The Weak Nuclear Force

Contents

- Fermi Theory and Parity Violation
- Higgs Mechanism
- Symmetry Breaking - Goldstone's theorem, massive Ws
- Hypercharge and the Z
- Fermion Masses and Cabibbo Mixing
- Higgs Searches

Learning Outcomes

- Know main experimental results on the weak force
- Be able to explain the $U(1)$ Higgs mechanism in detail
- Know $Q = T_3 + Y/2$
- Be able to write down W^3, B mass matrix and diagonalize it
- Be able to explain how the Higgs generates fermion masses
- Know the Higgs search methods.

Reference Books

- Introduction to High Energy Physics, Perkins
- Elementary Particle Physics , Kenyon.

The Weak Force

We now turn our attention to the description of the weak nuclear force and the associated gauge theory. We will have to explore the Higgs mechanism of symmetry breaking to explain the gauge boson masses.

1 Introduction

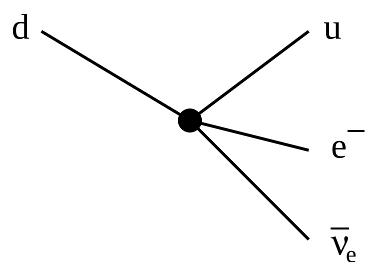
The weak nuclear force is responsible for β decay of nuclei

$$n \to p \ + \ e^- \ + \ \bar{\nu}_e \tag{1}$$

This was the process which lead Pauli to predict the existence of the neutrino in order to reconcile the experimental data with energy and momentum conservation. At the quark level the process is

$$d \to u \ + \ e^- \ + \ \bar{\nu}_e \tag{2}$$

The force is a weak force in day to day life because it is very short range. Fermi first successfully described the force by postulating a Feynman rule for a four fermion interaction at a point. In modern language we would write



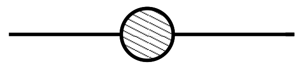
$$\mathcal{M} = (\bar{u}_p \gamma^\mu u_n) G_F (\bar{u}_e \gamma_\mu u_\nu) \tag{3}$$

where G_F is the Fermi constant. Note it replaces the $1/q^2$ type bit of the gauge boson propagator we have previously seen in this type of expression. G_F therefore has dimension of mass^{-2} - experimentally it is about $10^{-5}\mathrm{GeV}^{-2}$.

Such a theory on its own though is not a renormalizable theory - at loop level we get many divergences that may not be absorbed into parameters of the theory. The story will get considerably more complicated but let's start to see the mix of ideas that will enter.

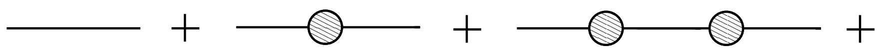
1.1 A Massive Propagator

The Feynman propagator for a massless particle controlled by the Klein Gordon equation ($\Box\phi = 0$) is $-i/q^2$. If we introduce a mass term ($(\Box + m^2)\phi = 0$) we may treat it as a perturbation to the massless theory. The Feynman rule that emerges turns out to be



im^2

Now we can work out the massive propagator by resumming the perturbation series for this particle



$$\begin{aligned}
&\tfrac{-i}{q^2} \quad + \quad \tfrac{-i}{q^2}(-im^2)\tfrac{-i}{q^2} \quad + \quad \tfrac{-i}{q^2}(-im^2)\tfrac{-i}{q^2}(-im^2)\tfrac{-i}{q^2} \quad + \quad ... \\
&= \tfrac{-i}{q^2}\left(1 + \tfrac{m^2}{q^2} + \tfrac{m^4}{q^4} + ...\right) \\
&= \tfrac{-i}{q^2}\left(1 - \tfrac{m^2}{q^2}\right)^{-1} \\
&= \tfrac{-i}{q^2 - m^2}
\end{aligned} \tag{4}$$

Note that at small momentum the propagator becomes a constant i/m^2. This suggest that G_F in the description of the weak force should correspond to the mass of a massive exchange particle.

1.2 SU(2)

There are hints of an SU(2) gauge theory in the weak force β-decay process. SU(2) transformations act on a doublet of wave functions as

$$\psi \rightarrow e^{i\theta^a T^a}\psi \tag{5}$$

where the T^a are the generators of SU(2) which are just the Pauli matrices

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{6}$$

However, to place u, d or ν_e, e in doublets we would require them to be identical particles which they manifestly are not in low energy experiments!

