SEMESTER 2 EXAMINATION 2011/12

CLASSICAL MECHANICS

Duration: 120 MINS

VERY IMPORTANT NOTE

Section A answers MUST BE in a separate blue answer book. If any blue answer booklets contain work for both Section A and B questions - the latter set of answers WILL NOT BE MARKED.

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A Sheet of Physical Constants will be provided with this examination paper.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators may be used.

Section A

A1. Define the centre of mass coordinate $\mathbf{R}$ for a collection of particles with masses m_i, and show that the total momentum $\mathbf{P} = M\,\dot{\mathbf{R}}$, where M is the total mass. [4]

A2. Define torque and state how it affects angular momentum. Hence show that if a particle is orbiting a central point subject to a central force, its angular momentum is conserved. [4]

A3. Show that conservation of angular momentum for the particle in **A2**, implies that it obeys Kepler's second law. [4]

A4. A ball dropped from a certain point on the leaning tower of Pisa, would land a little to the East of the point vertically below it. Explain this effect *qualitatively* from the point of view of an inertial observer. [4]

A5. Define the Coriolis force. State briefly how Foucault's pendulum demonstrates its effects. [4]

Section B

B1. Show that the moment of inertia of a thin uniform rod of mass m and length $2a$ about an axis through its centre perpendicular to its length is

$$\frac{1}{3}ma^2.$$

[6]

A thin uniform rod of length $2a$ stands vertically on a smooth (i.e., frictionless) horizontal table. The rod is slightly disturbed and falls down.

What path does the rod's centre of mass follow during the fall? [2]

Does the reaction force of the table on the rod do any work? Is mechanical energy conserved during the fall? [2]

Show that the rod hits the table with angular speed

$$\sqrt{\frac{3g}{2a}}.$$

[10]

B2. The minimum distance of a comet from the Sun is $R/4$, where R is the radius of the Earth's orbit (assumed circular), and its speed at that point is $2\sqrt{2}\,v_E$, where v_E is the orbital speed of the Earth. The Earth's and comet's orbits are coplanar.

If the mass of the comet is m, determine its angular momentum. [5]

By energy considerations, determine its speed when it crosses the Earth's orbit. [7]

Hence find the angle at which the orbits cross. [5]

Will the comet subsequently escape from the solar system? Explain how you reach your answer. [3]

B3. The equation of motion of a particle of mass m and position vector $\mathbf{r}$ under the influence of gravity and an additional force $\mathbf{F}$, as measured in a reference frame rotating with constant angular velocity $\boldsymbol{\omega}$, is

$$m\ddot{\mathbf{r}} = \mathbf{F} + m\mathbf{g} - 2m\,\boldsymbol{\omega}\times\dot{\mathbf{r}} - m\,\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}).$$

A bug crawls outward with constant speed u along the spoke of a wheel that is rotating with constant angular velocity ω about a vertical axis. With reference to the above equation of motion, describe the *inertial* (or *fictitious*) forces acting on the bug when it is at a distance r from the rotation axis, giving both their magnitude and direction. [10]

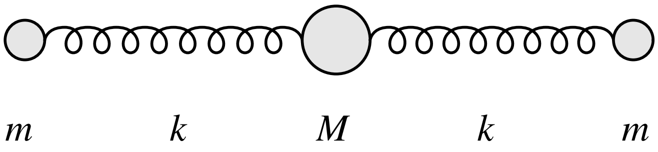
If the coefficient of friction between the bug and the spoke is μ, show that it can crawl to a distance

$$\frac{(\mu^2 g^2 - 4\omega^2 u^2)^{1/2}}{\omega^2}$$

from the rotation axis before it slips. [10]

B4. Explain what is meant by a *normal mode* for an oscillating system. [3]

A model for a triatomic molecule comprises two masses m, each connected to a central mass M by a spring. The two springs are identical and each has spring constant k. Consider the motion of this system when all the masses are constrained to lie along the same straight line, as shown below.



Show that the normal mode (angular) frequencies for this system are

$$0, \quad (k/m)^{1/2} \quad \text{and} \quad (k/m + 2k/M)^{1/2}.$$

[11]

Find and describe the displacements of the masses in each of the three normal modes. [6]

[Ignore any effects of gravity and of quantum mechanics in this problem.]

END OF PAPER