

**TTCM Assisted Genetic-Algorithm Aided  
Reduced-complexity Multiuser Detection**

**S. X. Ng, K. Yen and <sup>1</sup>L. Hanzo**

Department of Electronics and Computer Science, University of Southampton,  
Southampton SO17 1BJ, United Kingdom

<sup>1</sup>lh@ecs.soton.ac.uk, <http://www-mobile.ecs.soton.ac.uk>

**Abstract**

Turbo Trellis Coded Modulation assisted Genetic-Algorithm aided reduced-complexity Multiuser Detection (TTCM-GA-MUD) is capable of providing a considerable coding gain of 2.9 dB without any bandwidth expansion at a BER of  $10^{-4}$ , while maintaining a complexity reduction factor  $8.59 \times 10^6$  compared to the optimum multiuser detector in the context of 16QAM transmissions, while supporting  $K = 10$  users.

**Introduction:** The optimal Code Division Multiple Access (CDMA) Multiuser Detector (MUD)[1] based on the Maximum-Likelihood (ML) detection rule performs an exhaustive search of all the possible combinations of the  $K$  number of users'  $M$ -ary symbol sequences and then selects the most likely combination from the set of  $M^K$  legitimate combinations as the  $K$ -component  $M$ -ary detected symbol sequence. Since an exhaustive search is conducted, the computational complexity of the detector increases exponentially with the number of users  $K$  as well as with the number of phasors in the modulation constellation employed. Genetic Algorithms (GAs) have been used for efficiently solving combinatorial optimisation problems in numerous applications [2]. Recently, GA assisted MUD (GA-MUD) has been studied using Binary-Phase-Shift-Keying (BPSK) modulation in the context of a CDMA system [3, 4], which exhibits a substantially reduced complexity compared to the optimum MUD, while maintaining a similar performance. Turbo Trellis Coded Modulation (TTCM) [5] is a channel coding scheme, which has a structure similar to that of the family of power efficient binary turbo codes, but avoids extending the the required bandwidth by absorbing the parity bits upon extending the phasor constellation.

**TTCM-GA-MUD:** We consider a synchronous CDMA uplink system as illustrated in Figure 1, where  $K$  users simultaneously transmit data packets of 1000 TTCM encoded symbols using  $M$ -ary modulation to a MUD over AWGN channels. The TTCM scheme employed a code memory of three and four turbo decoding iterations. The received signal can be written as :  $r(t) = s(t) + n(t)$ , where  $s(t)$  is the sum of the transmitted signals of all users and  $n(t)$  is the zero-mean complex AWGN with

independent real and imaginary components, each having a double-sided power spectral density of  $\sigma^2 = \frac{N_0}{2}$ . It can be shown that the Log-Likelihood Function (LLF) for a vector  $\mathbf{b}$  is given by [1]:  $LLF(\mathbf{b}) = -\frac{1}{2\sigma^2} \int_0^{T_b} |r(t) - s(t)|^2 dt = -\frac{1}{2\sigma^2} \left\{ \int_0^{T_b} |r(t)|^2 dt - \Omega(\mathbf{b}) \right\}$ . where  $\Omega(\mathbf{b}) = 2\Re \left[ \mathbf{b}^H \mathbf{C}^* \mathbf{Z} \right] - \mathbf{b}^H \mathbf{C}^* \mathbf{R} \mathbf{C} \mathbf{b}$ ,  $\mathbf{R}$  is the  $(K \times K)$ -dimensional user signature sequence cross-correlation matrix of the  $K$  different signature sequences  $a_k(t)$ ,  $k \in \{1, \dots, K\}$ ,  $\mathbf{Z}$  is the output vector of the matched filters,  $\mathbf{b} = [b_1, \dots, b_K]^T$  is the complex  $K$ -component  $M$ -ary symbol vector of the  $K$  users, and  $\mathbf{C}$  is the complex channel impulse response vector of the links between the transmitters and receiver. More specifically, for transmission over a non-dispersive AWGN channel,  $\mathbf{C}$  is a diagonal unity matrix. The notations  $(\cdot)^H$  and  $(\cdot)^*$  represent the complex conjugate transpose and complex conjugate of the matrix  $(\cdot)$ , respectively.

