Adaptive Minimum Bit Error Rate Beamforming Assisted QPSK Receiver

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ABSTRACT

A novel adaptive beamforming technique is proposed for wireless communication with quadrature phase shift keying signalling based on the minimum bit error rate (MBER) criterion. It is shown that the MBER approach provides significant performance gain in terms of smaller bit error rate over the standard minimum mean square error approach. Using the classical Parzen window estimate of probability density function, both the block-data and sample-by-sample adaptive implementations of the MBER solution are developed.

I. INTRODUCTION

Spatial processing with adaptive antenna array has shown real promise for substantial capacity enhancement in wireless communication [1]–[5]. Adaptive beamforming can separate signals transmitted on the same carrier frequency, provided that they are separated in the spatial domain. The beamforming processing, which combines the signals received by the different elements of an antenna array to form a single output, is classically done by minimizing the mean square error (MSE) between the desired and actual array outputs or other related criteria such as the signal to interference plus noise ratio (SINR). Adaptive implementation of the minimum MSE (MMSE) beamforming solution can readily be realized using temporal reference techniques [6]–[10]. Specifically, block-data based beamformer weight adaptation can be achieved using the sample matrix inversion algorithm, while sample-by-sample adaptation can be carried out using the least mean square algorithm.

For a communication system, it is the bit error rate (BER) that really matters. Ideally, the system design should be based directly on minimizing the BER, rather than the MSE or SINR. A recent work [11] has suggested an adaptive minimum BER (MBER) beamforming assisted receiver for binary phase shift keying communication systems. This paper first presents a novel beamforming technique based directly on minimizing the system's BER for wireless systems with quadrature phase shift keying (QPSK) modulation. Adaptive implementation of the MBER beamforming solution is then studied. Adopting a Parzen window or kernel density estimation [12]–[14] to approximate the p.d.f. of the beamformer output, a block-data adaptive MBER algorithm is derived. This is then further simplified to develop a stochastic gradient adaptive MBER algorithm which we refer to as the least bit error rate (LBER).

II. SYSTEM MODEL

The system consists of M users (sources), and each user transmits a QPSK signal on the same carrier frequency $\omega = 2\pi f$. The baseband complex-valued signal of user i is:

$$m_i(k) = A_i b_i(k),\ 1 \le i \le M, \tag{1}$$

where $b_i(k) \in \{\pm 1 \pm j\}$ are QPSK symbols and $2|A_i|^2$ denotes the signal power of user i. The source 1 is assumed to be the desired user and the rest of the sources are the interfering users. The linear antenna array considered consists of L uniformly spaced elements, and the signals received by the antenna array are given by:

$$x_l(k) = \sum_{i=1}^{M} m_i(k) \exp\left(j\omega t_l(\theta_i)\right) + n_l(k) = \bar{x}_l(k) + n_l(k), \tag{2}$$

for $1 \le l \le L$, where $t_l(\theta_i)$ is the relative time delay at element l for source i, θ_i is the direction of arrival for source i, and $n_l(k)$ is a complex white Gaussian noise with zero mean and variance $E[|n_l(k)|^2] = 2\sigma_n^2$. The desired signal to noise ratio is SNR$=$ A_1^2/σ_n^2 and the desired signal to interferer i ratio is $\text{SIR}_i = A_1^2/A_i^2$, for $2 \le i \le M$. In vector form, the array input $\mathbf{x}(k) = [x_1(k) \cdots x_L(k)]^T$ can be expressed as

$$\mathbf{x}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k) \tag{3}$$

where the noise vector $\mathbf{n}(k)$ satisfies $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$, the system matrix $\mathbf{P} = [A_1\mathbf{s}_1 \cdots A_M\mathbf{s}_M]$ with the steering vector for source i $\mathbf{s}_i = [\exp(j\omega t_1(\theta_i)) \cdots \exp(j\omega t_L(\theta_i))]^T$, and the transmitted QPSK symbol vector is $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^T$.