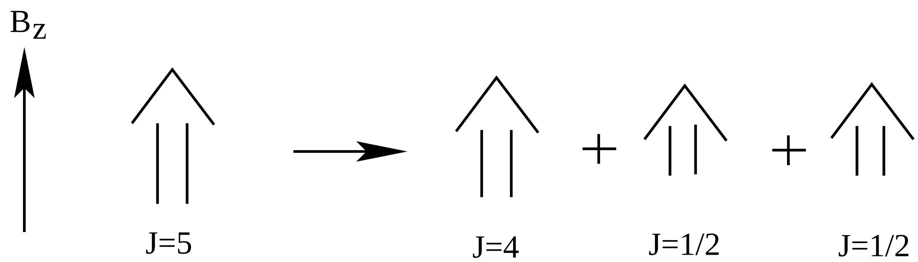
A gauge theory would also predict the existence of three massless SU(2) gauge bosons which we do not observe.

1.3 Parity Violation

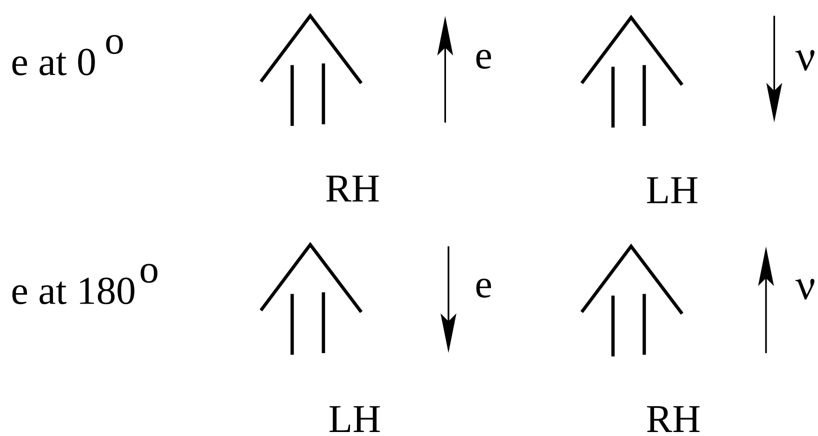
An added confusion to the mix is provided by Mdm Wu's experiment that shows the weak force violates parity. She studied the decay of Cobalt 60 in a magnetic field

$$^{60}Co \; \rightarrow \; ^{60}Ni^{*} + e^{-} + \bar{\nu}_e \tag{7}$$

The spins in the experiment align as



One would expect the electrons to emerge at all angles. If we take the massless limit and think about helicity in this experiment we can see that at 0^o and 180^o there could be processes



Experimentally though one measures an angular distribution of the emerging electrons that fits the profile $1 - \cos\theta$. In other words at $\theta = 0^o$ there is no production. This means that the weak force violates parity and only couples to left handed particles.

In the massless limit where left and right handed particles decouple in the Dirac equation there is no reason why this should not be the case. The confusion we will have to resolve is how this can be consistent with the electron having as mass that mixes these different helicity states.

2 The Higgs Mechanism

The key additional idea we need to make sense of the weak force is the Higgs Mechanism. We will now explore these ideas in simple environments before returning to the full theory of nature.

2.1 Introduction

Our first task will be to understand how to give masses to gauge bosons. This seems like a hard task given that the much prized gauge symmetry principle actively forbids such masses. The Higgs mechanism introduces an extra ingredient - we will fill space with matter charged under the gauge symmetry. A gauge boson then trying to propagate through this "goo" will interact continually and we will see that the energy of interaction will become the effective mass of the gauge boson. Mass is a scalar quantity and this tells us the background goo must be a scalar field (or wave function), Φ, the Higgs.

Lets do the basics of the maths in QED where we will give the photon a mass. We have seen how such a charged particle with a scalar wave function (satisfying the Klein Gordon equation) would enter into the QED Maxwell equations

$$\Box A^{\mu} = J^{\mu} = iq(\Phi^* D^{\mu}\Phi - (D^{\mu}\Phi)^*\Phi) \tag{8}$$

the current is just the charge times the number density current but where we have set $\partial^{\mu} \to D^{\mu} = \partial^{\mu} + iqA^{\mu}$ as minimal substitution requires.

Now imagine that throughout space $\Phi = v$ - we say it has a vacuum expectation value (vev) often written

$$\langle \Phi \rangle = v \tag{9}$$

Spatial derivatives of v are just zero and the only surviving bits in the wave equation above are

$$\Box A^{\mu} = -2q^2 v^2 A^{\mu} \tag{10}$$

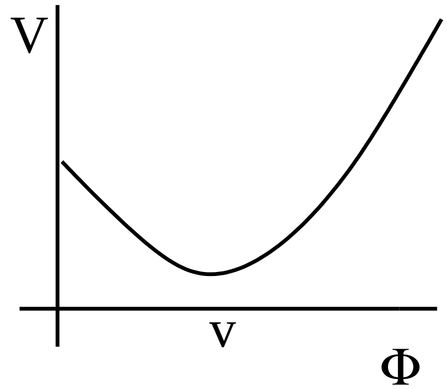
or

$$(\Box + m^2)A^{\mu} = 0, \qquad m^2 = 2q^2 v^2 \tag{11}$$

which makes the gauge field massive as we wanted!

