k) = \sum_{i=0}^{n_{\text{eq}}-1} w_i^* x(k-i) = \mathbf{w}^H \mathbf{x}(k), \quad (4)$$

where the equaliser's weight vector $\mathbf{w} = [w_0\ w_1 \cdots w_{n_{\text{eq}}-1}]^T$ and the channel observation vector $\mathbf{x}(k) = [x(k)\ x(k-1) \cdots x(k - n_{\text{eq}} + 1)]^T$. The equaliser output $y(k)$ is passed to the decision device to produce an estimate $\hat{s}(k - \tau)$ of the transmitted symbol $s(k - \tau)$, where $0 \le \tau \le \tau_{\max} \triangleq n_{\text{eq}} + n_{\text{ch}} - 2$ is the equaliser's unknown decision delay.

The equaliser's input vector $\mathbf{x}(k)$ can be expressed as

$$\mathbf{x}(k) = \mathbf{C}\ \mathbf{s}(k) + \mathbf{e}(k) \quad (5)$$

where $\mathbf{e}(k) = [e(k)\ e(k-1) \cdots e(k - n_{\text{eq}} + 1)]^T$, $\mathbf{s}(k) = [s(k)\ s(k-1) \cdots s(k - \tau_{\max})]^T$ and the $n_{\text{eq}} \times (\tau_{\max} + 1)$ CIR matrix $\mathbf{C}$ has a Toeplitz form

$$\begin{aligned} \mathbf{C} &\triangleq \begin{bmatrix} c_0 & c_1 & \cdots & c_{n_{\text{ch}}-1} & 0 & \cdots & 0 \\ 0 & c_0 & c_1 & \cdots & c_{n_{\text{ch}}-1} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & c_0 & c_1 & \cdots & c_{n_{\text{ch}}-1} \end{bmatrix} \\ &= [\mathbf{c}_0\ \mathbf{c}_1 \cdots \mathbf{c}_{\tau_{\max}}], \end{aligned} \quad (6)$$

with $\mathbf{c}_i$ denoting the i-th column of $\mathbf{C}$. With the perfect channel information, the MMSE solution that minimises the MSE $J_{\text{MSE}}(\mathbf{w}) \triangleq E[|s(k-\tau) - y(k)|^2]$ is given by [12]

$$\mathbf{w}_{\text{MMSE}} = \left(\mathbf{C}\mathbf{C}^H + \frac{2\sigma_e^2}{\sigma_s^2} \mathbf{I}_{n_{\text{eq}}} \right)^{-1} \mathbf{c}_\tau, \quad (7)$$

where $\mathbf{I}_{n_{\text{eq}}}$ denotes the $n_{\text{eq}} \times n_{\text{eq}}$ identity matrix. Define the combined equaliser and channel impulse response as

$$\mathbf{f}^T = [f_0\ f_1 \cdots f_{\tau_{\max}}] \triangleq \mathbf{w}^H \mathbf{C}, \quad (8)$$

and let

$$i_{\max} = \arg \max_{0 \le i \le \tau_{\max}} |f_i|. \quad (9)$$

The equaliser's decision delay is in fact $\tau = i_{\max}$. In simulation, the quality of equalisation can be judged using the maximum distortion (MD) measure defined by

$$\text{MD}(\mathbf{w}) \triangleq \left(\sum_{i=0}^{\tau_{\max}} |f_i| - |f_{i_{\max}}| \right) / |f_{i_{\max}}|. \quad (10)$$

Alternatively, the equalisation performance can be assessed using the MSE criterion given by

$$\begin{aligned} J_{\text{MSE}}(\mathbf{w}) \quad \triangleq \quad & \sigma_s^2 \Big(\left(1 - \mathbf{w}^H \mathbf{c}_\tau - \mathbf{w}^T \mathbf{c}_\tau^*\right) \\ & + \mathbf{w}^H \left(\mathbf{C}\mathbf{C}^H + \frac{2\sigma_e^2}{\sigma_s^2} \mathbf{I}_{n_{\text{eq}}} \right) \mathbf{w} \Big). \end{aligned} \quad (11)$$

Ultimately, the SER can be simulated to assess the equalisation performance.

