Phenomenology course: Problem Sheet 2

1. Solve the differential equation

$$\mu^2 \frac{d\alpha_s(\mu^2)}{d\mu^2} = \beta(\alpha_s(\mu^2)) = -\beta_0 \alpha_s^2(\mu^2) + \mathcal{O}(\alpha_s^3)$$

(also called the α_s evolution equation) through first order: i.e., ignore the higher order terms $\mathcal{O}(\alpha_s^3)$. Recall that in QCD:

$$\beta_0 \equiv \frac{11N_C - 2n_f}{12\pi} = \frac{33 - 2n_f}{12\pi}.$$

Determine the integration constant by setting $\alpha_s(\mu^2 = M_Z^2) \equiv \alpha_s(M_Z^2) = 0.12$ and calculate the value of α_s at $\mu = 10$ GeV. (Use $n_f = 5$ and $M_Z = 91.1$ GeV.)

2. In the lecture, I said that F_2 and F_L depend only on x and Q^2, and that the cross section is given by the linear combination:

$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[(1 + (1-y)^2)F_2(x, Q^2) + y^2 F_L(x, Q^2)\right].$$

How would you measure F_2 and F_L separately?

3. The most general form of the DIS cross section has three non-zero structure functions, F_1, F_2 and F_3, while the e^+e^- cross section only has one (which I called B). Why the difference ?

4. Given that the +-distribution is defined by:

$$\int_0^1 \frac{f(x)}{(1-x)_+} dx = \int_0^1 \frac{f(x) - f(1)}{(1-x)} dx,$$

and

$$\frac{1}{(1-x)_+} = \frac{1}{1-x} \quad \text{for } 0 \le x < 1,$$

show that

$$\int_0^1 P_{qq}^{(0)}(x) dx = 0,$$

where

$$P_{qq}^{(0)}(x) = C_F \left[\frac{1 + x^2}{(1 - x_+)} + \frac{3}{2}\delta(1-x)\right].$$

What is the significance of this result ?