When we give Φ a vev we are effectively changing the normalization of the wave function so there are a large number of particles per unit volume - we can think of Φ as a classical field in this limit.

Why would Φ have this non-zero vacuum value though? Well we'll just cook things so it does without answering that question! Here's an example potential for Φ that does the trick



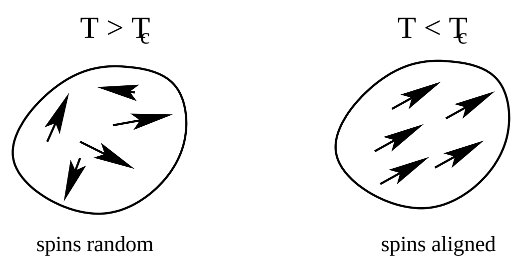
This looks a bit adhoc but so far so good.

We will see that, since this background goo has electric charge, all electrically charged particles will end up coupling to it and get a mass as a result. We will generate the electron mass etc in the SU(2) version of this story.

This all looks quite simple but to understand how we have got a mass and gauge invariance together we will need to think more deeply about what we're doing.

2.2 Symmetry Breaking

The reason we can suddenly have a mass for the gauge field in the Higgs Mechanism is because the symmetry is being broken by the vacuum of the theory. To begin to understand this idea think about a ferromagnet. Above the critical temperature the atomic spins are randomly pointing due to thermal excitation. Below the critical temperature the thermal energy is insufficient to disrupt a natural alignment of the spins due to their interactions.



In the absence of defects, the direction in which the spins choose to finally lie arranged, below T_c, is truly random in a quantum theory. The final arrangement picks out a particular direction in space and therefore breaks rotational symmetry that existed at high temperature or without the atoms present. This is the basic idea of spontaneous symmetry breaking.

2.2.1 Breaking an Internal Symmetry

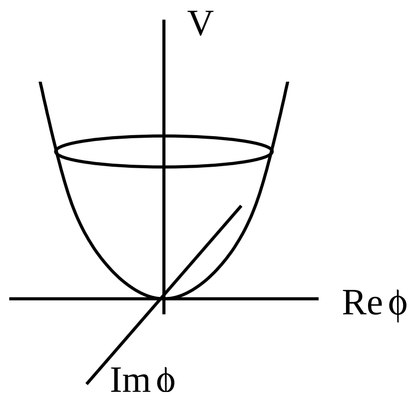
Consider a complex scalar field Φ with a global U(1) internal symmetry

$$\Phi \rightarrow e^{i\alpha}\Phi \tag{12}$$

We can write a potential for the field

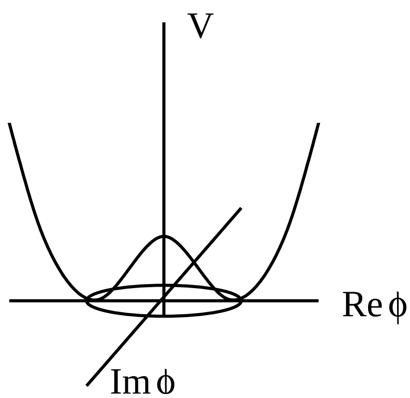
$$V = \frac{1}{2}\mu^2\Phi^*\Phi + \frac{\lambda}{4}(\Phi^*\Phi)^2 \tag{13}$$

When μ^2 is positive the potential looks like



The minimum is at $\Phi = 0$. The potential clearly exhibits the U(1) symmetry.