III. Blind Equalisation Algorithms

Before blind adaptation, the middle tap of $\mathbf{w}(0)$ is initialised to $1 + j0$ and the rest of the weights are set to $0 + j0$.

A. Constant modulus algorithm

Given the equaliser output $y(k) = \mathbf{w}^H(k-1)\mathbf{x}(k)$ at the sample k, the CMA adapts $\mathbf{w}$ according to [2], [3]

$$\left\{ \begin{array}{l} \varepsilon(k) = y(k) \left(\Delta - |y(k)|^2 \right), \\ \mathbf{w}(k) = \mathbf{w}(k-1) + \mu_{\text{CMA}} \varepsilon^*(k) \mathbf{x}(k), \end{array} \right. \quad (12)$$

where $\Delta = E\left[|s(k)|^4\right] / E\left[|s(k)|^2\right]$ and μ_{CMA} is the step size of the CMA. Typically, a very small μ_{CMA} has to be used to ensure convergence. The computational complexity of this CMA is summarised in Table I.

B. Combined CMA and SDD scheme

Set $\mathbf{w} = \mathbf{w}_c + \mathbf{w}_d$, with $\mathbf{w}_c(0) = \mathbf{w}_d(0) = 0.5\mathbf{w}(0)$. The weight vector $\mathbf{w}_c$ is updated using the CMA of (12) by substituting $\mathbf{w}_c$ in the place of $\mathbf{w}$. The weight vector $\mathbf{w}_d$ is updated using the SDD scheme [9]–[11]. Specifically, the complex plane is divided into the $M/4$ square regions, and each region $\mathcal{S}_{i,l}$ contains four symbol points as defined by

$$\mathcal{S}_{i,l} = \{s_{r,m},\ r = 2i-1, 2i, m = 2l-1, 2l\}, \quad (13)$$

where $1 \le i, l \le \sqrt{M}/2$. If the equaliser's output $y(k) \in \mathcal{S}_{i,l}$, a local approximation of the marginal probability density function (PDF) of $y(k)$ is given by [9]–[11]

$$\hat{p}(\mathbf{w}, y(k)) \approx \sum_{r=2i-1}^{2i} \sum_{m=2l-1}^{2l} \frac{1}{8\pi\rho} e^{-\frac{|y(k) - s_{r,m}|^2}{2\rho}}, \quad (14)$$

where ρ defines the cluster width associated with the four clusters of each region $\mathcal{S}_{i,l}$. The SDD algorithm is designed to maximise the log of the local marginal PDF criterion $E[J_{\text{LMAP}}(\mathbf{w}, y(k))]$, where $J_{\text{LMAP}}(\mathbf{w}, y(k)) =$

algorithm	multiplications	additions	$e^{\{\bullet\}}$ evaluations
CMA	$8 \times n_{\text{eq}} + 6$	$8 \times n_{\text{eq}}$	–
CMA+SDD	$12 \times n_{\text{eq}} + 29$	$14 \times n_{\text{eq}} + 21$	4
FIS	$2 + 22/N_{\text{sm}}$	$2 + 22/N_{\text{sm}}$	$6/N_{\text{sm}}$

TABLE I

Computational requirements per weight update. The symbol rate is N_{sm} times faster than the operational rate of the fuzzy inference system, and n_{eq} is the equaliser length.

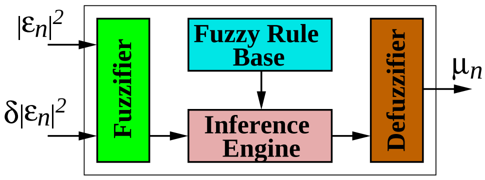


Fig. 1. Schematic of the fuzzy inference system for tuning the step size of the CMA. The operational rate n of the FIS is N_{sm} times slower than the symbol rate k.

