Dervivation of the two equations (3.2) inside the box in page 26

Consider the first one

$$E = \frac{1}{2}\mu\dot{\mathbf{r}}^2 + V(r).$$

Recall that we are assuming to be in the CM of the two-body system ($\mathbf{R} = 0$). Hence, in the equation in page 26

$$E = \frac{1}{2}M\dot{\mathbf{R}}^2 + \frac{1}{2}\mu\dot{\mathbf{r}}^2 + V(r) = \frac{1}{2}\mu\dot{\mathbf{r}}^2 + V(r),$$

as the first term on the RHS is zero. But one also has that

$$E = T + V(r) = \frac{1}{2}M\dot{\mathbf{R}}^2 + T_{\text{CM}} + V(r) = T_{\text{CM}} + V(r),$$

so that what we have to prove is

$$T_{\text{CM}} = \frac{1}{2}\mu\dot{\mathbf{r}}^2.$$

This has already been done: see Sect. 1.1.2, pages 3 and 4. There, take ($\dot{\mathbf{R}} = 0$)

$$\dot{\mathbf{r}}_1 = \dot{\mathbf{R}} + \dot{\boldsymbol{\rho}}_1 = \dot{\boldsymbol{\rho}}_1 \qquad \text{and} \qquad \dot{\mathbf{r}}_2 = \dot{\mathbf{R}} + \dot{\boldsymbol{\rho}}_2 = \dot{\boldsymbol{\rho}}_2.$$

Subtracting these two equations gives $\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2 = \dot{\boldsymbol{\rho}}_1 - \dot{\boldsymbol{\rho}}_2$ (this results is actually independent of $\dot{\mathbf{R}}$ being zero), while the usual CM condition states that

$$m_1\dot{\boldsymbol{\rho}}_1 + m_2\dot{\boldsymbol{\rho}}_2 = 0.$$

We can thus solve for $\dot{\boldsymbol{\rho}}_1$ and $\dot{\boldsymbol{\rho}}_2$:

$$\dot{\boldsymbol{\rho}}_1 = \frac{m_2(\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)}{m_1 + m_2}, \qquad \dot{\boldsymbol{\rho}}_2 = \frac{-m_1(\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)}{m_1 + m_2}.$$

Substituting these in the kinetic energy expression in the center of mass, i.e.,

$$T_{\text{CM}} = \frac{1}{2}m_1\dot{\boldsymbol{\rho}}_1^2 + \frac{1}{2}m_2\dot{\boldsymbol{\rho}}_2^2,$$

gives

$$T = \frac{1}{2}\frac{m_1 m_2}{m_1+m_2}(\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2)^2 = \frac{1}{2}\mu\dot{\mathbf{r}}^2,$$

from the definition of μ and $\mathbf{r}$.

To prove

$$\mathbf{L} = \mu \mathbf{r} \times \dot{\mathbf{r}}$$

is a bit more complicated. But we should certainly try. Comparing eq. (1.6) with the expression just before box (3.2), it is clear that

$$\mathbf{L} = \mathbf{L}_{\rm CM}$$

for $\mathbf{R} = 0$ (and $\dot{\mathbf{R}} = 0$). Hence, let us start from the definition

$$\mathbf{L}_{\rm CM} = \boldsymbol{\rho}_1 \times m_1 \dot{\boldsymbol{\rho}}_1 + \boldsymbol{\rho}_2 \times m_2 \dot{\boldsymbol{\rho}}_2$$

for a two-body system. Now, let us play a trick. Again, recalling that $m_1\dot{\boldsymbol{\rho}}_1 + m_2\dot{\boldsymbol{\rho}}_2 = 0$, let us replace in the above expression, once $-m_2\dot{\boldsymbol{\rho}}_2$ for $m_1\dot{\boldsymbol{\rho}}_1$ and then vice versa, to obtain the twin expressions

$$\begin{aligned} \mathbf{L}_{\rm CM} &= -\boldsymbol{\rho}_2 \times m_2 \dot{\boldsymbol{\rho}}_1 + \boldsymbol{\rho}_2 \times m_2 \dot{\boldsymbol{\rho}}_2 \\ \mathbf{L}_{\rm CM} &= \boldsymbol{\rho}_1 \times m_1 \dot{\boldsymbol{\rho}}_1 - \boldsymbol{\rho}_1 \times m_1 \dot{\boldsymbol{\rho}}_2, \end{aligned}$$

which I can also rewrite as

$$\begin{aligned} \mathbf{L}_{\rm CM} &= -m_2\boldsymbol{\rho}_2 \times (\dot{\boldsymbol{\rho}}_1 - \dot{\boldsymbol{\rho}}_2), \\ \mathbf{L}_{\rm CM} &= m_1\boldsymbol{\rho}_1 \times (\dot{\boldsymbol{\rho}}_1 - \dot{\boldsymbol{\rho}}_2), \end{aligned}$$

respectively. Now, sum the two to obtain

$$2\mathbf{L}_{\rm CM} = (m_1\boldsymbol{\rho}_1 - m_2\boldsymbol{\rho}_2) \times (\dot{\boldsymbol{\rho}}_1 - \dot{\boldsymbol{\rho}}_2).$$

Clearly,

$$\dot{\boldsymbol{\rho}}_1 - \dot{\boldsymbol{\rho}}_2 = \dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2$$

(this would also be true for $\dot{\mathbf{R}} \neq 0$). The assumption $\dot{\mathbf{R}} = 0$ simplifies though the way we can deal with

$$m_1\boldsymbol{\rho}_1 - m_2\boldsymbol{\rho}_2,$$

as this can be rewritten as

$$m_1\mathbf{r}_1 - m_2\mathbf{r}_2 = \frac{m_1 m_2}{m_1 + m_2}(\mathbf{r}_1 - \mathbf{r}_2) + \frac{m_1^2\mathbf{r}_1 - m_2^2\mathbf{r}_2}{m_1 + m_2},$$

having multiplied and divided by $m_1 + m_2$. For $\mathbf{R} = 0$, one has

$$m_1\mathbf{r}_1 + m_2\mathbf{r}_2 = 0,$$

that is,

$$m_1\mathbf{r}_1 = -m_2\mathbf{r}_2.$$

Now, multiply both sides once by m_1 and a second time by m_2, to obtain the twin expressions

$$\begin{aligned} m_1^2\mathbf{r}_1 &= -m_1 m_2 \mathbf{r}_2, \\ m_2^2\mathbf{r}_2 &= -m_1 m_2 \mathbf{r}_1. \end{aligned}$$

By subtraction, one gets

$$m_1^2\mathbf{r}_1 - m_2^2\mathbf{r}_2 = m_1 m_2 (\mathbf{r}_1 - \mathbf{r}_2).$$

Therefore,

$$m_1\mathbf{r}_1 - m_2\mathbf{r}_2 = 2\frac{m_1 m_2}{m_1 + m_2}(\mathbf{r}_1 - \mathbf{r}_2)$$

and

$$2\mathbf{L}_{\rm CM} = 2\frac{m_1 m_2}{m_1 + m_2}(\mathbf{r}_1 - \mathbf{r}_2) \times (\dot{\mathbf{r}}_1 - \dot{\mathbf{r}}_2),$$

which means

$$\mathbf{L}_{\rm CM} = \mu \mathbf{r} \times \dot{\mathbf{r}}.$$

After this, I agree that one would deserve a beer !