SEMESTER 2 EXAMINATION 2008/09

PHYS6011 PARTICLE PHYSICS

Duration: 120 MINS

*Answer **all** questions in **Section A** and two **and only two** questions in **Section B**.*

Section A carries 1/3 of the total marks for the exam paper and you should aim to spend about 40 mins on it. Section B carries 2/3 of the total marks for the exam paper and you should aim to spend about 80 mins on it.

A sheet of Physical Constants will be provided with this exam paper. An outline marking scheme is shown in brackets to the right of each question.

Only University approved calculators may be used.

Section A

A1. Dirac proposed the energy (E) momentum ($\mathbf{p}$) relation for a particle of mass m

$$E = \alpha.\mathbf{p} + \beta m$$

What conditions must the constants α and β satisfy for this equation to describe a relativistic particle for which $E^2 - |\mathbf{p}|^2 = m^2$? [4]

A2. The wave equation describing non-interacting photons is just Maxwell's equation in free space. In Lorentz gauge, in which $\partial^\mu A_\mu = 0$, it is

$$\partial_\nu \partial^\nu A^\mu = 0$$

Show that, even though the index μ on A^μ runs from 0..3, this equation is consistent with there being just two physical polarizations of the photon. [4]

A3. The Υ(4S) particle has a mass of 10.6 GeV/c^2 and can be created by colliding an electron and positron (with mass of 0.51 MeV/c^2).

a) What energy must the electron and/or positron have in the centre of mass to create the Υ(4S)? [1]

b) If the positron is at rest in the laboratory, what energy must the electron now have to create the Υ(4S)? [3]

A4. Write down the Feynman rule for the coupling between a quark and a gluon being careful to define each quantity. [3]

Briefly describe one piece of experimental evidence for the existence of gluons [1]

A5. In the electroweak theory there is a Higgs field which is a complex doublet of the SU(2) gauge symmetry and hence has four real degrees of freedom. Briefly explain why there is only one physical Higgs particle in the spectrum. [4]

Section B

B1. A massless Dirac fermion with momentum $\mathbf{p}$ is described by a two-component spinor, ψ, that satisfies the spinor equation

$$H\psi = \sigma.\mathbf{p}\psi$$

where H is the Hamiltonian and σ^i are the Pauli matrices.

(a) Show that the commutator between H and the angular momentum operators $\mathbf{L}$ are

$$[H, \mathbf{L}] = -i\sigma \times \mathbf{p}$$

[7]

(b) Show that $\mathbf{L} + \sigma/2$ commutes with H. You may use the result

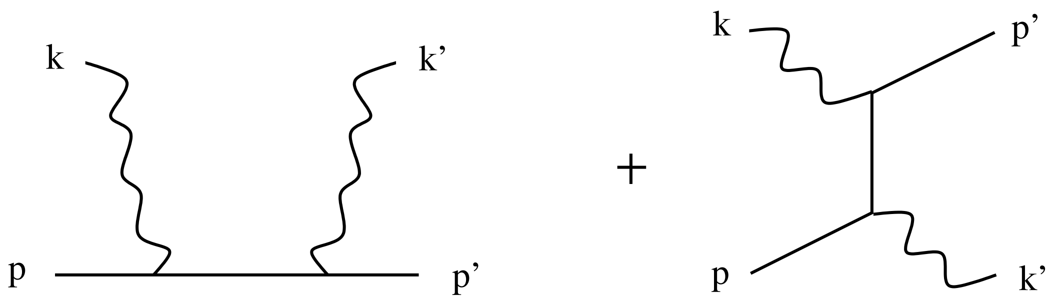
$$[\sigma^i, \sigma^j] = 2i\varepsilon^{ijk}\sigma^k$$

where ε^{ijk} is a totally anti-symmetric tensor of magnitude one. [7]

(c) What are the physical consequences of the results in parts (a) and (b)? [2]

(d) In the case of a particle moving in the z-direction with momentum p_z the eigenvectors of the spinor equation are just $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Find the energy eigenvalues and the eigenvalues of the helicity operator $\sigma.\mathbf{p}/|\mathbf{p}|$. Physically interpret these states. [4]

B2. Compton scattering (the scattering of an electron and a photon) is described by the two Feynman diagrams



