SEMESTER 1 EXAMINATION 2006/07

CLASSICAL MECHANICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. The kinetic energy of a system of particles with positions $\mathbf{r}_i$ and masses m_i can be written as

$$T = \sum_i \frac{1}{2} m_i \dot{\mathbf{R}}^2 + \sum_i \frac{1}{2} m_i \dot{\boldsymbol{\rho}}_i^2,$$

where $\mathbf{r}_i = \mathbf{R} + \boldsymbol{\rho}_i$, and $\mathbf{R}$ is the centre of mass coordinate. Give the definition of $\mathbf{R}$ in terms of $\mathbf{r}_i$ and m_i, and a full interpretation of the two terms contributing to T. [4]

A2. Define the angular velocity ω of a spinning body, and state the formula for the rotational kinetic energy of the body in terms of ω and its moment of inertia I. Use formulae in all cases. (You may also sketch a diagram.) [4]

A3. What is meant by the term "apparent gravity" as seen by an observer in the rotating frame of the Earth ? Write down an expression for this quantity and identify each term in it. [4]

A4. A ball dropped from a certain point on the Sears tower in Chicago would land a little to the East of the point vertically below it. Explain this effect *qualitatively* from the point of view of an inertial observer using a diagram. [4]

A5. Express the principle of Time Translation Invariance in the context of simple or damped harmonic motion. [4]

Section B

B1. (a) A rocket whose casing and payload have negligible mass, has a mass M of fuel, and is at rest in deep space. It then burns fuel in such a way that the escaping gases have a constant speed u relative to the rocket. Show that when the rocket has mass m at a generic time t its speed is given by

$$v = u \ln\left(\frac{M}{m}\right).$$ [8]

(b) When an identical rocket is launched (from rest) on the surface of the Earth, vertically, it is clearly subject to the action of gravity. In this case, show that the velocity attained by the rocket when it has mass m' at a generic time t' (while still under the effect of the Earth's gravitational field of intensity g, to be taken as constant) is

$$v' = u \ln\left(\frac{M}{m'}\right) - gt'.$$ [6]

(c) Prove that in order for the two rockets to attain the same velocity $v = v'$ the left-over fuel mass in case (b), namely m', must necessarily be less than that in (a), namely m. [6]

B2. (a) Show that the moment of inertia of a thin uniform rod of length a and mass m about an axis through one end, perpendicular to the rod, is $\frac{1}{3}ma^2$. [6]

(b) A wheel of radius a consists of a thin rim of mass M and n spokes, each of mass m, which may be considered as thin rods terminating at the centre of the wheel. Calculate the moment of inertia of the wheel about an axis through the rim, perpendicular to the plane of the wheel. [6]

(c) If the wheel rolls without slipping down a plane inclined at an angle θ to the horizontal, show that the linear acceleration of its centre of mass is [8]

$$\frac{3(M+nm)g\sin\theta}{6M+4nm}$$

B3. (a) Derive the expression for the gravitational potential $\Phi(r)$ at distance r from the centre of a thin uniform spherical shell of mass M and radius R. Take the potential to be zero at infinite distance and be sure to give Φ for r both inside and outside the shell. [10]

(b) The magnitude g of the acceleration due to gravity is larger down a mine than it is on the Earth's surface. Show that this can be explained if

$$\rho_{\rm s} < \frac{2}{3}\rho_{\rm ave},$$

where $\rho_{\rm s}$ is the density of the Earth at its surface and $\rho_{\rm ave}$ is the Earth's average density. [10]

[Assume the Earth to be spherically symmetric.]

B4. (a) A planet orbits the Sun under the influence of the Sun's gravitational attraction. Explain why the vector angular momentum is conserved and why this means that the orbit lies in a plane. What other quantity or quantities are conserved? [6]

(b) The planet moves in an elliptical orbit with semi-major axis a. Prove that the product of the minimum and maximum speeds in the orbit is equal to

$$\frac{GM}{a},$$ [14]

where G is Newton's constant of universal gravitation and M is the mass of the Sun.

[Ignore the attraction of the planet to any other planets.]

END OF PAPER