If μ^2 is negative though the potential looks like



Close to the origin the $|\Phi|^2$ term dominates, whilst at large values the $|\Phi|^4$ term dominates. There is a ring of minima. To find the value of $|\Phi|$ look on the real axis and require

$$\frac{dV}{d\Phi} = 0 = \mu^2\Phi + \lambda\Phi^3 \tag{14}$$

so the minima is at

$$|\Phi|^2 = -\frac{\mu^2}{\lambda} \tag{15}$$

The value of the potential at the minimum is

$$V_{min} = -\frac{\mu^4}{4\lambda} \tag{16}$$

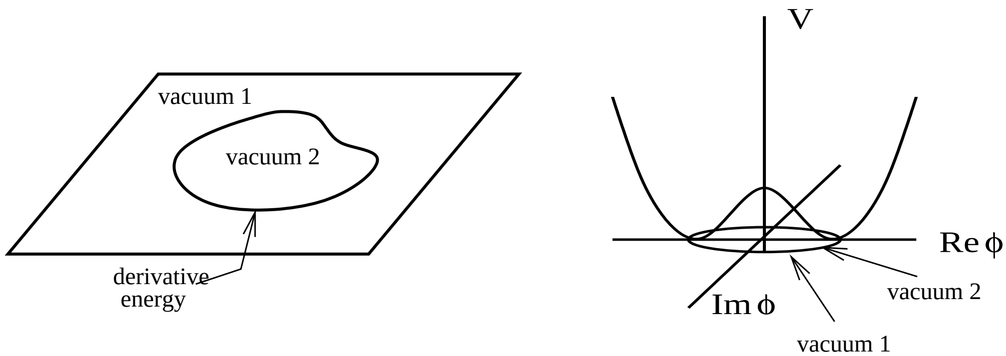
Globally the potential again exhibits the U(1) symmetry (the potential does not depend on the phase of Φ). However, the minima have a symmetry breaking form. there are many degenerate vacua and which one we end up in will be random. When one looks locally around a minimum there is not a rotational symmetry.

2.2.2 Goldstone's Theorem

In the above example there is a massless state. We can see this by writing

$$\Phi = re^{i\theta}, \qquad \Phi^*\Phi = r^2 \tag{17}$$

The potential does not depend on θ. Thus there is no energy cost to a configuration where θ changes in space



The only energy cost is due to the derivative from the changing field. In terms of the Klein Gordon equation though

$$(\Box + m^2)\Phi = 0 \tag{18}$$

we know that derivatives of Φ are not a mass but momentum. So this state is massless.

This is not true for the r degree of freedom. If we change r there is a direct energy cost from the potential which corresponds to a mass.

Goldstone's Theorem: there is a massless degree of freedom for every broken generator of a group.

In the above case the U(1) degree of freedom is broken so there is one massless mode. If we were to break an SU(2) symmetry with three generators we would therefore expect to find 3 massless modes in the theory. Is this a block to describing the weak force as a broken symmetry? - there are no physical massless modes. In fact in a gauge theory something extra happens.

2.3 Superconductivity

The first example of a broken gauge symmetry to be understood was in the phenomena of superconductivity. In a superconductor interactions with the lattice of ions produces forces that bind electrons into Cooper pairs - a charge -2, spinless bound state of two electrons. We can treat the Cooper pair as a scalar field.

Let's look at a relativistic version of this theory. We will introduce some potential as above that gives the scalar a non-zero vacuum expectation value

$$\langle \Phi \rangle = v \tag{19}$$

This will enter into the photon's Klein Gordon equation

$$\Box A^\mu = iq\Phi^* D^\mu \Phi - iq(D^\mu \Phi)^* \Phi \tag{20}$$

giving

$$(\Box + m^2)A^\mu = 0, \qquad m^2 = 2q^2v^2 \tag{21}$$

The spatial behaviour of a constant magnetic field described by a vector potential is then the solution of

$$\nabla^2 \vec{A} = 2q^2v^2\vec{A} \tag{22}$$

with solution

$$\vec{A} = \vec{A}_0 e^{-\sqrt{2}qvx} \tag{23}$$

The vector potential therefore decays away exponentially in the material and magnetic fields are expelled by the superconductor.

We wanted to ask whether there was a massless mode associated with the flat direction in the scalar potential. In fact a spatial fluctuation in the phase of the scalar field can be gauged away. If we write a configuration

$$\Phi = re^{i\theta(x)} \tag{24}$$

then we can make a gauge transformation

$$\Phi \rightarrow re^{i\theta(x)}e^{iq\alpha(x)} \tag{25}$$

Clearly we can choose α to cancel θ. In this way we can always remove the massless fluctuations - it is not physical.

We should think about the degrees of freedom in this problem. Initially we had a massless photon with two polarizations. Now it is massive it becomes possible for there to be fluctuations in the longitudinal direction of motion. Where has this extra degree of freedom come from? - it is precisely the degree of freedom corresponding to the θ fluctuation that the gauge theory has cancelled. The gauge field has "eaten" the Goldstone boson to become massive!

This model therefore produces a massive photon and a massive degree of freedom associated with fluctuations in r. This later field is called the Higgs Boson.

