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Particle Swarm Optimisation Aided Semi-blind Joint Maximum Likelihood Channel Estimation and Data Detection for MIMO Systems

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Abbreviations

- MIMO → multiple-input multiple-output
- ML → maximum likelihood
- OHRSA → optimised hierarchy reduced search algorithm
- PSO → particle swarm optimisation
- LSCE → least squares channel estimate
- MSE → mean square error
- MCE → mean channel error
- BER → bit error rate
- QPSK → quadrature phase shift keying
- RWBS → repeated weighted boosting search

MIMO System Model

- n_T-transmitters n_R-receivers MIMO model

$$\mathbf{y}(k) = \mathbf{H}\,\mathbf{s}(k) + \mathbf{n}(k)$$

  - ☞ QPSK data symbols vector $\mathbf{s}(k) = [s_1(k)\ s_2(k) \cdots s_{n_T}(k)]^T$ with $E\left[|s_m(k)|^2\right] = \sigma_s^2$
  - ☞ Complex-valued Gaussian white noise vector given by $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_{n_R}(k)]^T$ with $E\left[\mathbf{n}(k)\mathbf{n}^H(k)\right] = 2\sigma_n^2 \mathbf{I}_{n_R}$.
  - ☞ MIMO channel matrix $\mathbf{H} = [h_{p,m}]$, $1 \le p \le n_R$ and $1 \le m \le n_T$, with $h_{p,m}$ being channel coefficient linking mth transmitter to pth receiver

- Channel taps $h_{p,m}$ are independent of each other, complex-valued Gaussian distributed with $E\left[|h_{p,m}|^2\right] = 1$

- Signal-to-noise ratio (SNR) is defined by $\mathrm{E_b/N_o} = \sigma_s^2 / 2\sigma_n^2$

Joint ML Blind Scheme

- ❏ PDF of Rx data $\mathbf{Y}$ conditioned on channel $\mathbf{H}$ and Tx symbols $\mathbf{S}$:

$$p(\mathbf{Y}|\mathbf{H},\mathbf{S}) = \frac{1}{\left(2\pi\sigma_n^2\right)^{n_R\times L}} e^{-\frac{1}{2\sigma_n^2}\sum_{k=1}^{L}\|\mathbf{y}(k)-\mathbf{H}\mathbf{s}(k)\|^2}$$

  - ☞ Received $n_R \times L$ data matrix $\mathbf{Y} = [\mathbf{y}(1)\ \mathbf{y}(2)\cdots\mathbf{y}(L)]$
  - ☞ Transmitted $n_T \times L$ symbol matrix $\mathbf{S} = [\mathbf{s}(1)\ \mathbf{s}(2)\cdots\mathbf{s}(L)]$

- ❏ Joint ML channel and data estimation:

$$(\hat{\mathbf{S}},\hat{\mathbf{H}}) = \arg\left\{\min_{\check{\mathbf{S}},\check{\mathbf{H}}} J_{\text{ML}}(\check{\mathbf{S}},\check{\mathbf{H}})\right\}$$

with cost function

$$J_{\text{ML}}(\check{\mathbf{S}},\check{\mathbf{H}}) = \frac{1}{n_R\times L}\sum_{k=1}^{L}\left\|\mathbf{y}(k)-\check{\mathbf{H}}\ \check{\mathbf{s}}(k)\right\|^2$$

  - ☛ Computationally prohibitive

Iterative Decomposition

- *Outer-level Optimisation*:
  - A search algorithm, such as PSO, searches MIMO channel space
  - to find a global optimal channel estimate $\hat{\mathbf{H}}$ by minimising MSE

$$J_{\text{MSE}}(\check{\mathbf{H}}) = J_{\text{ML}}(\hat{\mathbf{S}}(\check{\mathbf{H}}), \check{\mathbf{H}})$$

  - $\hat{\mathbf{S}}(\check{\mathbf{H}})$ is ML data estimate given channel $\check{\mathbf{H}}$, provided by inner level

- *Inner-level Optimisation*:
  - Given channel $\check{\mathbf{H}}$ by outer level
  - OHRSA detector finds ML data estimate $\hat{\mathbf{S}}(\check{\mathbf{H}})$
  - and feeds back ML metric $J_{\text{MSE}}(\check{\mathbf{H}})$ to outer level