$\rho \log(\hat{p}(\mathbf{w}, y(k)))$, using a stochastic gradient optimisation. That is, $\mathbf{w}_d$ is updated according to [9]–[11]

$$\mathbf{w}_d(k) = \mathbf{w}_d(k-1) + \mu_{\text{SDD}} \frac{\partial J_{\text{LMAP}}(\mathbf{w}(k-1), y(k))}{\partial \mathbf{w}_d}, \quad (15)$$

where μ_{SDD} is the step size of the SDD, and

$$\frac{\partial J_{\text{LMAP}}(\mathbf{w}, y(k))}{\partial \mathbf{w}_d} = \frac{1}{Z_N} \sum_{r=2i-1}^{2i} \sum_{m=2l-1}^{2l} e^{-\frac{|y(k)-s_{r,m}|^2}{2\rho}}$$
$$\times (s_{r,m} - y(k))^* \mathbf{x}(k), \quad (16)$$

with the normalisation factor

$$Z_N = \sum_{r=2i-1}^{2i} \sum_{m=2l-1}^{2l} e^{-\frac{|y(k)-s_{r,m}|^2}{2\rho}}. \quad (17)$$

The computational complexity of this combined CMA and SDD scheme (CMA+SDD) is also given in Table I. When the objective of equalisation is accomplished, $y(k) \approx s(k-\tau) + \hat{e}(k)$, where $\hat{e}(k)$ is Gaussian distributed with zero mean. Therefore, the value of ρ is related to the variance of $\hat{e}(k)$, which is $2\sigma_e^2 \mathbf{w}^H \mathbf{w}$. Soft decision nature becomes explicit in (16), because rather than committing to a single hard decision $\mathcal{Q}[y(k)]$, where $\mathcal{Q}[\bullet]$ denotes the quantisation operator, as the hard DD scheme would, alternative decisions are also considered in the local region $\mathcal{S}_{i,l}$ that includes $\mathcal{Q}[y(k)]$, and each tentative decision is weighted by an exponential term $e^{\{\bullet\}}$, which is a function of the distance between the equaliser's soft output $y(k)$ and the tentative decision $s_{r,m}$. This soft decision nature substantially reduces the risk of error propagation and achieves faster convergence, compared with the hard DD scheme [9]–[11].

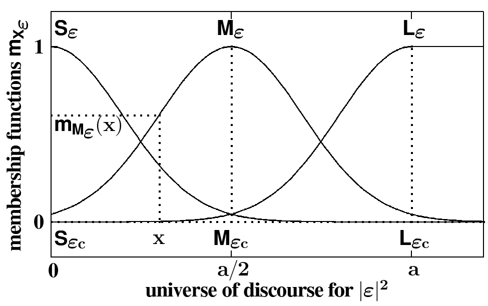


Fig. 2. Membership functions over the universe of discourse for $|\varepsilon_n|^2$, where the fuzzy variable $\mathsf{X} = \mathsf{S}$, M or L.

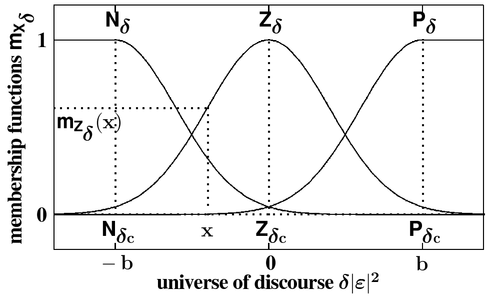


Fig. 3. Membership functions over the universe of discourse for $\delta|\varepsilon_n|^2$, where the fuzzy variable $\mathsf{X} = \mathsf{N}$, Z or P.