3 A Toy Model of the Weak Force

Let's try to put all of our pieces together to make a theory of the weak force - it will turn out we're still lacking one ingredient though.

$SU(2)_L$

If all the fermions were massless we could place left handed helicity spinors in SU(2) gauge doublets

$$\left(\begin{array}{c} \nu_e \\ e \end{array} \right)_L, \qquad \left(\begin{array}{c} u \\ d \end{array} \right)_L \tag{26}$$

the right handed fields u_r, d_r... are singlets of SU(2).

Since we're in an SU(2) gauge theory we have three gauge fields $W^{1\mu}, W^{2\mu}, W^{3\mu}$ associated with the three generators

$$\frac{1}{2}\left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right), \qquad \frac{1}{2}\left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right), \qquad \frac{1}{2}\left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right) \tag{27}$$

rather than W^1, W^2 it's more physical to think about the W bosons associated with

$$T^1 \pm iT^2 = \left(\begin{array}{cc} 0 & 1 \\ 0 & 0 \end{array} \right) \quad \text{or} \quad \left(\begin{array}{cc} 0 & 0 \\ 1 & 0 \end{array} \right) \tag{28}$$

These change the top component of a doublet into the bottom and vice versa. We call this basis the W^+ and W^- gauge bosons.

The Higgs

We next introduce an SU(2) doublet scalar field

$$\Phi = \left(\begin{array}{c} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{array} \right) \tag{29}$$

with a symmetry breaking potential

$$V = -\frac{\mu^2}{2}|\Phi|^2 + \frac{\lambda}{4}|\Phi|^4 \tag{30}$$

We can look at the vacuum where

$$\Phi = \left(\begin{array}{c} v \\ 0 \end{array} \right) \tag{31}$$

(Note you can get to any other vacuum by making a gauge transformation so there is no lack of generality here!).

We will have 3 Goldstone Bosons that are eaten by the 3 W gauge bosons to get masses

$$M_W^2 \simeq g_W^2 v^2 \tag{32}$$

and finally we will have a Higgs field whose mass depends on the choice of μ and λ.

What's missing: we have not yet included QED in this discussion and the three W particles have ended up degenerate whilst in nature we see two degenerate $W^\pm$ and the Z.

4 Hypercharge

Including the U(1) of QED into our model is not straightforward. The complete set of symmetry transformations we can make on a left handed doublet are

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \to e^{i\theta^a T^a} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \tag{33}$$

where the full set of possible T^a are

$$T^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad T^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad T^3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad T^4 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{34}$$

The first three are the generators of SU(2). The last though can not be QED because ν_e and e do not have the same electric charge. The electric charge matrix is given by

$$Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} = T^3 - T^4 \tag{35}$$

We have no choice but to gauge the U(1) associated with T^4 - it's called *hypercharge*. In order to remove this symmetry at low energies we will make the Higgs have hypercharge so the associated gauge boson, B^μ, becomes massive.

To obtain QED the Higgs vev will have zero $T^3 - T^4$ charge so a mixture of $W^{3\mu}$ and B^μ will be able to propagate and become the massless photon. We will thus require

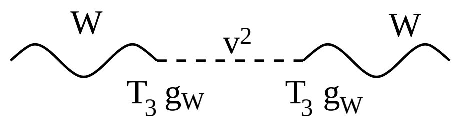
$$Y_{Higgs} = -1, \qquad T^3_{Higgs} = \frac{1}{2}, \qquad Q_{Higgs} = 0 \tag{36}$$

The T^3 charge means we are placing the vev in the top element of it's doublet - $(v, 0)$. Note we are using conventions where

$$Q = T^3 + \frac{1}{2}Y \tag{37}$$

4.1 The Massless Photon

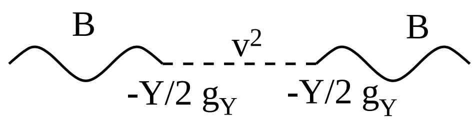
The 3 W bosons get mass from their interaction with the Higgs. We can draw this as



from which we see that

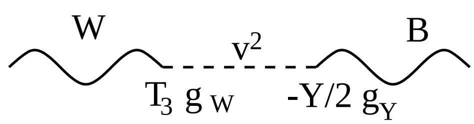
$$M_W^2 = \frac{1}{4} g_W^2 v^2 \tag{38}$$

Similarly the B^μ gauge boson gets a mass



$$M_B^2 = \frac{1}{4} Y^2 g_Y^2 v^2 = \frac{1}{4} g_Y^2 v^2 \tag{39}$$

There can also be a mass mixing between the W^3 and B



$$M_{mix}^2 = \frac{1}{4} g_W g_Y Y v^2 = \ + \ \frac{1}{4} g_W g_Y v^2 \tag{40}$$