Semi Blind Scheme

- ❑ Use K training data to provide LSCE

$$\check{\mathbf{H}}_{\text{LSCE}} = \mathbf{Y}_K \mathbf{S}_K^H \left(\mathbf{S}_K \mathbf{S}_K^H\right)^{-1}$$

  for adding search algorithm at outer level

  - ☞ $\mathbf{Y}_K = [\mathbf{y}(1)\ \mathbf{y}(2) \cdots \mathbf{y}(K)]$
  - ☞ $\mathbf{S}_K = [\mathbf{s}(1)\ \mathbf{s}(2) \cdots \mathbf{s}(K)]$

- ❑ To maintain throughput,
  - ☞ use minimum number of training symbols, $K = n_T$
- ❑ Design $\mathbf{S}_K$ to have n_T orthogonal rows
  - ☞ most efficient estimate and no need for matrix inversion
- ❑ Semi blind method can resolve
  - ☞ estimation and decision ambiguities inherented in pure blind method

Particle Swarm Optimisation

- Solving optimisation problem

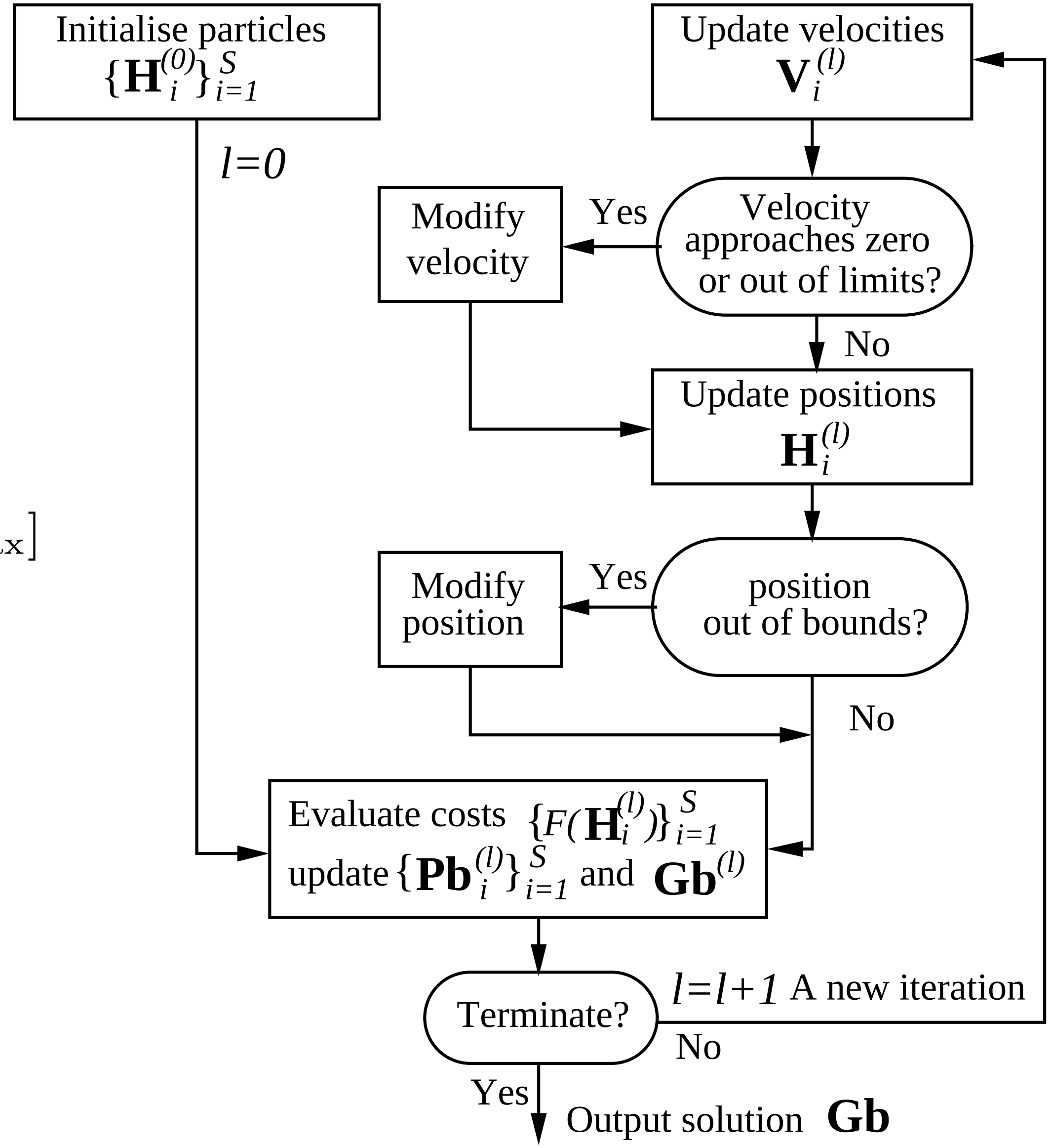
$$\hat{\mathbf{H}} = \arg \min_{\check{\mathbf{H}} \in \mathsf{P}^{n_R \times n_T}} F(\check{\mathbf{H}})$$

