UNIVERSITY OF SOUTHAMPTON PHYS2006W1

SEMESTER 2 EXAMINATION 2005/2006

CLASSICAL MECHANICS

Duration: 120 MINS

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*Answer **all** questions in **Section A** and **two and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

Only non-preprogrammed calculators may be used.

Section A

A1. In a system of N particles, the force acting on the ith particle is written as

$$\mathbf{F}_i = \mathbf{F}_i^{\text{ext}} + \sum_{j \neq i} \mathbf{F}_{ij},$$

where $\mathbf{F}_i^{\text{ext}}$ is the external force on the ith particle and $\mathbf{F}_{ij}$ is the force of the jth particle on the ith one.

The total linear momentum of the system and the total force acting on it are:

$$\mathbf{P} = \sum_{i=1}^{N} \mathbf{p}_i \quad \text{and} \quad \mathbf{F} = \sum_{i=1}^{N} \mathbf{F}_i.$$

Show that

$$\mathbf{F}^{\text{ext}} = \dot{\mathbf{P}},$$

where

$$\mathbf{F}^{\text{ext}} = \sum_{i=1}^{N} \mathbf{F}_i^{\text{ext}}.$$

Which law or laws of motion did you have to invoke to obtain the final result ? [4]

A2. A rocket of mass m_i is moving at velocity v_i in deep space (i.e., completely isolated). After burning all its remaining fuel, what is the total momentum of the rocket plus that of all exhaust fumes ?

If

$$v = v_i + u \ln(m_i/m)$$

is the velocity of the rocket when its mass is m, for what value of m_i/m is the rocket momentum maximised ? (Assume that u, the modulus of the velocity at which the gases are expelled relative to the rocket, is constant.) [4]

A3. Explain why spinning skaters rotate faster as they draw their arms in. [4]

A4. Find an expression relating the acceleration g due to gravity on the Earth's surface to Newton's gravitational constant G and the mass and radius of the Earth (assumed spherically symmetric and with constant density). Define all quantities you introduce. [4]

A5. Assume that Foucault's pendulum motion is described by the equation:

$$\alpha = ae^{-i(\omega \sin\lambda)t} \cos(\omega_0 t),$$

where $\alpha = x + iy$, x and y being the displacements of the bob on the Earth's surface. Describe each of the three terms appearing in the above formula and define each quantity therein (except the time t). [4]

Section B

B1. (a) Define the moment of inertia and radius of gyration of a solid body about a fixed axis. [6]

(b) Show that the moment of inertia of a solid sphere of constant density, with radius a and mass m, is $\frac{2}{5}ma^2$. [6]

(c) If the solid sphere rolls without slipping down a plane inclined at an angle θ to the horizontal, show that the linear acceleration of its centre of mass is [8]

$$\frac{5}{7}g\sin\theta.$$

B2. (a) A planet of mass m orbits a sun of mass M subject only to the gravitational attraction of the sun. Explain why the angular momentum $\mathbf{L}$ is conserved, and why this means that the orbit lies in a plane. [4]

(b) Show that the planet's total energy is E can be written in the form

$$\frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} - \frac{GMm}{r} = E,$$

where G is the universal constant of gravitation and r is the distance from the sun to the planet. [4]

(c) Sketch a graph of the *effective potential*,

$$U(r) = \frac{L^2}{2mr^2} - \frac{GMm}{r},$$

and use your graph to illustrate how different kinds of orbital motion are possible, depending on the value of E. [6]

(d) What minimum total energy, E_{min} can the motion have for this value of L. What is the shape of the corresponding orbit ? [2]

(e) If $E_{\text{min}} < E < 0$, show that the motion is restricted to have $r_{\text{min}} \leq r \leq r_{\text{max}}$, where,

$$r_{\text{min}} + r_{\text{max}} = -\frac{GMm}{E}.$$

[Ignore any complications due to reduced mass in this question: work in the approximation that the sun remains stationary.] [4]

B3. The equation of motion of a particle of mass m moving near the surface of the Earth, acted on by gravity and an additional force $\mathbf{F}$ is

$$\ddot{\mathbf{r}} = \frac{\mathbf{F}}{m} - g\frac{\mathbf{r}}{r} - 2\boldsymbol{\omega}\times\dot{\mathbf{r}} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}),$$

where $\mathbf{r}$ is the particle's position vector measured from the centre of the Earth in a frame rotating with the Earth at angular velocity $\boldsymbol{\omega}$.

(a) Explain the significance of the terms in this equation, with particular reference to the two terms containing cross products. [8]

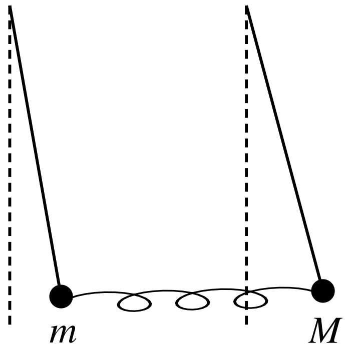
(b) A suspended plumb line does not point directly along the line to the Earth's centre, but suffers a small deflection owing to the centrifugal term in the equation of motion. Estimate this deflection in terms of ω, the Earth's radius R, and g for a plumb line suspended at a point at latitude λ. [10]

What is the deflection at Southampton (latitude $51°$)? [2]

[Latitude is $0°$ at the equator. Take the Earth to be a sphere of radius 6370 km.]

B4. (a) Explain what is meant by a *normal mode* for an oscillating system. [2]

(b) A pair of simple pendula of equal length ℓ have bobs of mass m and M. The pendula are coupled by a weak spring of spring constant k, as shown in the diagram below. Find the normal modes of this system for small oscillations in the plane of the diagram. [12]



(c) The system is released from rest with m in its equilibrium position and M displaced a small distance a directly away from the other pendulum. Find an expression describing the subsequent motion. [6]

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