The full W^3, B gauge boson mass matrix can then be written

$$(W^{3\mu}, B^\mu) \frac{v^2}{4} \begin{pmatrix} g_W^2 & + \ g_W g_Y \\ + \ g_W g_Y & g_Y^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \tag{41}$$

To find the mass eigenstates we can insert "one" written as $U^\dagger U$, with U a unitary matrix, to the left and right of the mass matrix, ie, we group things as

$$\left[(W^{3\mu}, B^\mu) U^\dagger \right] \left[U \frac{v^2}{4} \begin{pmatrix} g_W^2 & + \ g_W g_Y \\ + \ g_W g_Y & g_Y^2 \end{pmatrix} U^\dagger \right] \left[U \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} \right] \tag{42}$$

Now by choosing an appropriate U we can diagonalize the mass matrix to its eigenvalues. Factoring out the $v^2/4$

$$(g_W^2 - \lambda)(g_Y^2 - \lambda) - g_W^2 g_Y^2 = 0 \tag{43}$$

$$\lambda^2 - (g_W^2 - g_Y^2)\lambda = 0 \tag{44}$$

The eigenvalues are

$$\lambda = 0, \quad \lambda = \frac{v^2}{4}(g_W^2 + g_Y^2) \tag{45}$$

There is a massless state - the photon - and a massive state - the Z boson. The eigenfunctions are

$$Z^\mu = \frac{g_W W^{3\mu} \ + \ g_Y B^\mu}{\sqrt{g_W^2 + g_Y^2}} \tag{46}$$

$$A^\mu = \frac{g_W B^\mu \ -- \ g_Y W^{3\mu}}{\sqrt{g_W^2 + g_Y^2}} \tag{47}$$

We can think of this transformation as a rotation and write

$$Z^\mu = \cos\theta_W W^{3\mu} \ + \ \sin\theta_W B^\mu \tag{48}$$

$$A^\mu = \sin\theta_W W^{3\mu} \ -- \ \cos\theta_W B^\mu \tag{49}$$

with

$$\cos\theta_W = \frac{g_W}{\sqrt{g_W^2 + g_Y^2}}, \qquad \sin\theta_W = \frac{g_Y}{\sqrt{g_W^2 + g_Y^2}} \tag{50}$$

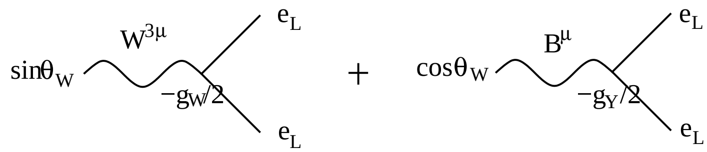
4.2 Fermion Charges

We have to assign all the chiral fermions appropriate charges to the hypercharge gauge boson so that their QED charges work out right. This gives

	T_3	Y	$Q = T_3 + \frac{1}{2}Y$
ν_L	$+\frac{1}{2}$	-1	0
e_l	$-\frac{1}{2}$	-1	-1
e_R	0	-2	-1
u_L	$+\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$
d_L	$-\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$
u_R	0	$+\frac{4}{3}$	$+\frac{2}{3}$
d_R	0	$-\frac{2}{3}$	$-\frac{1}{3}$

(51)

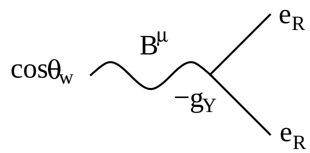
So for example the **photons coupling to the left handed electron** is given in terms of the $W^{3\mu}$ and B^{μ} gauge bosons by



We want to get $-e$ as the answer so we learn that

$$g_W \sin\theta_W = g_Y \cos\theta_W = e \tag{52}$$

Does the **right handed electron - photon** coupling work now? There is no coupling to the $W^{3\mu}$



We require

$$-g_Y \cos\theta_W = -e \tag{53}$$

which is consistent with what we have above.

The ν_L should not couple to the photon. We get the two contributions

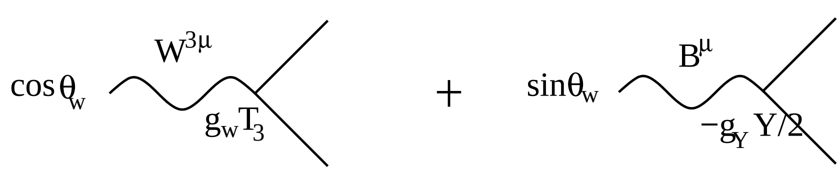
$$\frac{1}{2} g_W \sin\theta_W - \frac{1}{2} g_Y \cos\theta_W = 0 \tag{54}$$

using the relation above again. It works!