A linear beamformer is employed, whose output is given by:

$$y(k) = \mathbf{w}^H \mathbf{x}(k) = \bar{y}(k) + e(k) \tag{4}$$

where $\mathbf{w} = [w_1 \cdots w_L]^T$ is the complex beamformer weight vector, and $e(k) = \mathbf{w}^H \mathbf{n}(k)$ is Gaussian with zero mean and $E[|e(k)|^2] = 2\sigma_n^2 \mathbf{w}^H \mathbf{w}$. Define the combined impulse response of the beamformer and the system as $\mathbf{w}^H \mathbf{P} = [c_1 \cdots c_M]$. The beamformer's output can alternatively be expressed as

$$y(k) = c_1 b_1(k) + \sum_{k=2}^{M} c_i b_i(k) + e(k) \tag{5}$$

where the first term is the desired signal and the second term the residual interference. Provided that c_1 is real and positive, the optimal decision regarding the transmitted symbol $b_1(k)$ is:

$$\hat{b}_1(k) = \text{sgn}(y_R(k)) + j\text{sgn}(y_I(k)) \tag{6}$$

where $y_R(k) = \Re[y(k)]$ and $y_I(k) = \Im[y(k)]$ are the real and imaginary parts of $y(k)$, respectively, and $\text{sgn}(\cdot)$ the sign function.

The first column of $\mathbf{P}$ is $\mathbf{p}_1 = A_1 \mathbf{s}_1$. In general, $c_1 = \mathbf{w}^H \mathbf{p}_1$ is complex-valued. The following rotating operation

$$\mathbf{w}^{\text{new}} = \frac{c_1^{\text{old}}}{\left|c_1^{\text{old}}\right|} \mathbf{w}^{\text{old}} \tag{7}$$

can be used to make c_1 real and positive. This rotation is a linear transformation and does not alter the BER, but it enables the optimal decision rule (6) to be adopted. The implication of this rotation is that, for QPSK modulation, the steering vector $\mathbf{s}_1$ of the desired user is required at the receiver. Note that this is different from the BPSK case, where the receiver does not require the steering vector of the desired user in order to make a decision [11].

III. MBER Beamforming Solution

Note that $\bar{\mathbf{x}}(k) \in \mathcal{X} \triangleq \{\bar{\mathbf{x}}^{(q)} = \mathbf{P}\mathbf{b}^{(q)},\ 1 \le q \le N_b\}$, where $N_b = 4^M$ and $\mathbf{b}^{(q)}$, $1 \le q \le N_b$, are all the possible sequences of $\mathbf{b}(k)$. Thus, $\bar{y}(k) \in \mathcal{Y} \triangleq \{\bar{y}^{(q)} = \mathbf{w}^H \bar{\mathbf{x}}^{(q)},\ 1 \le q \le N_b\}$. $\mathcal{Y}$ can be divided into the four subsets conditioned on the value of $b_1(k)$:

$$\mathcal{Y}_{\pm,\pm} \triangleq \{\bar{y}^{(q)} \in \mathcal{Y}:\ b_1(k) = \pm 1 \pm j\}. \tag{8}$$

It can be seen that the conditional p.d.f. of $y(k)$ given $b_1(k) = +1 + j$ is a Gaussian mixture given by:

$$p(y|+,+) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} \frac{1}{2\pi\sigma_n^2 \mathbf{w}^H\mathbf{w}} \exp\left(-\frac{\left|y - \bar{y}^{(q)}\right|^2}{2\sigma_n^2 \mathbf{w}^H\mathbf{w}}\right) \tag{9}$$

where $N_{sb} = N_b/4$ is the number of the points in $\mathcal{Y}_{+,+}$. Obviously, the two marginal conditional p.d.f.s for $y_R(k)$ and $y_I(k)$ are also Gaussian mixtures. Define $P_{E_R}(\mathbf{w}) \triangleq \mathrm{Prob}(\hat{b}_{R,1}(k) \neq b_{R,1}(k))$ and $P_{E_I}(\mathbf{w}) \triangleq \mathrm{Prob}(\hat{b}_{I,1}(k) \neq b_{I,1}(k))$, where $b_1(k) = b_{R,1}(k) + jb_{I,1}(k)$ and $\hat{b}_1(k) = \hat{b}_{R,1}(k) + j\hat{b}_{I,1}(k)$. The BER of the beamformer (4) is:

$$P_E(\mathbf{w}) = \frac{1}{2}\left(P_{E_R}(\mathbf{w}) + P_{E_I}(\mathbf{w})\right). \tag{10}$$

