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# **Determination of Dynamic Flexure Model Parameters for Ship Angular Deformation Measurement**

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# Background

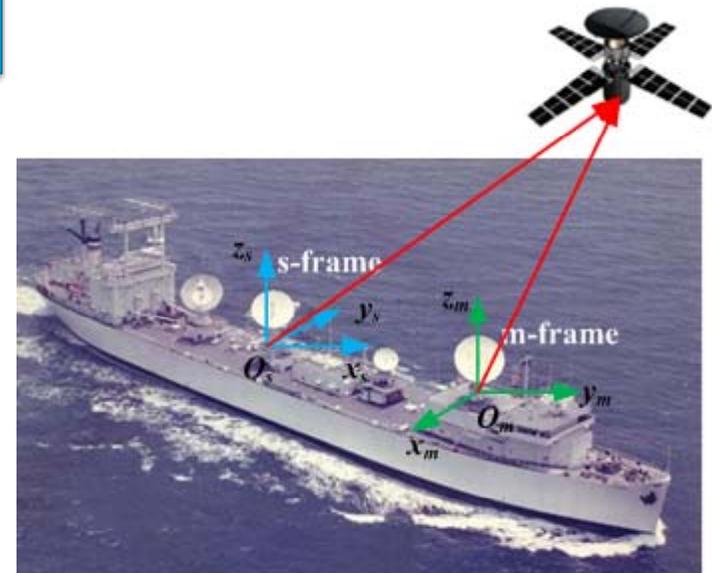
## ● Ship Angular Deformation

Ship angular deformation refers to the two frames angle displacement

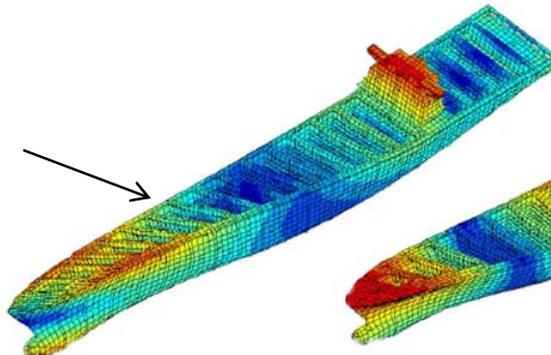
**Pitching:** cross  $x$ -axis

**Rolling:** cross  $y$ -axis

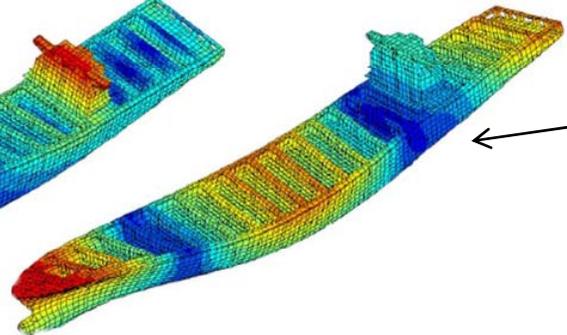
**Yawing:** cross  $z$ -axis



rolling  
deformation



pitching  
deformation



# Background

## ● Measurement Approach

Ship deformation:

$$\varphi(t) = \varphi_0 + \theta(t)$$

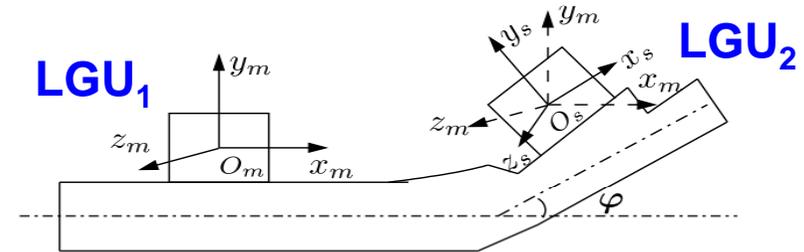
where  $\varphi_0$  is time-invariant component,