Finally we should mention the right handed neutrino which is not in the list above. It has no weak interactions and QED charge zero, so it's hypercharge is zero as well. If this particle exists we can not tell from electroweak interactions. Further the neutrinos were thought to be massless until very recently so right handed neutrinos were not included in the original "Standard Model".

4.3 Z Interactions

Each chiral fermion has a contribution to its coupling to the Z one from $W^{3\mu}$ and one from B^{μ}



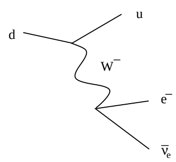
$$g_Z = g_W \cos\theta_W T_3 - g_Y \sin\theta_W \frac{Y}{2} \tag{55}$$

5 Experimental Confirmation of the Electroweak Theory

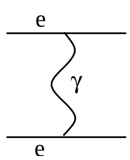
The theory described above works very well (in fact astonishing well!) and combined with QCD is called the "Standard Model" of particle physics. Let's look at a few tested predictions (there are very many).

Firstly one must fix the parameters of the theory which are g_W, g_Y and v. Three standard measurements are

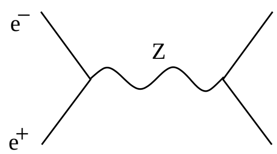
G_F determines v $\qquad\qquad G_F \sim \frac{g_W^2}{M_W^2} \sim \frac{1}{v^2}$

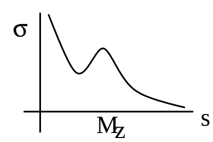


$\alpha_{EM}(q^2 = 0)$ determines e and hence one of g_W and g_Y



M_Z determines the other coupling





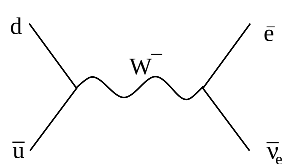
These measurements tell us

$$v = 246GeV, \qquad g_Y = 0.356, \qquad g_W = 0.668, \qquad \sin^2\theta_W = 0.231 \tag{56}$$

Now we can make some predictions

5.1 W Mass

We can look for W production in $p\bar{p}$ collisions



We find a resonant peak at about 81 GeV. We predicted that

$$\frac{M_W}{M_Z} = \frac{g_W}{\sqrt{g_W^2 + g_Y^2}} = \cos\theta_W \tag{57}$$

Current precision measurements confirm this prediction at around the 0.1% level.

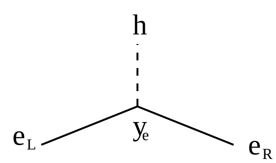
5.2 Z Width

The decay rate of the Z boson (which determines the width of the Z resonance peak in e^+e^- scattering data) is proportional to the sum of the squares of the couplings to all its possible decay products - see above for how to work out these couplings. Clearly this tests a large number of parameters of the model and also counts the number of each sort of fermion. Again experiment matches prediction at the 0.1% level.

6 Fermion Masses

So far we have worked in the ultra-relativistic limit which allowed us to treat left and right handed fermions as separate. Lets now see how the Higgs couples these particles by giving them mass.

The masses arise through interactions with the Higgs that pictorially look like



when the Higgs gets a vev there is a left right coupling

$$m_e = y_e v \tag{58}$$

There is such an interaction linking each familiar massive particle's left and right handed partners. Note that in this interaction the hypercharge of the left handed particle minus the right handed particle equals +1 for all the fermion cases. Combining in the -1 hypercharge of the Higgs we see that the interaction conserves hypercharge.

SU(2) charge is also conserved because the Higgs and left handed particle are both doublets and

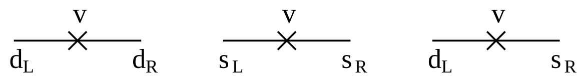
$$2 \otimes 2 = 3 \oplus 1 \tag{59}$$

In other words they produce a singlet which matches the right handed particles quantum numbers.

The couplings y_f are called "Yukawa couplings" and are free choices for each of the fermions. So we can generate all the fermion masses in this way but we are just parameterizing them not explaining them.