It can easily be shown that

$$P_{E_R}(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} Q\left(g_R^{(q)}(\mathbf{w})\right) \tag{11}$$

and

$$P_{E_I}(\mathbf{w}) = \frac{1}{N_{sb}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} Q\left(g_I^{(q)}(\mathbf{w})\right), \tag{12}$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^{\infty} \exp\left(-\frac{v^2}{2}\right) dv, \tag{13}$$

$$g_R^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(\Re[b_1^{(q)}])\bar{y}_R^{(q)}}{\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} = \frac{\mathrm{sgn}(b_{R,1}^{(q)})\Re[\mathbf{w}^H\bar{\mathbf{x}}^{(q)}]}{\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}}, \tag{14}$$

$$g_I^{(q)}(\mathbf{w}) = \frac{\mathrm{sgn}(\Im[b_1^{(q)}])\bar{y}_I^{(q)}}{\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} = \frac{\mathrm{sgn}(b_{I,1}^{(q)})\Im[\mathbf{w}^H\bar{\mathbf{x}}^{(q)}]}{\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} \tag{15}$$

and $b_1^{(q)} = b_{R,1}^{(q)} + jb_{I,1}^{(q)}$ is the first element of $\mathbf{b}^{(q)}$. Note that the BER is invariant to a positive scaling of $\mathbf{w}$. Similarly, the BER can be calculated alternatively based on any of the other three subsets $\mathcal{Y}_{+,-}$, $\mathcal{Y}_{-,+}$ and $\mathcal{Y}_{-,-}$.

The MBER beamforming solution is then defined as

$$\mathbf{w}_{\mathrm{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w}). \tag{16}$$

The gradient of $P_E(\mathbf{w})$ with respect to $\mathbf{w}$ is

$$\nabla P_E(\mathbf{w}) = \frac{1}{2}\left(\nabla P_{E_R}(\mathbf{w}) + \nabla P_{E_I}(\mathbf{w})\right), \tag{17}$$

and it can be shown that

$$\nabla P_{E_R}(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} \exp\left(-\frac{\left(\bar{y}_R^{(q)}\right)^2}{2\sigma_n^2\mathbf{w}^H\mathbf{w}}\right) \times \mathrm{sgn}\left(b_{R,1}^{(q)}\right)\left(\frac{\bar{y}_R^{(q)}\mathbf{w}}{\mathbf{w}^H\mathbf{w}} - \bar{\mathbf{x}}^{(q)}\right) \tag{18}$$

and

$$\nabla P_{E_I}(\mathbf{w}) = \frac{1}{2N_{sb}\sqrt{2\pi}\sigma_n\sqrt{\mathbf{w}^H\mathbf{w}}} \sum_{\bar{y}^{(q)} \in \mathcal{Y}_{+,+}} \exp\left(-\frac{\left(\bar{y}_I^{(q)}\right)^2}{2\sigma_n^2\mathbf{w}^H\mathbf{w}}\right) \times \mathrm{sgn}\left(b_{I,1}^{(q)}\right)\left(\frac{\bar{y}_I^{(q)}\mathbf{w}}{\mathbf{w}^H\mathbf{w}} + j\bar{\mathbf{x}}^{(q)}\right). \tag{19}$$

Given the gradient, the optimization problem (16) can be solved for iteratively using the simplified conjugated gradient algorithm, which is detailed in [15],[16].

IV. Adaptive MBER Beamforming

The p.d.f. of $y(k)$ can be shown to be explicitly given by:

$$p(y) = \frac{1}{N_b 2\pi\sigma_n^2\mathbf{w}^H\mathbf{w}} \sum_{q=1}^{N_b} \exp\left(-\frac{\left|y - \bar{y}^{(q)}\right|^2}{2\sigma_n^2\mathbf{w}^H\mathbf{w}}\right) \tag{20}$$

and the BER can alternatively be calculated by

$$P_E(\mathbf{w}) = \frac{1}{2N_b} \sum_{q=1}^{N_b} \left(Q\left(g_R^{(q)}(\mathbf{w})\right) + Q\left(g_I^{(q)}(\mathbf{w})\right)\right) \tag{21}$$

where the computation is over all the $\bar{y}^{(q)} \in \mathcal{Y}$.

