SEMESTER 2 EXAMINATION 2012-2013

CLASSICAL MECHANICS

Duration: 120 MINS (2 hours)

This paper contains 9 questions.

Answers to Section A and Section B must be in separate answer books

Answer **all** questions in **Section A** and **only two** questions in **Section B**.

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it.

Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

An outline marking scheme is shown in brackets to the right of each question.

A Sheet of Physical Constants is provided with this examination paper.

Only university approved calculators may be used.

A foreign language translation dictionary (paper version) is permitted provided it contains no notes, additions or annotations.

Section A

A1. The kinetic energy of a system of particles with positions $\mathbf{r}_i$ and masses m_i can be written

$$T = \sum_i \frac{1}{2} m_i \dot{\mathbf{R}}^2 + \sum_i \frac{1}{2} m_i \dot{\boldsymbol{\rho}}_i^2,$$

where $\mathbf{r}_i = \mathbf{R} + \boldsymbol{\rho}_i$, and $\mathbf{R}$ is the centre of mass coordinate. Give the definition of $\mathbf{R}$ in terms of $\mathbf{r}_i$ and m_i, and a full interpretation of the two terms contributing to T. [4]

A2. Two circular loops are made from lengths of the same thin uniform wire. One is twice the radius of the other. What is the ratio of the moments of inertia of the two loops about axes through the centres perpendicular to the plane of the loops? Give details of your working. [4]

A3. State Kepler's three laws. Which one does remain true for all central forces (not just inverse-square law)? [5]

A4. When a heavy ball is dropped from, say, the North side of the top of the Sears tower, a tall building in Chicago, it lands horizontally displaced by $15\,\mathrm{cm}$. In which horizontal direction is it displaced and why? Explain this qualitatively by mentioning which effect is resposible for this motion. [4]

A5. Explain what is meant by a *normal mode* for an oscillating system. [3]

Section B

B1. (a) A rocket at rest in deep space has a total mass M. It then burns fuel in such a way that the escaping gases have a constant speed u relative to the rocket. Show that when the rocket has mass m the rocket's speed is given by

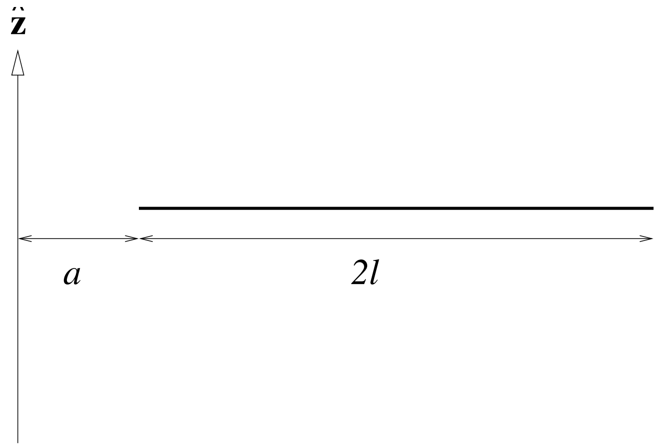
$$u \ln\left(\frac{M}{m}\right).$$ [7]

(b) What fraction of the mass M should be fuel in order that the momentum of the rocket after all the fuel is burned is maximised? [7]

(c) What fraction of the mass M should be fuel in order that the kinetic energy of the rocket after all the fuel is burned is maximised? [6]

TURN OVER

B2. (a) Define the moment of inertia of an object about a *fixed* rotation axis, paying particular attention to defining precisely all symbols. [5]



(b) Show that the moment of inertia of a thin horizontal rod of length $2l$ and mass m, about a vertical axis $\hat{\mathbf{z}}$ in the vertical plane of the rod a distance a from the rod (see diagram above) is

$$\frac{1}{3}ml^2 + m(a+l)^2.$$ [7]

(c) The moment of inertia of a spinning skater with arms outstretched can be modelled by attaching two such horizontal rods to a uniform right circular cylinder of radius a, with its axis coinciding with $\hat{\mathbf{z}}$. If the length and mass of each of her arms are $70\,\mathrm{cm}$ and $3\,\mathrm{kg}$, the radius of her torso can be taken to be $a = 20\,\mathrm{cm}$ and her *total* weight is $60\,\mathrm{kg}$, how much faster does she spin when she brings her arms down to lie vertically by her sides? [8]

[The moment of inertia of a uniform right circular cylinder of mass M and radius a about its central axis is $\frac{1}{2}Ma^2$.]

B3. (a) Find an expression relating the acceleration g due to gravity at the earth's surface to Newton's gravitational constant G and the mass and radius of the earth (assumed spherically symmetric and nonrotating). [3]

(b) A satellite is launched by rocket into earth orbit. When released by the rocket, the satellite is $200\,\mathrm{km}$ above the earth's surface with a velocity of $8\,\mathrm{km\,s^{-1}}$ perpendicular to a line drawn back to the earth's centre. By considering two conservation laws, find the satellite's furthest distance from the earth's centre during its subsequent orbital motion. [12]

(c) What is the value of the eccentricity of the orbit? [5]

[The earth's radius is $6370\,\mathrm{km}$.]

TURN OVER

B4. The equation of motion of a particle moving under gravity with position $\mathbf{x}$ measured from a point on or near the Earth's surface is

$$\ddot{\mathbf{x}} = \mathbf{g}^* - 2\,\boldsymbol{\omega} \times \dot{\mathbf{x}}.$$

where $\mathbf{g}^*$ is the effective local gravitational field and $\boldsymbol{\omega}$ is the angular velocity of the Earth's rotation. Suppose the particle is projected from $\mathbf{x} = 0$ with velocity $\mathbf{v}$ at time $t = 0$.

(a) Explaining any approximations you make in reaching your answer, show that the subsequent position of the particle is given by,

$$\mathbf{x} = \mathbf{v}t + \frac{1}{2}\mathbf{g}^* t^2 - \frac{1}{3}\boldsymbol{\omega} \times \mathbf{g}^* t^3 - \boldsymbol{\omega} \times \mathbf{v}\, t^2.$$ [8]

(b) A shell is fired due East from a gun at latitude λ with muzzle velocity v and elevation angle α. Show that the lateral (North or South) deflection when the shell strikes the Earth has magnitude

$$\frac{4\omega v^3}{g^2} \sin\lambda \sin^2\alpha \cos\alpha.$$ [10]

(c) If the shell is fired in the Northern hemisphere, is the deflection to the North or South? Explain your reasoning. [2]

[Neglect air resistance in this question.]

END OF PAPER