6.1 Cabibbo Mixing

The Higgs can generate masses between any left handed fermion and a right handed fermion such that the interaction conserves $SU(2)_L \otimes U(1)_Y$ charges. However there are a number of particles with identical quantum numbers - for example the d and s quarks are identical. In fact since these extra masses are possible they occur ie there are interactions of the form



We are left with an off-diagonal mass matrix again

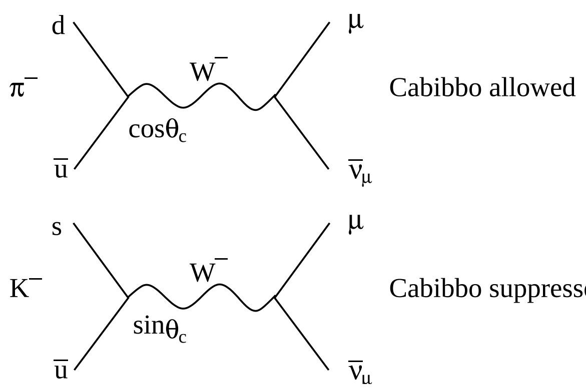
$$(d_L, s_L) \begin{pmatrix} m_{dd} & m_{ds} \\ m_{sd} & m_{ss} \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix} \tag{60}$$

Here the s and d notation is for *weak eigenstates* - ie the states that the W boson produces. Now though we should diagonalize the matrix to obtain the *mass eigenstates*. The weak eigenstates are found by making a rotation on the mass eigenstates

$$\begin{pmatrix} d^w \\ s^w \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d^m \\ s^m \end{pmatrix} \tag{61}$$

θ_c is called the Cabibbo angle and $\sin\theta_c \simeq 0.23$.

The upshot is that as well as the decay $W^+ \to u\bar{d}^m$ there is also a small chance of seeing $W^+ \to u\bar{s}^m$. We talk about Cabibbo allowed and suppressed interactions depending on whether there is a large $\cos\theta_c$ or a small $\sin\theta_c$. eg

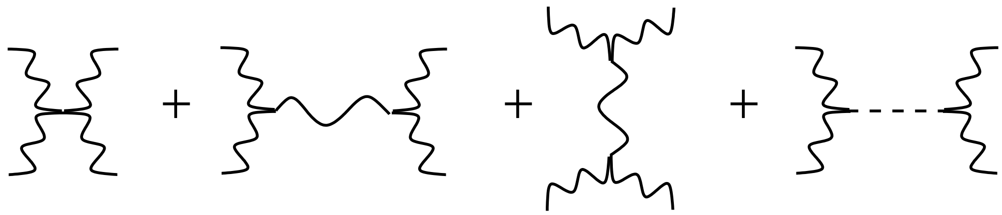


In the full model there is a 3x3 matrix called the CKM matrix that mixes in the b quark too. (Note for reasons we will not address here the charge -1/3 quark sector is the only sector of the Standard Model where there is this distinction between weak and mass eigenstates).

7 Higgs Searches

We have not to date discovered the Higgs boson but are closing in on it we believe. The Higgs potential involves the parameters μ and λ that determine the vev and m_h - we can't predict the Higgs mass therefore. If we want the Higgs to have perturbative couplings though $m_h < 1.5$ TeV.

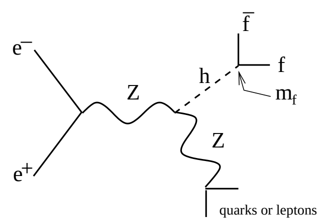
The Higgs also plays a crucial role in WW scattering. In the Standard Model there are the diagrams



If we only include the first three diagrams we find the probability of scattering rises with energy until it becomes greater than one! The fourth diagram damps this behaviour and provides a sensible answer. Again the Higgs needs to be below a TeV or so for things to make sense. We expect therefore to find something soon.

7.1 Direct Searches

The e^+e^- machine LEP was used to search for the Higgs through the process



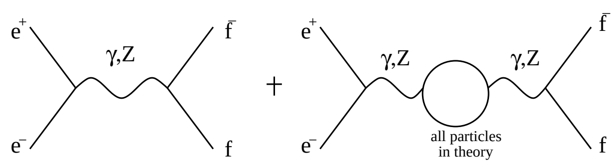
One looks for a peak in the cross section indicating one has passed the Higgs mass threshold. The main decay product will be the b quark since the higsg fermion coupling is proportional to the fermion mass. The experiment is now over and did not find the Higgs, only placing the limit

$$m_h \geq 115\ GeV \tag{62}$$

We wait on the LHC in 2008 which can search upto and above a TeV.

7.2 Indirect Searches

At loop level there are corrections to the LEP processes of the form



When we square the amplitude we get a cross term that is order α_{EW} smaller than the tree level process. In other words this is a 1% effect we can hope to probe with precision data. Such computations have been made and we find a limit

$$70\ GeV \geq m_h \leq 250\ GeV \tag{63}$$

This provides more evidence for optimism that LHC will find the Higgs.