C. Fuzzy step size CMA

For the fuzzy step size CMA, we choose the fuzzy inference system (FIS) of Fig. 1, which maps the two input variables, $|\varepsilon_n|^2$ and $\delta|\varepsilon_n|^2$, into an appropriate step size μ_n. The operation of the FIS is based on the principle of fuzzy logic [22], [23]. The two input variables are defined respectively as

$$|\varepsilon_n|^2 = \frac{1}{N_{\text{sm}}} \sum_{l=0}^{N_{\text{sm}}-1} |\varepsilon(k-l)|^2, \quad (18)$$

$$\delta|\varepsilon_n|^2 = |\varepsilon_n|^2 - |\varepsilon_{n-1}|^2, \quad (19)$$

where $n = \lfloor k/N_{\text{sm}} \rfloor$ with $\lfloor \bullet \rfloor$ denoting the integer floor operator, and N_{sm} is the short-term average length. The FIS operates once every N_{sm} samples, and the output μ_n is used as the step size of the CMA for the subsequent N_{sm}

$$\mu_{\text{CMA}} = \mu_n, \; n \cdot N_{\text{sm}} \leq k < (n+1) \cdot N_{\text{sm}}. \quad (20)$$

The initial conditions can be set to $|\varepsilon_0|^2 = 0$ and $\mu_0 = \mu_{\min}$, where $\mu_{\min}$ represents the smallest value for the step size.

The two crisp input variables are transformed separately to the respective degrees, to which they belong to the corresponding fuzzy sets via appropriate membership functions (MBFs). The fuzzy sets used to partition the universe of discourse for $|\varepsilon_n|^2$ are labelled as small (S_ε), medium (M_ε) and large (L_ε), and their associate MBFs are shown in Fig. 2, where $\mathsf{S}_{\varepsilon c}$, $\mathsf{M}_{\varepsilon c}$ and $\mathsf{L}_{\varepsilon c}$ are the centroids of S_ε, M_ε and L_ε, respectively. The Gaussian MBFs

$$\mathsf{m}_{\mathsf{X}_\varepsilon}(x) = e^{-\frac{(x-\mathsf{X}_{\varepsilon c})^2}{2\rho_\varepsilon}} \quad (21)$$

are used, where X_ε represents S_ε, M_ε or L_ε, with the exception that $\mathsf{m}_{\mathsf{L}_\varepsilon}(x) = 1$ for $x \geq a$. The fuzzy sets used to partition the universe of discourse for $\delta|\varepsilon_n|^2$ are labelled as negative (N_δ), zero (Z_δ) and positive (P_δ), with the related MBFs shown in Fig. 3, where $\mathsf{N}_{\delta c}$, $\mathsf{Z}_{\delta c}$ and $\mathsf{P}_{\delta c}$ are the centroids of N_δ, Z_δ and P_δ, respectively. Again the Gaussian MBFs

$$\mathsf{m}_{\mathsf{X}_\delta}(x) = e^{-\frac{(x-\mathsf{X}_{\delta c})^2}{2\rho_\delta}} \quad (22)$$

Fuzzy set	S_μ	M_μ	L_μ
centroid	$\mu_{\min}$	$2\mu_{\min}$	$\mu_{\max} = 4\mu_{\min}$
universe of discourse		$[\mu_{\min}, \;\; \mu_{\max}]$	

TABLE II

FUZZY SETS FOR CRISP μ_n.

μ_n		$\delta\lvert\varepsilon_n\rvert^2$ N_δ	Z_δ	P_δ
$\lvert\varepsilon_n\rvert^2$	S_ε	1 L_μ	2 M_μ	3 S_μ
	M_ε	4 M_μ	5 S_μ	6 S_μ
	L_ε	7 M_μ	8 S_μ	9 S_μ

Fig. 4. Fuzzy rule table for the step size μ_n.

are used, where X_δ represents N_δ, Z_δ or P_δ. But we have $\mathsf{m}_{\mathsf{N}_\delta}(x) = 1$ for $x \le -b$ and $\mathsf{m}_{\mathsf{P}_\delta}(x) = 1$ for $x \ge b$.