A. Block-Data Gradient Adaptive MBER Algorithm

Given a block of K training samples $\{\mathbf{x}(k), b_1(k)\}$, a Parzen window estimate of the p.d.f. (20) is given by:

$$\hat{p}(y) = \frac{1}{K 2\pi\rho_n^2\mathbf{w}^H\mathbf{w}} \sum_{k=1}^{K} \exp\left(-\frac{|y - y(k)|^2}{2\rho_n^2\mathbf{w}^H\mathbf{w}}\right) \tag{22}$$

where the kernel width ρ_n is related to the noise standard deviation σ_n. From this estimated p.d.f., the estimated BER is given by:

$$\hat{P}_E(\mathbf{w}) = \frac{1}{2K} \sum_{k=1}^{K} \left(Q\left(\hat{g}_R^{(k)}(\mathbf{w})\right) + Q\left(\hat{g}_I^{(k)}(\mathbf{w})\right)\right) \tag{23}$$

with

$$\hat{g}_R^{(k)}(\mathbf{w}) = \frac{\mathrm{sgn}(b_{R,1}(k))y_R(k)}{\rho_n\sqrt{\mathbf{w}^H\mathbf{w}}} \tag{24}$$

and

$$\hat{g}_I^{(k)}(\mathbf{w}) = \frac{\text{sgn}(b_{I,1}(k))y_I(k)}{\rho_n\sqrt{\mathbf{w}^H\mathbf{w}}}. \tag{25}$$

The gradient of $\hat{P}_E(\mathbf{w})$ can readily be calculated with

$$\nabla\hat{P}_{E_R}(\mathbf{w}) = \frac{1}{2K\sqrt{2\pi}\rho_n\sqrt{\mathbf{w}^H\mathbf{w}}}\sum_{k=1}^{K}\exp\left(-\frac{y_R^2(k)}{2\rho_n^2\mathbf{w}^H\mathbf{w}}\right)$$
$$\times\text{sgn}(b_{R,1}(k))\left(\frac{y_R(k)\mathbf{w}}{\mathbf{w}^H\mathbf{w}} - \mathbf{x}(k)\right) \tag{26}$$

and

$$\nabla\hat{P}_{E_I}(\mathbf{w}) = \frac{1}{2K\sqrt{2\pi}\rho_n\sqrt{\mathbf{w}^H\mathbf{w}}}\sum_{k=1}^{K}\exp\left(-\frac{y_I^2(k)}{2\rho_n^2\mathbf{w}^H\mathbf{w}}\right)$$
$$\times\text{sgn}(b_{I,1}(k))\left(\frac{y_I(k)\mathbf{w}}{\mathbf{w}^H\mathbf{w}} + j\mathbf{x}(k)\right). \tag{27}$$

By substituting $\nabla P_E(\mathbf{w})$ with $\nabla\hat{P}_E(\mathbf{w})$ in the conjugate gradient updating mechanism, a block-data adaptive MBER algorithm is readily obtained. The step size μ for the simplified conjugate gradient updating and the kernel width ρ_n are the two algorithm parameters.

B. Stochastic Gradient Adaptive MBER Algorithm

An alternative Parzen window estimate to the true density (20) is:

$$\tilde{p}(y) = \frac{1}{K2\pi\rho_n^2}\sum_{k=1}^{K}\exp\left(-\frac{|y-y(k)|^2}{2\rho_n^2}\right) \tag{28}$$

and a BER estimate is

$$\tilde{P}_E(\mathbf{w}) = \frac{1}{2K}\sum_{k=1}^{K}\left(Q\left(\tilde{g}_R^{(k)}(\mathbf{w})\right) + Q\left(\tilde{g}_I^{(k)}(\mathbf{w})\right)\right) \tag{29}$$

with

$$\tilde{g}_R^{(k)}(\mathbf{w}) = \frac{\text{sgn}(b_{R,1}(k))y_R(k)}{\rho_n} \tag{30}$$

and

$$\tilde{g}_I^{(k)}(\mathbf{w}) = \frac{\text{sgn}(b_{I,1}(k))y_I(k)}{\rho_n}. \tag{31}$$

This approximation is valid provided that the width ρ_n is chosen appropriately.