  - **Cost function** $F(\check{\mathbf{H}}) = J_{\text{MSE}}(\check{\mathbf{H}})$
  - **Search space** $\mathsf{P}^{n_R \times n_T}$ with

$$\mathsf{P} = [-P_{\max},\ P_{\max}] + j[-P_{\max},\ P_{\max}]$$

- A **swarm** of **particles**, $\{\mathbf{H}_i^{(l)}\}_{i=1}^{S}$, are evolved in search space
  - S is swarm size
  - Index l denotes iteration step
- Each particle has a velocity $\mathbf{V}_i^{(l)} \in \mathsf{V}^{n_R \times n_T}$ to direct its flying, where

$$\mathsf{V} = [-V_{\max},\ V_{\max}] + j[-V_{\max},\ V\max]$$



PSO Aided Scheme

- a) **Initialisation** Set iteration index $l = 0$,
  - ☞ $\mathbf{H}_1^{(l)} = \check{\mathbf{H}}_{\text{LSCE}}$
  - ☞ Randomly generate rest of particles:

$$\mathbf{H}_i^{(l)} = \check{\mathbf{H}}_{\text{LSCE}} + \eta(\mathbf{1}_{n_R \times n_T} + j\mathbf{1}_{n_R \times n_T}), \ 2 \le i \le S$$

    where η is a uniformly distributed random variable in $[-\alpha, \ \alpha]$

- b) **Evaluation** Particle $\mathbf{H}_i^{(l)}$ has cost $F(\mathbf{H}_i^{(l)})$
  - ☞ Each particle remembers its best position visited so far, which defines *cognitive information* $\mathbf{Pb}_i^{(l)}$
  - ☞ Every particle knows best position visited among entire swarm, which provides *social information* $\mathbf{Gb}^{(l)}$
  - ☞ Cognitive information $\mathbf{Pb}_i^{(l)}$, $1 \le i \le S$, and social information $\mathbf{Gb}^{(l)}$ are updated, given new costs $\{F(\mathbf{H}_i^{(l)})\}_{i=1}^{S}$

PSO Aided Scheme (continue)

- **c) Update** Velocities and positions are updated

$$\mathbf{V}_i^{(l+1)} = w_{\mathrm{I}} * \mathbf{V}_i^{(l)} + rand() * c_1 * (\mathbf{Pb}_i^{(l)} - \mathbf{H}_i^{(l)}) + rand() * c_2 * (\mathbf{Gb}^{(l)} - \mathbf{H}_i^{(l)})$$

$$\mathbf{H}_i^{(l+1)} = \mathbf{H}_i^{(l)} + \mathbf{V}_i^{(l+1)}$$

  - ☞ If $\mathbf{V}_i^{(l+1)} \rightarrow$ zero, it is reinitialised randomly within $\mathsf{V}^{n_R \times n_T}$
  - ☞ If $\mathbf{V}_i^{(l+1)}$ is outside $\mathsf{V}^{n_R \times n_T}$, it is moved back inside velocity space
  - ☞ If $\mathbf{H}_i^{(l+1)}$ is outside search space, it is moved back inside $\mathsf{P}^{n_R \times n_T}$

- **d) Termination** If maximum number of iterations, $I_{\max}$, is reached, terminate with solution $\mathbf{Gb}^{(I_{\max})}$; otherwise, $l = l + 1$ and go to **b)**