The universe of discourse for the step size μ_n is defined by $[\mu_{\min},\ \mu_{\max}]$, and the fuzzy sets used to partition it are labelled as small (S_μ), medium (M_μ) and large (L_μ), as summarised in Table II. The fuzzy inference engine constructs a set of fuzzy `IF-THEN` rules. Since there are 3 fuzzy sets for each of $|\varepsilon_n|^2$ and $\delta|\varepsilon_n|^2$, the number of fuzzy `IF-THEN` rules is 9. These fuzzy `IF-THEN` rules are shown in Fig. 4. Rule 1, for example, reads like: `IF` $|\varepsilon_n|^2$ is S_ε `AND` $\delta|\varepsilon_n|^2$ is N_δ `THEN` μ_n is L_μ. Let $\mathsf{m}_{\mathsf{X}_\mu}(\mu_n[i])$ be the MBF value at location $\mu_n[i]$, where $1 \le i \le 9$. The 9 locations $\mu_n[i]$, $1 \le i \le 9$, are specified by Fig. 4 and Table II. For example, from Fig. 4 and Table II, we have $\mu_n[1] = \mu_{\max}$. The min operator is applied to truncate the output fuzzy set for each rule. According to the fuzzy rule table of Fig. 4, the MBF value at $\mu_n[1]$ is

$$\mathsf{m}_{\mathsf{X}_\mu}(\mu_n[1]) = \min\{\mathsf{m}_{\mathsf{S}_\varepsilon}(|\varepsilon_n|^2), \mathsf{m}_{\mathsf{N}_\delta}(\delta|\varepsilon_n|^2)\}, \tag{23}$$

and so on. The defuzzification method used to obtain a crisp value for the step size is the following centroid calculation

$$\mu_n = \frac{\sum_{i=1}^{9} \mu_n[i] \cdot \mathsf{m}_{\mathsf{X}_\mu}(\mu_n[i])}{\sum_{i=1}^{9} \mathsf{m}_{\mathsf{X}_\mu}(\mu_n[i])}, \tag{24}$$

which returns the centre of area under the aggregated MBF curve.

The extra computational complexity imposed by this FIS is given in Table I. Suitable values for $N_{\rm sm}$ can typically be chosen in the range of 10 to 20. The range of $|\varepsilon_n|^2$ is simply $a \approx \max|\varepsilon(k)|^2$, and our experience suggests that the variance of the Gaussian MBFs for $|\varepsilon_n|^2$ can be set to $\rho_\varepsilon = (0.01a)^2$. For better efficiency, $\mathsf{P}_{\delta c}$ should be relatively small, and we find by experiment that $b = 0.01a$ to $0.001a$ are appropriate depending on the size of QAM constellation M. The variance of the Gaussian MBFs for $\delta|\varepsilon_n|^2$ can be set to $\rho_\delta = (0.2b)^2$. The minimum value of the step size $\mu_{\min}$ is simply chosen to be the value for the constant step-size CMA which produces satisfactory performance in terms of both steady-state error and convergence speed.

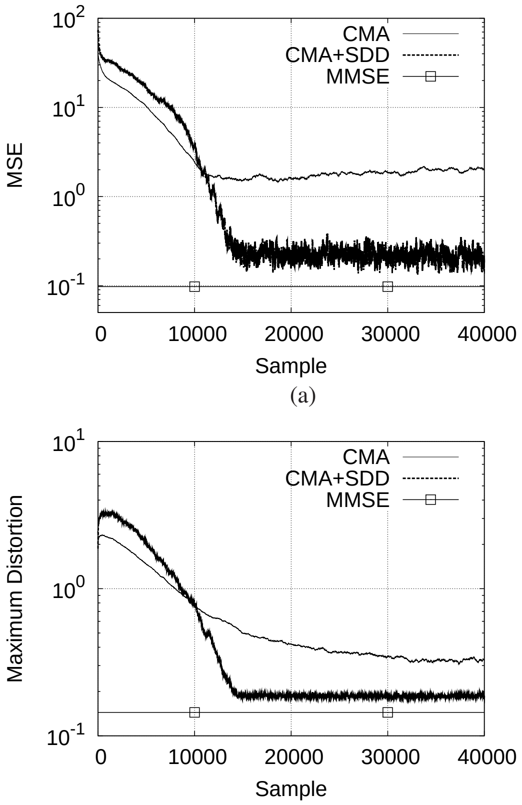
D. Combined Fuzzy step size CMA and SDD scheme