To derive a sample-by-sample adaptive algorithm, consider a single-sample estimate of $p(y)$, namely:

$$\tilde{p}(y,k) = \frac{1}{2\pi\rho_n^2}\exp\left(-\frac{|y-y(k)|^2}{2\rho_n^2}\right) \tag{32}$$

and the instantaneous BER "estimate" $\tilde{P}_E(\mathbf{w},k)$. Using the instantaneous stochastic gradient of:

$$\nabla\tilde{P}_E(\mathbf{w},k) = \frac{1}{4\sqrt{2\pi}\rho_n}\left(-\text{sgn}(b_{R,1}(k))\exp\left(-\frac{y_R^2(k)}{2\rho_n^2}\right)\right.$$
$$\left.+j\text{sgn}(b_{I,1}(k))\exp\left(-\frac{y_I^2(k)}{2\rho_n^2}\right)\right)\mathbf{x}(k) \tag{33}$$

gives rise to a stochastic gradient adaptive algorithm, which we referred to as the LBER algorithm:

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu\left(-\nabla\tilde{P}_E(\mathbf{w}(k),k)\right), \tag{34}$$

$$c_1 = \mathbf{w}^H(k+1)\mathbf{p}_1, \tag{35}$$

$$\mathbf{w}(k+1) = \frac{c_1}{|c_1|}\mathbf{w}(k+1), \tag{36}$$

where the adaptive gain μ and the kernel width ρ_n are the two algorithmic parameters that have to be set appropriately. A perfect estimation of the steering vector $\mathbf{s}_1$ is assumed here.

V. SIMULATION STUDY

The example consisted of four sources and a three-element antenna array. Fig. 1 shows the locations of the desired source and the interfering sources graphically. The simulated channel conditions were $A_i = 1 + j0$, $1 \le i \le 4$. Fig. 2 compares the BER performance of the MBER solution with that of the MMSE solution under three different conditions: **(a)** the desired user and all the three interfering sources had equal power, **(b)** the desired user and the interfering sources 2, 3 had equal power, but the interfering source 4 had 6 dB more power than the desired user, and **(c)** all the three interfering sources had 2 dB more power than the desired user. For this example, the MBER beamformer has significantly better performance than the MMSE beamformer. The results also demonstrate that the MBER beamforming is robust to the near-far effect.

Performance of the block-data gradient adaptive MBER algorithm was next studied. Fig. 3 illustrates the convergence rates of the algorithm given SNR= 17 dB, SIR_2 $=\text{SIR}_3 = 0$ dB, $\text{SIR}_4 = -6$ dB and the block size $K = 400$, and with the two different initial weight vectors. It can be seen that this block-data adaptive MBER algorithm converged rapidly. The effect of the block size K on the performance of this block-data adaptive MBER algorithm is investigated in Fig. 4, given the condition that the desired user and the interfering sources 2, 3 had an equal power, while the interfering source 4 had a 6 dB higher power than the desired user. Fig. 5 shows the learning curves of the LBER algorithm under the same conditions of Fig. 3, where DD denotes decision-directed adaptation with $\hat{b}_1(k)$ substituting $b_1(k)$. It can be seen that the

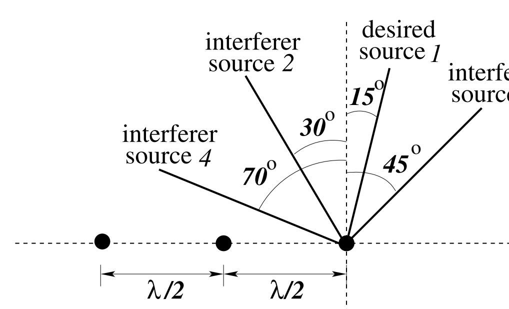

Fig. 1. Locations of the desired source and the interfering sources with respect to the three-element linear array with $\lambda/2$ element spacing, λ being the wavelength.