$\theta(t)$  is dynamic component, which is usually modeled as a

**second-order Gauss-Markov process**, the correlation function is

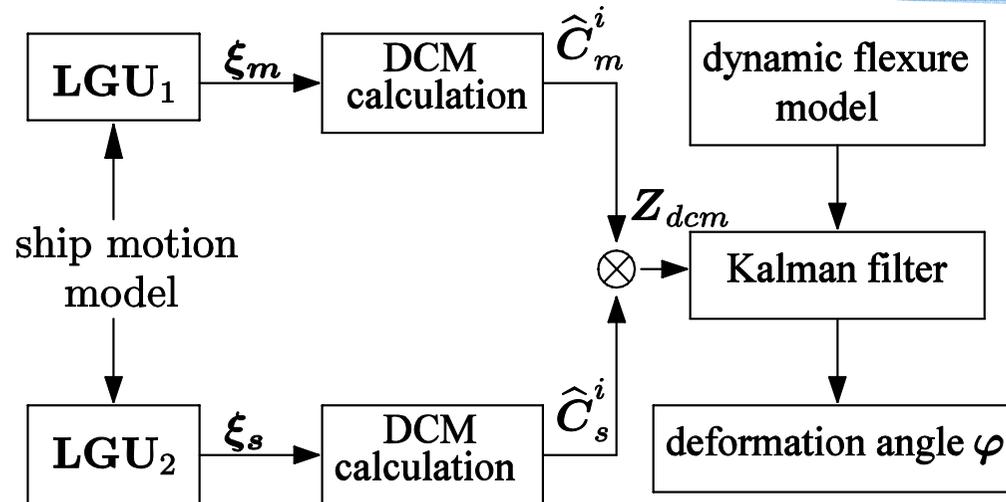
$$R_{\theta_i}(\tau) = \sigma_i^2 \exp(-\alpha_i |\tau|) \left( \cos \beta_i \tau + \frac{\alpha_i}{\beta_i} \sin \beta_i |\tau| \right), i = x, y, z$$

in which  $\sigma^2$  is the variance,  $\alpha$  is the damping factor,  $\beta$  is the circular frequency.



Measurement system

# Background



Schematic diagram of ship deformation measurement system

## Kalman Filter

**Measurement function:**  $Z_{dcm} = B\theta - A\phi_0 + B\left(\hat{C}_i^m \psi_m - \hat{C}_i^s \psi_s\right)$

**State function:**  $\dot{X} = FX + w$

**State vector:**  $X = [\phi_0^T \ \theta^T \ \dot{\theta}^T \ \psi_m^T \ \psi_s^T \ \tilde{\varepsilon}_m^T \ \tilde{\varepsilon}_s^T]^T$

# Background

Specifically, the measurement vector is given by

$$\mathbf{Z}_{dcm} = \begin{bmatrix} C_{13}C'_{12} + C_{23}C'_{22} + C_{33}C'_{32} \\ C_{13}C'_{11} + C_{23}C'_{21} + C_{33}C'_{31} \\ C_{11}C'_{12} + C_{21}C'_{22} + C_{31}C'_{32} \end{bmatrix},$$

and the matrices  $\mathbf{A}$  and  $\mathbf{B}$  are given by

$$\mathbf{A} = \begin{bmatrix} C_{33}C'_{22} - C_{23}C'_{32} & C_{13}C'_{32} - C_{33}C'_{12} \\ C_{33}C'_{21} - C_{23}C'_{31} & C_{13}C'_{31} - C_{33}C'_{11} \\ C_{31}C'_{22} - C_{21}C'_{32} & C_{11}C'_{32} - C_{31}C'_{12} \\ C_{23}C'_{12} - C_{13}C'_{22} \\ C_{23}C'_{11} - C_{13}C'_{21} \\ C_{21}C'_{12} - C_{11}C'_{22} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

where  $C_{ij}$  and  $C'_{ij}$  are the components of DCMs of  $\text{LGU}_1$  and  $\text{LGU}_2$ , respectively.