Complexity for block length L

$$C = N_{\mathrm{OHRSA}} \times C_{\mathrm{OHRSA}}(L) = S \times I_{\max} \times C_{\mathrm{OHRSA}}(L)$$

$C_{\mathrm{OHRSA}}(L)$: OHRSA complexity, and N_{OHRSA}: number of OHRSA evaluations

PSO Algorithmic Parameters

- Inertial weight $w_\mathrm{I} = 0$, other alternative is $w_\mathrm{I} = rand()$ or w_I set to a small positive constant
- Empirical time varying acceleration coefficients

$$\begin{aligned} c_1 &= (2.5 - 0.5) * l/I_{\max} + 0.5 \\ c_2 &= (0.5 - 2.5) * l/I_{\max} + 2.5 \end{aligned}$$

- Search limit $P_{\max} = 1.8$, which lies between 2 to 3 standard deviations of true channel tap distribution
- Empirical velocity limit $V_{\max} = 1.0$
- Empirical control parameter $\alpha = 0.15$ in channel population initialisation
- $S = 20$ is appropriate with $I_{\max} = 50$ sufficient

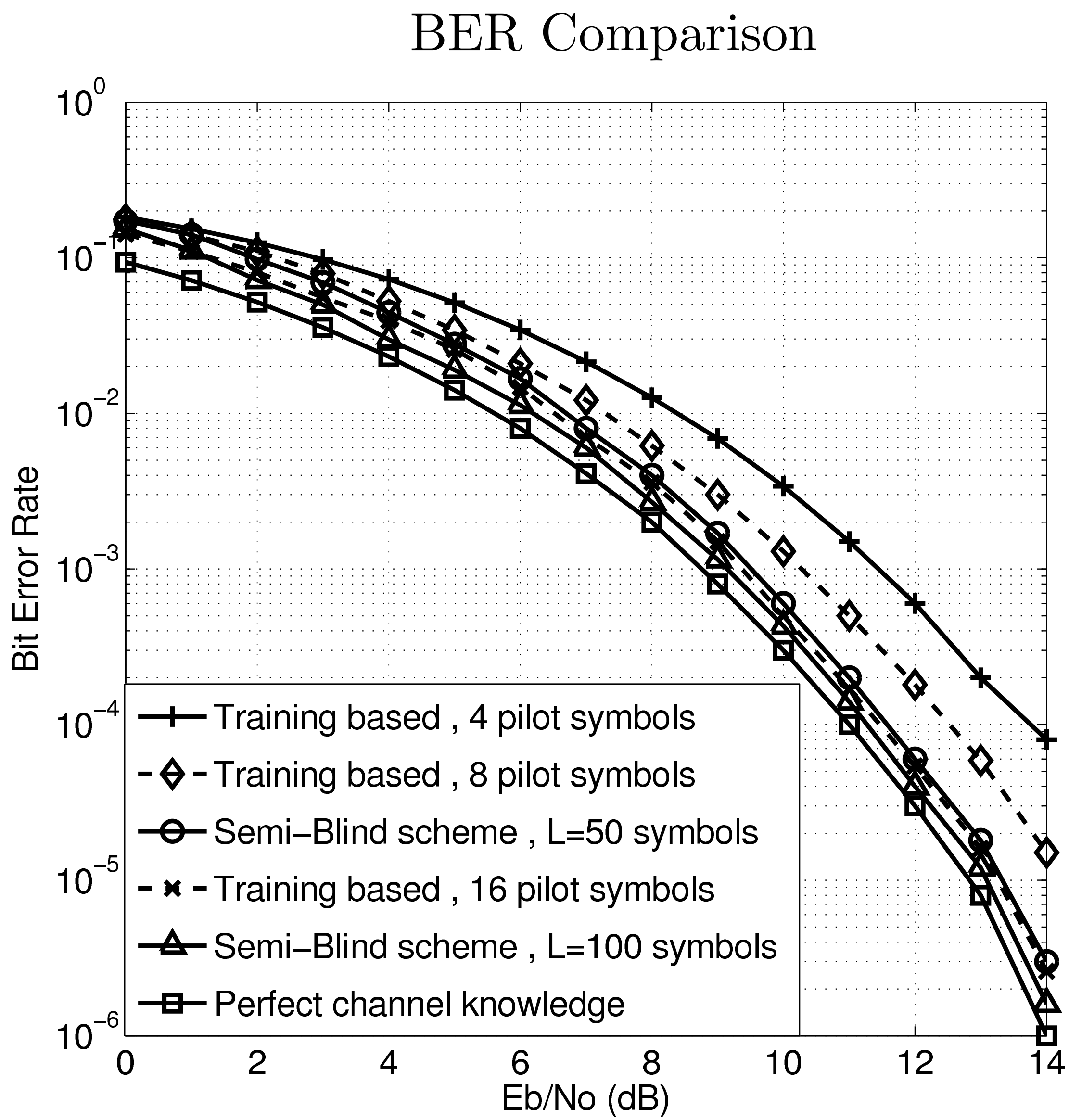
Simulation Example

- QPSK MIMO: $n_T = 4$ and $n_R = 4$
- $S = 20$ and $I_{\max} = 50$:

$$N_{\text{OHRSA}} = 1000$$

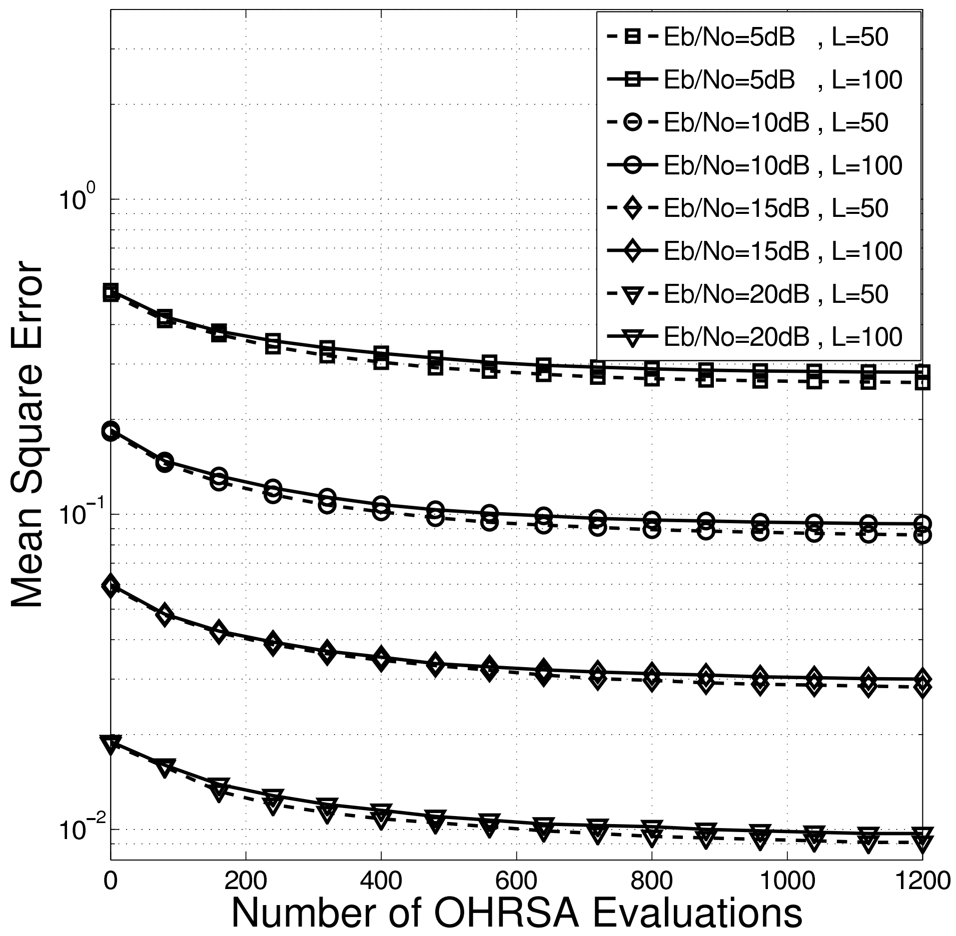
- Results averaged over 50 channel realisations
- Performance metrics:
  - BER
  - MSE $J_{\text{MSE}}(\check{\mathbf{H}})$
  - Mean channel error (MCE)