LBER algorithm has a reasonable convergence rate. Note that the convergence speed was slower when the initial weight vector was set to $\mathbf{w}_{\rm MMSE}$, compared with the other initial condition.

VI. Conclusions

An adaptive MBER beamforming technique has been proposed for wireless systems with QPSK signalling. It has been demonstrated that the MBER beamformer utilizes the system resource more intelligently than the standard MMSE beamformer and, consequently, achieves a better performance in terms of a smaller BER. The results also suggest that the MBER solution is robust to the near-far effect. Adaptive implementation of the MBER beamforming solution has been developed based on the classical approach of Parzen window estimate for the p.d.f. of the beamformer output. A block-data conjugate gradient adaptive MBER algorithm has been shown to converge rapidly and requires a reasonably small block size to accurately approximate the theoretical MBER solution. A stochastic gradient adaptive MBER algorithm called the LBER has been shown to perform well.

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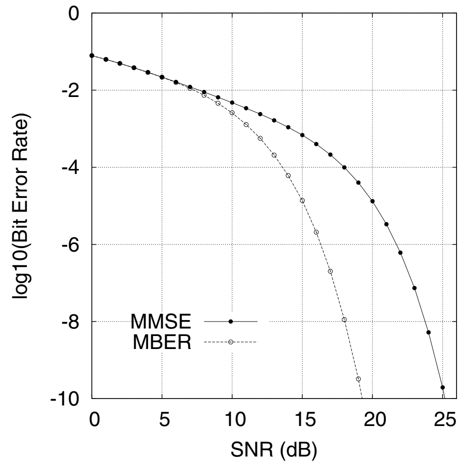
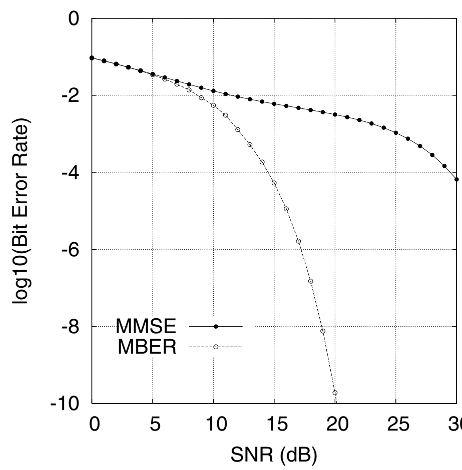
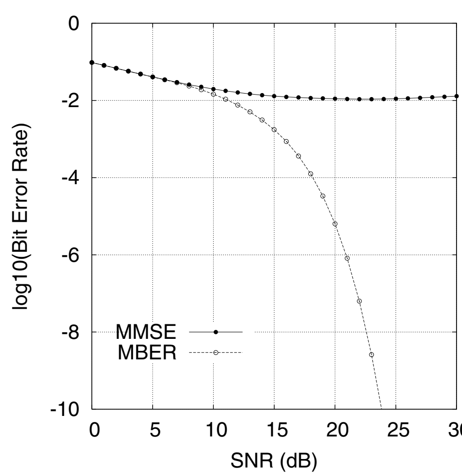


(a) $\text{SIR}_i = 0$ dB for $i = 2, 3, 4$.



(b) $\text{SIR}_2 = \text{SIR}_3 = 0$ dB and $\text{SIR}_4 = -6$ dB.



(c) $\text{SIR}_i = -2$ dB for $i = 2, 3, 4$.

Fig. 2. Comparison of bit error rate performance.

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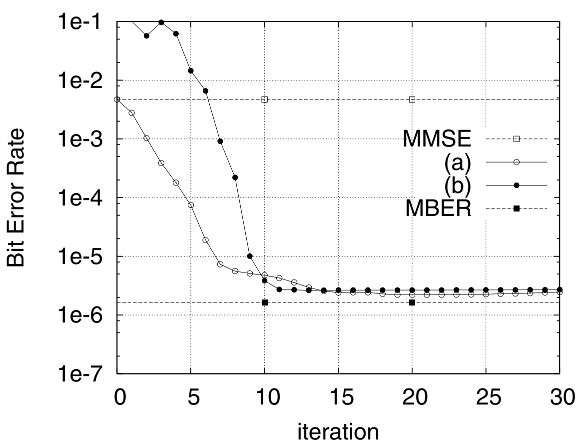