The above fuzzy step size CMA (FL-CMA) can be combined with the SDD adaptation to provide the concurrent fuzzy step size CMA and SDD scheme (FL-CMA+SDD). Note that it is not necessary to adopt a variable step size strategy for the SDD adaptation, since the "error" or the stochastic gradient used for correcting the weights is well "normalised" by the normalisation factor Z_N of (17).

IV. Simulation Study

The modulation scheme was 64-QAM, the channel length was $n_{\rm ch} = 5$ and the CIR $\mathbf{c}_{\rm CIR}$ was given by

$$[-0.2+j0.3\ \ -0.5+j0.4\ \ 0.7-j0.6\ \ 0.4+j0.3\ \ 0.2+j0.1]^T. \tag{25}$$



(a)

(b)

Fig. 5. Convergence performance comparison of the CMA and CMA+SDD, in terms of: (a) the MSE and (b) MD measure, averaged over 10 runs and given SNR= 38 dB.

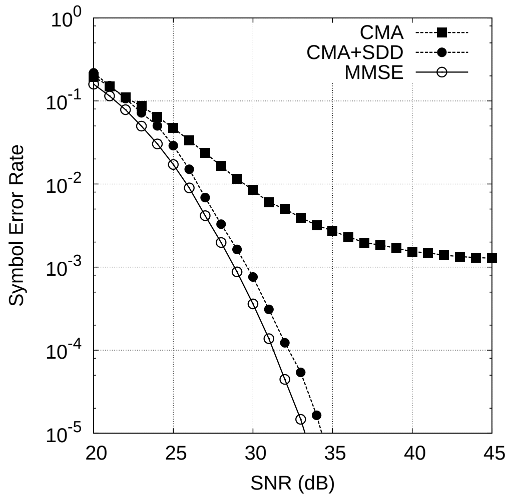


Fig. 6. Symbol error rate comparison of the three equalisers.

The equaliser length was chosen to be $n_{\rm eq} = 23$. With $\mathbf{w}(0)$ initialised to all zero elements except the middle tap to $1+j0$, the actual decision delay of the blind equaliser was $\tau = 13$. The appropriate step size of the CMA was found empirically to be $\mu_{\rm CMA} = 2 \times 10^{-7}$, while $\mu_{\rm SDD} = 2 \times 10^{-4}$ and $\rho = 0.6$ were found appropriate for the CMA+SDD. The learning curves of the blind CMA and CMA+SDD equalisers, averaged over 10 runs and quantified in terms of the MSE as well as MD measures, are depicted in Fig. 5 with the MMSE solution as the benchmark. The SER performance of the three equalisers, namely, the MMSE, the CMA and the CMA+SDD, are compared in Fig. 6.

The FL tuning unit for the step size of the CMA was next investigated. For 64-QAM

$$\max|\varepsilon(k)|^2 \approx \max\{|s(k)|^2(\Delta - |s(k)|^2)^2\} \approx 10^5, \quad (26)$$

and, therefore, we set the centroid of L_ε to $a = 10^5$ and chose $b = 0.01a = 10^3$ as the centroid of P_δ. The variances of the Gaussian MBFs were set to $\rho_\varepsilon = (0.01a)^2$ and $\rho_\delta = (0.2b)^2$ for $|\varepsilon_n|^2$ and $\delta|\varepsilon_n|^2$, respectively. The short-term average length for calculating $|\varepsilon_n|^2$ was chosen to be $N_{\rm sm} = 20$, while $\mu_{\rm min} = 2 \times 10^{-7}$ was adopted as the smallest value for μ_n. Given SNR= 38 dB, the convergence performance of this fuzzy step size CMA, labelled as the FL-CMA, is compared with that of the CMA with a constant step size $\mu_{\rm CMA} = 2\times 10^{-7}$ in Fig. 7, where it can be seen that this FL-CMA did achieve a significantly faster convergence. However, its steady-state performance was poorer than the CMA, since the step size of the FL-CMA was always larger than or equal to 2×10^{-7}.