$$J_{\text{MCE}}(\check{\mathbf{H}}) = \|\mathbf{H} - \check{\mathbf{H}}\|^2$$

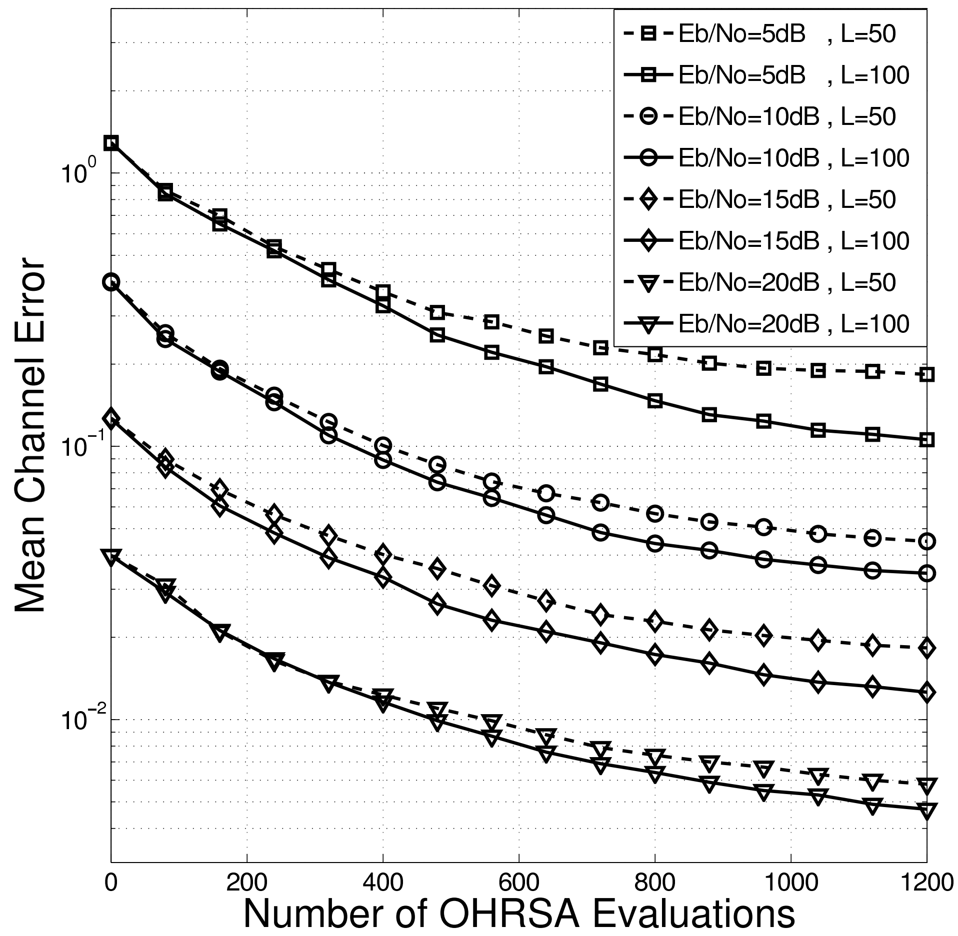


Convergence Performance

MSE



MCE



Comparison

- Previous RWBS based semi-blind scheme

  M. Abuthinien, S. Chen and L. Hanzo, "Semi-blind joint maximum likelihood channel estimation and data detection for MIMO systems," *IEEE Signal Processing Letters*, vol.15, pp.202–205, 2008.

  - Convergence in $N_{\text{OHRSA}} = 1200$

- Proposed PSO based semi-blind scheme

  - Convergence in $N_{\text{OHRSA}} = 1000$
  - Performance is slightly better

- Proposed PSO-based semi-blind method achieved 20% saving in computation, compared with RWBS based semi-blind scheme

Summary

- Novel semi-blind joint ML channel estimation and data detection has been proposed for MIMO
  - PSO algorithm is invoked at upper level to identify unknown MIMO channel
  - Enhanced ML sphere detector, OHRSA, is used at lower level for ML data detection
  - Minimum pilot overhead is employed to aid initialisation of PSO-based channel estimator
- Compared with existing state-of-the-art, PSO-aided semi-blind scheme imposes significantly lower complexity in attaining joint ML solution