Fig. 3. Convergence rate of the block-data gradient adaptive MBER algorithm with $K = 400$, SNR= 17 dB, $\text{SIR}_2 = \text{SIR}_3 = 0$ dB and $\text{SIR}_4 = -6$ dB, and given (a): initial $\mathbf{w} = \mathbf{w}_{\text{MMSE}}$, $\mu = 0.3$ and $\rho_n^2 = 0.06$; and (b): initial $\mathbf{w} = [0.0 + j0.1\ 0.1 + j0.0\ 0.1 + j0.0]^T$, $\mu = 0.7$ and $\rho_n^2 = 0.06$.

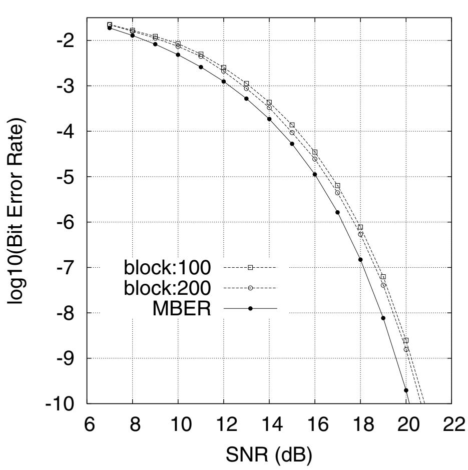
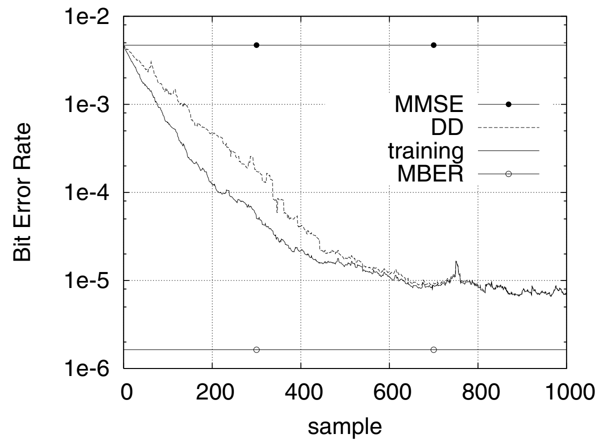
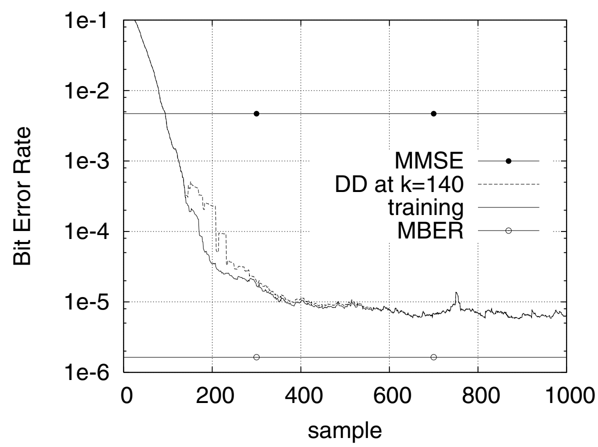


Fig. 4. Effect of block size on the performance of the block-data gradient adaptive MBER algorithm, given $\text{SIR}_2 = \text{SIR}_3 = 0$ dB and $\text{SIR}_4 = -6$ dB.



(a) $\mathbf{w}(0) = \mathbf{w}_{\text{MMSE}}$.



(b) $\mathbf{w}(0) = [0.0 + j0.1\ 0.1 + j0.0\ 0.1 + j0.0]^T$.

Fig. 5. Learning curves of the stochastic gradient adaptive MBER algorithm averaged over 100 runs, given SNR= 17 dB, $\text{SIR}_2 = \text{SIR}_3 = 0$ dB, $\text{SIR}_4 = -6$ dB, $\mu = 0.03$, and $\rho_n^2 = 0.06$, where DD: decision-directed adaptation with $\hat{b}_1(k)$ substituting $b_1(k)$.

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