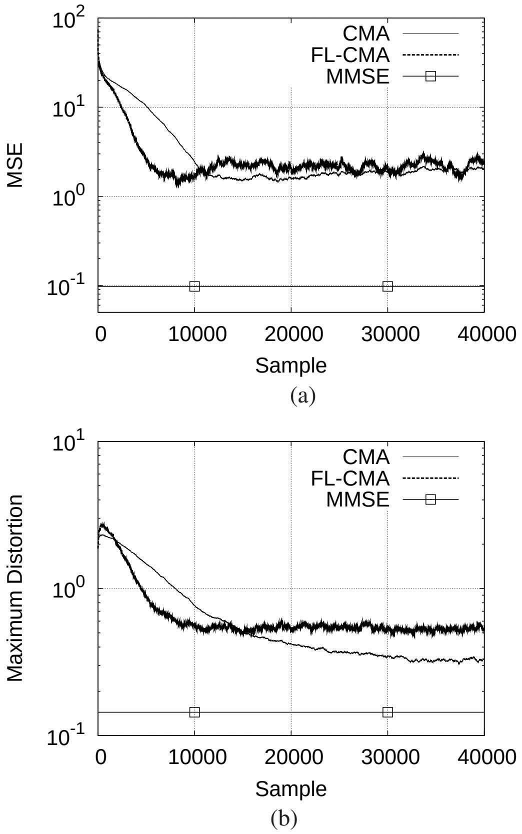


Fig. 7. Convergence performance comparison of the CMA and FL-CMA, in terms of: (a) the MSE and (b) MD measure, averaged over 10 runs and given SNR= 38 dB.

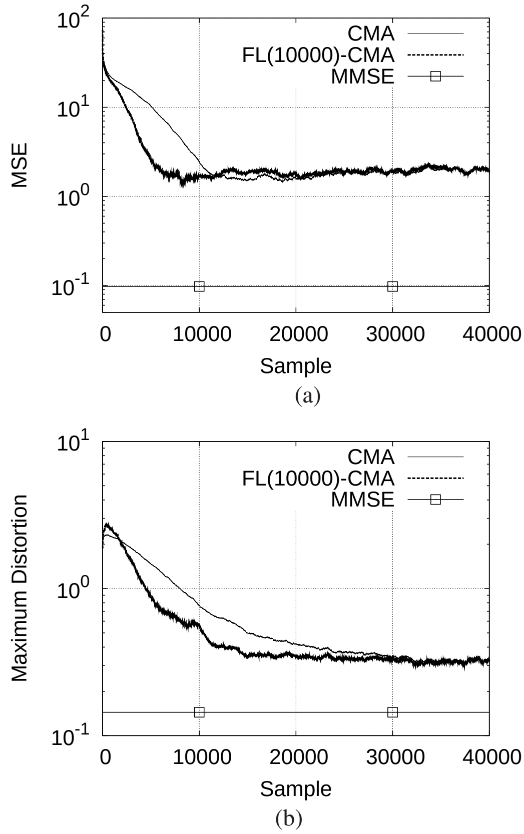


Fig. 8. Convergence performance comparison of the CMA and FL(10000)-CMA, in terms of: (a) the MSE and (b) MD measure, averaged over 10 runs and given SNR= 38 dB. The FL(10000)-CMA uses the fuzzy step size for the first 10000 samples and then switches to a constant step size.