# Background

The state transition matrix is given by

$$F = \begin{bmatrix} O_{3 \times 3} & & \\ & F_{6 \times 6}^1 & \\ & & F_{12 \times 12}^2 \end{bmatrix}$$

in which

$$F_{6 \times 6}^1 = \begin{bmatrix} O_{3 \times 3} & & & & & \\ -b_x^2 & 0 & 0 & -2\mu_x & 0 & 0 \\ 0 & -b_y^2 & 0 & 0 & -2\mu_y & 0 \\ 0 & 0 & -b_z^2 & 0 & 0 & -2\mu_z \end{bmatrix}$$

and

$$F_{12 \times 12}^2 = \begin{bmatrix} O_{3 \times 6} & -\hat{C}_m^i & O_{3 \times 3} \\ O_{3 \times 6} & O_{3 \times 3} & -\hat{C}_s^i \\ O_{6 \times 12} & & \end{bmatrix},$$

The state noise covariance is

$$E[ww^T] = \text{diag} \left\{ O_{1 \times 3}, 4b_x^2 \sigma_x^2 \alpha_x, 4b_y^2 \sigma_y^2 \alpha_y, 4b_z^2 \sigma_z^2 \alpha_z, O_{1 \times 9}, (\sigma_{mr}^2)^T, (\sigma_{sr}^2)^T \right\},$$

# Background

- **Determine the Dynamic Flexure Model Parameters**

- **Empirical method**

**The parameters are determined according to experience**

- **Statistical method**

**The parameters are obtained from previously recorded measurement data**

**In actual condition, the parameters depend on sea condition, ship velocity and ship structure, etc. It requires to estimate the parameters on-line.**

# Background

## ● Our Novelty

The dynamic flexure information is existing in attitude difference measured by LGU<sub>1</sub> and LGU<sub>2</sub>.

$$Z_{dcm} = B\theta - A\phi_0 + B\left(\hat{C}_i^m\psi_m - \hat{C}_i^s\psi_s\right)$$

Assume the dynamic flexure can be depicted as a **second-order Gauss-Markov process**, we developed an on-line dynamic flexure parameters estimation method by utilising **the attitude difference** measured by two LGUs, and **Tufts-Kumaresan (T-K) method** was applied to obtain a robust and accuracy estimates.

# Parameters Estimation Approach

The attitude matching function can be written as

$$Z_{dcm} = B\theta + (B - A)\phi_0 + B(\hat{C}_i^m \psi_m - \hat{C}_i^s \psi_s)$$



$$\tilde{Z}_{dcm} \approx B\theta$$

**Remove the second term  $(B - A)\phi_0$  :** for  $(B-A)$  is a small, and  $\phi_0$  can be compensated to several *mrads* using the course estimate results, so the multiply results are small and can be removed.

**Remove the third term  $B(\hat{C}_i^m \psi_m - \hat{C}_i^s \psi_s)$  :** for the frequency of attitude error caused by gyro bias and random walk noise is far less than  $\theta$ , this term can be removed through a high-pass filter.

# Parameters Estimation Approach

The correlation function of  $\tilde{Z}_{dcm}$  is given by

$$R_Z(\tau) = \langle \tilde{Z}_{dcm}(t), \tilde{Z}_{dcm}(t + \tau) \rangle = \langle \theta(t), \theta(t + \tau) \rangle$$

Recall that the correlation function of dynamic flexure  $\theta(t)$ , based on **the second-order Gauss-Markov process** assumption is

$$R_\theta(\tau) = \sigma^2 \exp(-\alpha|\tau|) \left( \cos \beta\tau + \frac{\alpha}{\beta} \sin \beta|\tau| \right)$$

Therefore, the parameters  **$\sigma^2$ ,  $\alpha$  and  $\beta$**  can be obtained from  **$R_Z(\tau)$** .

# Parameters Estimation Approach

## • T-K Method

The T-K methods is widely applied in estimation of parameters for closely spaced sinusoidal signals in noise

$$y(n) = \sum_{l=1}^M a_l \exp\left[(-\alpha_l + j\beta_l)n\right] + q(n), n = 1, 2, \dots, N$$

where  $M$  is the number of sinusoidal signals,  $N$  is the sample length,  $a_l$  is the amplitude,  $\alpha_l$  is the damping factor and  $\beta_l$  is circular frequency.

The parameters  $\alpha_l$  and  $\beta_l$  can be resolved by using **T-K method**. Then, substitute the estimate results  $\alpha_l$  and  $\beta_l$  to above equation, and the magnitude  $a_l$  can be resolved by using the **least square methods**.

