UNIVERSITY OF SOUTHAMPTON PHYS2006W1

SEMESTER 1 EXAMINATION 2008/09

CLASSICAL MECHANICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

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Section A

A1. Define momentum in terms of the total mass and velocity of a body. A rocket starts from rest in deep space. After burning all its fuel, what is the total momentum of the rocket plus all the exhaust gases? [4]

A2. Define the angular velocity $\boldsymbol{\omega}$ of a spinning body, and state the formula for the rotational kinetic energy of the body in terms of $\boldsymbol{\omega}$ and its moment of inertia I. [4]

A3. Explain why a spinning skater spins faster as she draws her arms in. [4]

A4. State Kepler's three laws and say which of them remain true for a general central force, i.e. not just in the case of an inverse square law. [4]

A5. Explain what is meant by a *normal mode* for an oscillating system. [4]

Section B

B1. Define the moment of inertia of an object about a *fixed* rotation axis. [4]

A uniform sphere has mass m and radius a. Show that its moment of inertia about any diameter is

$$\frac{2}{5}ma^2.$$ [6]

Material is removed from the sphere to make a concentric spherical cavity of radius $a/2$. What is the mass of the resulting hollow ball? Show that its moment of inertia about a diameter is

$$\frac{31}{80}ma^2.$$ [4]

Show that the acceleration of the hollow ball, when it is rolling without slipping down an inclined plane, is $98/101$ of the acceleration of the original uniform sphere rolling without slipping down the same plane. [6]

B2. In a system of N particles, the force on the ith particle is written as

$$\mathbf{F}_i = \mathbf{F}_i^{\text{ext}} + \sum_{j \neq i} \mathbf{F}_{ij},$$

where $\mathbf{F}_i^{\text{ext}}$ is the external force on the ith particle and $\mathbf{F}_{ij}$ is the force of the jth particle on the ith.

What does Newton's third law imply for the relation between $\mathbf{F}_{ij}$ and $\mathbf{F}_{ji}$? [2]

If the total external force is given by

$$\mathbf{F}^{\text{ext}} = \sum_{i=1}^{N} \mathbf{F}_i^{\text{ext}},$$

show that

$$\mathbf{F}^{\text{ext}} = \frac{d\mathbf{P}}{dt},$$ [7]

where $\mathbf{P}$ is the total linear momentum of the system, which you should define in terms of the individual particle masses m_i and positions $\mathbf{r}_i$.

A uniform heavy chain lies in a heap on a table. One end of the chain is raised vertically with constant speed u. When a length z of the chain has been lifted off the table, show that the upward force that must be applied at the end of the chain is equal to $\rho g(z + u^2/g)$, where ρ is the mass per unit length of the chain. [11]

B3. Find an expression relating the acceleration g due to gravity at the earth's surface to Newton's gravitational constant G and the mass and radius of the earth (assumed spherically symmetric and non-rotating). [4]

A satellite is launched by rocket into earth orbit. When released by the rocket, the satellite is $200\,\mathrm{km}$ above the earth's surface with a velocity of $8\,\mathrm{km\,s^{-1}}$ perpendicular to a line drawn back to the earth's centre. By considering two conservation laws, find the satellite's furthest distance from the earth's centre during its subsequent orbital motion. [12]

What is the value of the eccentricity e of the orbit? [4]

[Recall that the earth's radius is $6370\,\mathrm{km}$ and notice that the eccentricity of a closed conic section is the distance between its center and its focus (or one of its two foci) expressed in terms of its radius (or its semi-major axis).]

B4. The equation of motion of a particle moving under gravity with position $\mathbf{x}$ measured from a point on or near the Earth's surface is

$$\ddot{\mathbf{x}} = \mathbf{g}^* - 2\,\boldsymbol{\omega} \times \dot{\mathbf{x}},$$

where $\mathbf{g}^*$ is the effective local gravitational field and $\boldsymbol{\omega}$ is the angular velocity of the Earth's rotation. Suppose the particle is projected from $\mathbf{x} = 0$ with velocity $\mathbf{v}$ at time $t = 0$. Explaining any approximations you make in reaching your answer, show that the subsequent position of the particle is given by,

$$\mathbf{x} = \mathbf{v}t + \frac{1}{2}\mathbf{g}^* t^2 - \frac{1}{3}\boldsymbol{\omega} \times \mathbf{g}^* t^3 - \boldsymbol{\omega} \times \mathbf{v}\, t^2. \qquad [8]$$

A shell is fired due East from a gun at latitude λ with muzzle velocity v and elevation angle α. Show that the lateral (North or South) deflection when the shell strikes the Earth has magnitude

$$\frac{4\omega v^3}{g^2} \sin\lambda \sin^2\alpha \cos\alpha. \qquad [10]$$

If the shell is fired in the Northern hemisphere, is the deflection to the North or South? [2]

[Neglect air resistance in this question.]

END OF PAPER