It was not difficult to re-design the parameters of the FL tuning unit so that the resulting FL-CMA could achieve the same steady-state performance as the CMA but the gain in convergence speed would somewhat diminish. A better strategy is to use this FL-CMA in the initial stage of blind adaptation for the maximum benefit in convergence rate and then to switch to the constant step size CMA for the same good steady-state performance. Fig. 8 shows learning curve of this switched FL-CMA, labelled as the FL(10000)-CMA, where the FL-CMA was used for the initial adaptation of 10000 samples and the CMA of a constant step size $\mu_{\rm CMA} = 2\times 10^{-7}$ was used afterward. The choice of 10000 was based on the observation that the MSE of the CMA converged approximately after 10000 samples. The results of Fig. 8 confirm that the FL(10000)-CMA had the same steady-state performance as the CMA, but the former achieved considerably faster convergence. The SER performance of the FL(10000)-CMA, not shown, is similar to that of the CMA depicted in Fig. 8. The learning curve of the combined FL(10000)-CMA and SDD scheme, labelled as the FL(10000)-CMA+SDD, is compared with that of the CMA+SDD in Fig. 9, where it can be seen that both the

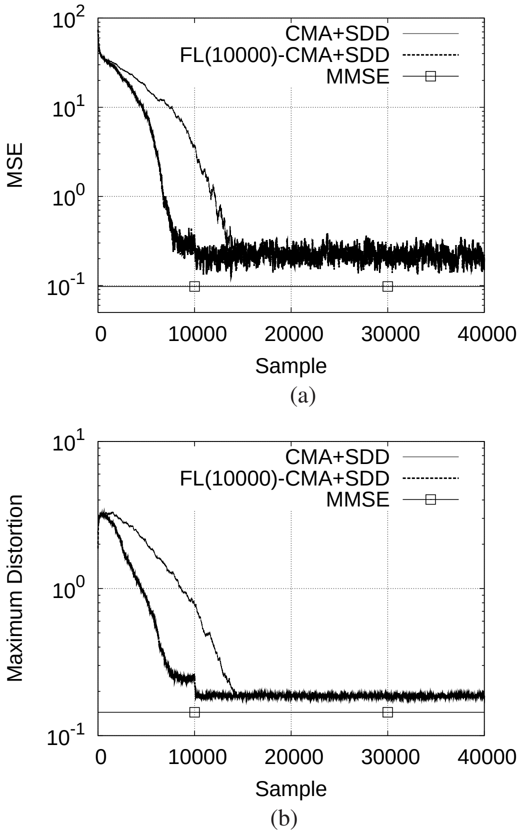


Fig. 9. Convergence performance comparison of the CMA+SDD and FL(10000)-CMA+SDD, in terms of: (a) the MSE and (b) MD measure, averaged over 10 runs and given SNR= 38 dB. The FL(10000)-CMA uses the fuzzy step size for the first 10000 samples and then switches to a constant step size.

blind equalisers achieved the same steady-state performance but the FL(10000)-CMA+SDD converged faster than the latter, requiring approximately 4000 fewer samples to converge. The SER of the FL(10000)-CMA+SDD , not shown, is the same as the CMA+SDD shown in Fig. 6.

V. Conclusions

Blind equalisation of high-order QAM systems has been revisited using the concurrent CMA and SDD scheme. A detailed design of a fuzzy step size CMA has been given and the advantages of using this fuzzy step size approach have been investigated. It has been demonstrated that, in order to achieve the maximum benefit in convergence speed and yet not to sacrifice any steady-state equalisation performance, a good strategy is to use the fuzzy step size CMA in the initial stage of blind adaptation and to switch to the CMA with a small constant step size afterward. This switched fuzzy step size CMA has been combined with the SDD adaptation, and the resulting concurrent blind equaliser has been shown to achieve significantly faster convergence with the same excellent steady-state equalisation performance, in comparison with the previous concurrent CMA and SDD scheme that employs a constant step size for the CMA. More specifically, the FL assisted CMA and SDD scheme requires several thousands fewer samples than the constant step-size CMA and SDD scheme to converge.

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