(a) We normally assume that the external photons in the process satisfy the free Maxwell's equation

$$\partial^\nu \partial_\nu A^\mu = 0$$

Show that there are solutions of the form $A^\mu = \varepsilon^\mu e^{-iq^\lambda x_\lambda}$ with x^λ the position four-vector and ε^μ any polarization vector. [2]

(b) Similarly the external electron fields satisfy the free Dirac equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

There are solutions of the form $\psi = u(p)e^{-ip^\lambda x_\lambda}$ - find the equation that $u(p)$ satisfies. [1]

(c) The result at leading order in perturbation theory for the scattering of an electron of charge q from a state a to a state c as the result of the interaction with a U(1) gauge potential A^μ is

$$\mathcal{M}_{ac} = -i \int J^\mu_{ac}\, A_\mu\, d^4x \qquad \text{where} \qquad J^\mu_{ac} = iq(\bar{\psi}_c \gamma^\mu \psi_a)$$

The internal electron wave function may be found by solving the Dirac equation in the presence of the photon gauge field for ψ_{int}

$$(i\gamma^\mu \partial_\mu - m)\psi_{\text{int}} = q\gamma_\mu A^\mu_{ext} \psi_{ext}$$

where the subscript *ext* corresponds to an external particle at the vertex.

Using these results show that the contribution to the scattering amplitude from the first diagram above is given by

$$\begin{aligned} -i\mathcal{M}_1 \;&=\; \bar{u}(p')\left(\varepsilon^*_\nu(k')\;\; iq\gamma^\nu\; \frac{i(\not{p}'+\not{k}')+m}{(p'+k')^2-m^2}\;\; iq\gamma^\mu\;\; \varepsilon_\mu(k)\right) u(p) \\ &\qquad (2\pi)^4 \delta^4(p+k-p'-k') \end{aligned}$$

[10]

(d) Show how this expression can be associated with a set of Feynman rules for the external electron and photon states, the vertices and the internal fermion propagator. [4]

(e) Using your Feynman rules write down the contribution to the scattering amplitude for the second diagram above. [3]

B3. (a) How does a Cyclotron work? [8]

(b) What are the limitations of a Cyclotron? [2]

(c) How do the Synchrocyclotron and the Isocyclotron overcome the limitations of the Cyclotron? [2]

(d) What are the benefits and drawbacks of circular and linear colliders? [6]

(e) Name two modern uses (in addition to particle physics research) for Cyclotrons. [2]

B4. A complex scalar field, ϕ, with mass m and charge q with respect to a U(1) gauge field, A^μ, satisfies the wave equation

$$D^\mu D_\mu \phi + m^2 \phi = 0, \qquad D^\mu = \partial^\mu + iqA^\mu$$

(a) Show that under the gauge transformation

$$\phi \to e^{iq\theta(x)}, \qquad A^\mu \to A^\mu - \partial^\mu \theta(x)$$

$D^\mu \phi$ transforms to $e^{iq\theta(x)} D^\mu \phi$. [6]

(b) Hence, explain why the gauge transformation is a symmetry of the ϕ wave equation. [2]

(c) We can introduce a potential for the field ϕ of the form

$$V = -\frac{\mu^2}{2}|\phi|^2 + \frac{\lambda}{4}|\phi|^4$$

where μ and λ are positive constants.

Show that this potential is gauge invariant, sketch the potential and find the value of ϕ at the minimum. [4]

(d) The gauge field's wave equation in Lorentz gauge is given by

$$\partial^\nu \partial_\nu A^\mu = J^\mu, \qquad J^\mu = iq(\phi^* D^\mu \phi - (D^\mu \phi)^* \phi)$$

Show that if ϕ lies at the minimum of the potential in (c) that the photon effectively develops a mass and satisfies the Klein Gordon equation

$$(\partial^\mu \partial_\mu + M^2) A^\mu = 0$$

[3]

(e) Find a solution of this massive Klein Gordon equation for which $A^0 = 0$ and $\mathbf{A}$ is time independent. Explain, using your solution, why magnetic fields are expelled from the volume of space in which ϕ is non-zero and why the theory has zero electrical resistance. [5]

END OF PAPER