# Parameters Estimation Approach

## ● Parameters Estimation Procedure

### ● Initialization

- Calculate the DCMs of  $\hat{C}_m^i$  and  $\hat{C}_s^i$ , derive  $Z_{dcm}$
- Compensate the  $\hat{\phi}_0$  using course estimation results
- Remove gyro errors through high-pass filter

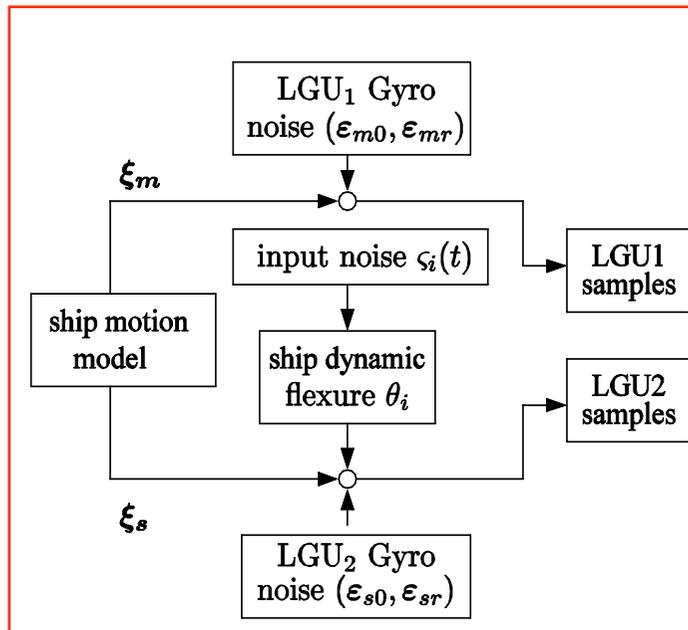
### ● T-K Based Parameters Estimation

- Calculate the correlation function  $R_Z(\tau)$  of  $Z_{dcm}$
- Construct the T-K prediction function and evaluate the **frequency  $\beta/2\pi$**  and **damping factor  $\alpha$**
- Calculate the **variance  $\sigma^2$**  using the least square algorithm

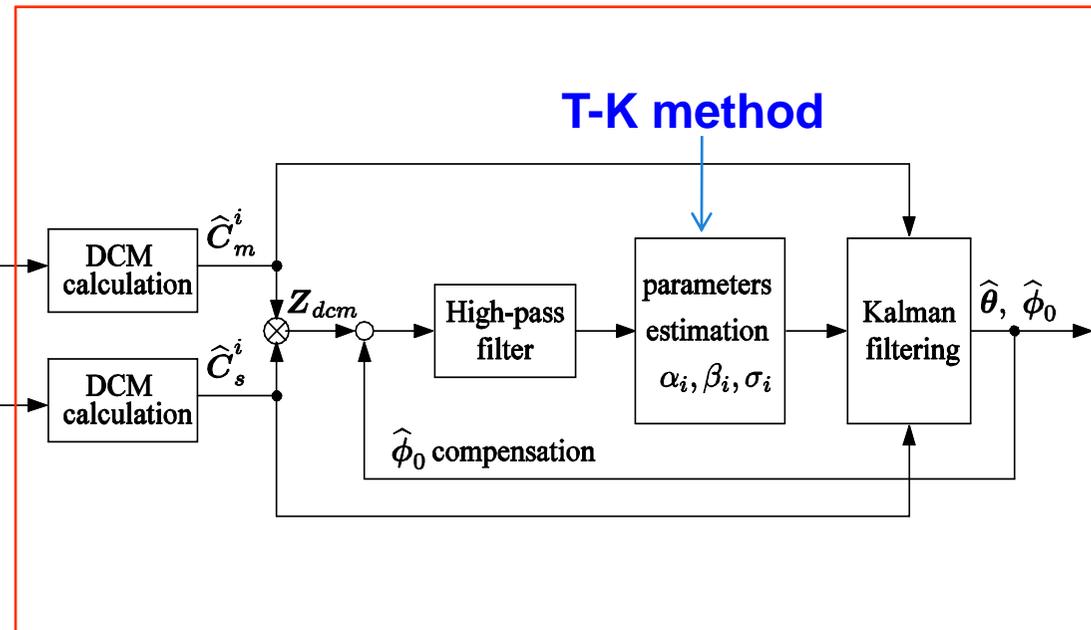
### ● KF Based Angular Deformation Measurement

# Simulation System

## Gyro samples generation



## Parameters estimation and KF based deformation measurement



Schematic diagram of gyro samples generation and dynamic flexure parameters estimation

# Simulation System

The ship attitude can also be modeled as a second-order Gauss-Markov process, whose correlation function takes the form

$$R_{\xi_i}(\tau) = \sigma_{\xi_i}^2 \exp(-\alpha_{\xi_i} |\tau|) \left( \cos \beta_{\xi_i} \tau + \frac{\alpha_{\xi_i}}{\beta_{\xi_i}} \sin \beta_{\xi_i} |\tau| \right)$$

## Ship attitude parameters

	Magnitude $\sigma_{\xi_i}$ (deg)	Frequency $\beta_{\xi_i} / 2\pi$ (Hz)	Damping factor $\alpha_{\xi_i}$ (s <sup>-1</sup> )
Pitch	2.2	0.18	0.10
Roll	3.4	0.07	0.06
Yaw	0.8	0.05	0.12

Set according  
to experience

Identified from  
experiment data

# Simulation System

## True dynamic flexure parameters

Set according  
to experience

	Magnitude $\sigma_i$ (mrad)	Frequency $\beta_i / 2\pi$ (Hz)	Damping factor $\alpha_i$ (s <sup>-1</sup> )
Pitch	0.40	0.19	0.13
Roll	0.68	0.17	0.11
Yaw	0.50	0.18	0.10

Identified from  
experiment data

In order to reflect actual measurement environment, we add Gaussian white noise with variance  $\sigma_{\zeta_i}^2$  in dynamic flexure signal. The SNR is defined by

$$SNR_i = 10 \log_{10} \frac{\sigma_i^2}{\sigma_{\zeta_i}^2}$$

# Results and Analysis

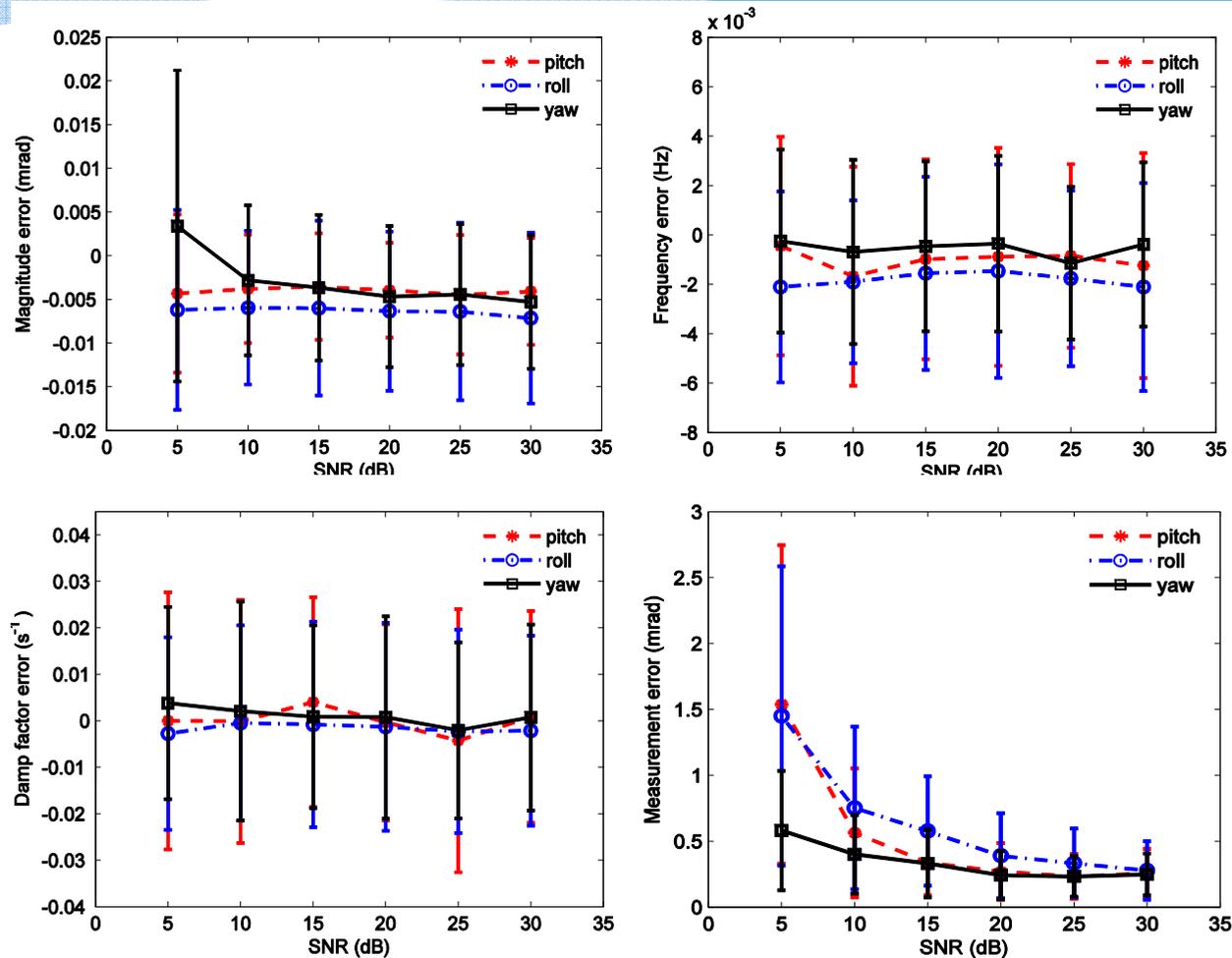
MEANS AND STANDARD DEVIATIONS OF THE ESTIMATED DYNAMIC FLEXURE PARAMETERS OBTAINED UNDER THE CONDITION OF  $\sigma_{\zeta_i}^2 = 0$ , GIVEN  $T = 600$  s,  $N = 20$  s,  $L = 6$  s AND  $M = 2$ .

	Magnitude $\sigma_i$ (mrad)	Frequency $\beta_i/2\pi$ (Hz)	Damping factor $\alpha_i$ ( $s^{-1}$ )
Pitch	0.3950 (0.0064)	0.1892 (0.0041)	0.1272 (0.0252)
Roll	0.6722 (0.0089)	0.1685 (0.0039)	0.1054 (0.0188)
Yaw	0.4961 (0.0075)	0.1798 (0.0034)	0.1013 (0.0183)

PERFORMANCE OF THE KALMAN FILTER BASED SHIP ANGULAR DEFORMATION MEASUREMENT OBTAINED BASED ON THE DYNAMIC FLEXURE MODEL IDENTIFIED UNDER THE CONDITION OF  $\sigma_{\zeta_i}^2 = 0$ .

	Mean and standard deviation of true deformation angle (mrad)	Mean and standard deviation of KF estimated deformation angle (mrad)	Mean and standard deviation of KF based measurement error (mrad)
Pitch	3.4981 (0.4267)	3.5179 (0.5525)	0.2626 (0.2005)
Roll	3.4792 (0.7209)	3.5131 (0.7776)	0.3259 (0.2369)
Yaw	3.5027 (0.4840)	3.5483 (0.5626)	0.1944 (0.1382)

# Results and Analysis



Mean estimate errors for dynamic flexure parameters magnitude  $\sigma^2$ , frequency  $\beta/2\pi$  and damping factor  $\alpha$  as well as measurement error with different SNR

# Conclusions

- we have developed an on-line dynamic flexure parameters estimation approach based on T-K method for KF based ship angular deformation measurement
- Compared with previous methods, the proposed method offers:
  - on-line estimation (not require *a priori* knowledge)
  - accurate estimation
  - robust to noise and work conditions



**Thank You For Your  
Attention**