The decision rule for the optimum-MUD scheme based on the ML criterion is to choose the specific  $M$ -ary symbol combination  $\mathbf{b}$  from the set of  $M^K$  possible combinations of the  $K$  number of  $M$ -ary users, which maximises the correlation metric  $\Omega(\mathbf{b})$ , yielding  $\hat{\mathbf{b}} = \arg \{ \max_{\mathbf{b}} [\Omega(\mathbf{b})] \}$ . Here, the optimum decision vector  $\hat{\mathbf{b}}$  represents the *hard decision* values for a specific  $K$ -symbol combination of the  $K$  users during an  $M$ -ary symbol period. Based on the *hard decision* vector component  $\hat{b}_k$  of vector  $\hat{\mathbf{b}}$ , we derived the log-likelihood channel metrics for the  $k^{th}$  user's TTCM decoder for all the  $M$  possible  $M$ -ary modulated symbols as:  $P_{k,m}(\hat{b}_k | b_{k,m}) = -\frac{|\hat{b}_k - b_{k,m}|^2}{2\sigma^2}$ , where  $b_{k,m}$  is the  $m^{th}$  phasor in the constellation space and  $m \in \{0, \dots, M-1\}$ . Note that it is possible to obtain the *soft decision* metrics for the optimum-MUD, although its employment imposes a higher complexity. Specifically, given  $r(t)$  and all possible  $LLF(\mathbf{b})$  values, we derived the *soft decision* metrics as:  $P_{k,m}(r(t) | b_{k,m}) = \ln \left\{ \sum_{\substack{\text{(all possible } \mathbf{b}) \\ \text{(if } b_{k,m} = b_k)}} \exp(LLF(\mathbf{b})) \right\}$ .

Our proposed reduced-complexity TTCM-GA-MUD scheme employed the *hard decision* metric and performed only a partial search, rather than full search, using the objective function of  $\exp(\Omega(\mathbf{b}))$ . The configuration of the GA employed in our system is shown in Table 1. For a detailed description of the GA-MUD, the readers are referred to [3]. In general, the detection complexity of the GA-MUD is governed by the required number of GA generations  $Y$  and populations  $P$ , which ensure a reliable decision. The computational complexity of the GA, quantified in the context of the total number of objective function evaluations is related to  $P \times Y$ .

**Simulation results:** We found -although not explicitly shown here due to lack of space- that for detecting  $K$  users each employing  $M$ -ary modulation, the rule of thumb quantifying the required complexity of the TTCM-GA-MUD scheme is given by  $P \times Y \approx \frac{1}{2} K^2 M^2$ , where  $P \approx KM$  and

$Y \approx \frac{1}{2}KM$ , which attains a performance similar to that of the optimum MUD. **Hence, the computational complexity reductions obtained by the TTCM-GA-MUD compared to that of the optimum MUD can be construed to be about  $F = \frac{M^K}{\frac{1}{2}K^2M^2} = 2M^{K-2}K^{-2}$ .**

For the specific example of  $K = 10$  and  $M = 16$ , the complexity reduction factor  $F$  was  $8.59 \times 10^6$ .

In Figure 2, we show the Bit Error Ratio (BER) versus signal to noise ratio per bit, namely  $E_b/N_0$ , performance of the TTCM-GA-MUD and that of the TTCM-optimum-MUD schemes, when communicating over AWGN channels using both Quadrature-PSK (QPSK) and 16-level Quadrature Amplitude Modulation (16QAM) TTCM. It is shown in Figure 2 that the optimum-MUD exhibits an approximately 0.3 dB performance loss both in conjunction with *hard-* and *soft decisions*, when  $K$  increased from one to four. At  $K = 10$ , the performance loss of TTCM-GA-MUD is about 1 and 3 dB, as shown in Figure 2 compared to the TTCM-assisted single user schemes employing *hard-* and *soft decisions*, respectively, regardless whether QPSK or 16QAM modulation schemes were used.

**Conclusions:** In this contribution, we proposed a reduced-complexity TTCM-GA-MUD scheme, which is capable of maintaining a performance similar to that of the optimum-MUD. For  $K = 10$  users and a BER of  $10^{-4}$ , the TTCM-GA-MUD aided QPSK and 16QAM systems achieved an SNR-reduction of 3.8 and 2.9 dBs at the effective throughputs of 1 and 3 bit/symbol in comparison to the single user bounds of uncoded BPSK and uncoded 8-level PSK (8PSK), respectively, without extending the required bandwidth. The complexity reduction factors of  $F = 1.31 \times 10^3$  and  $F = 8.59 \times 10^6$  were achieved for  $M = 4$  and  $M = 16$ , respectively, when supporting  $K = 10$  users,

## References

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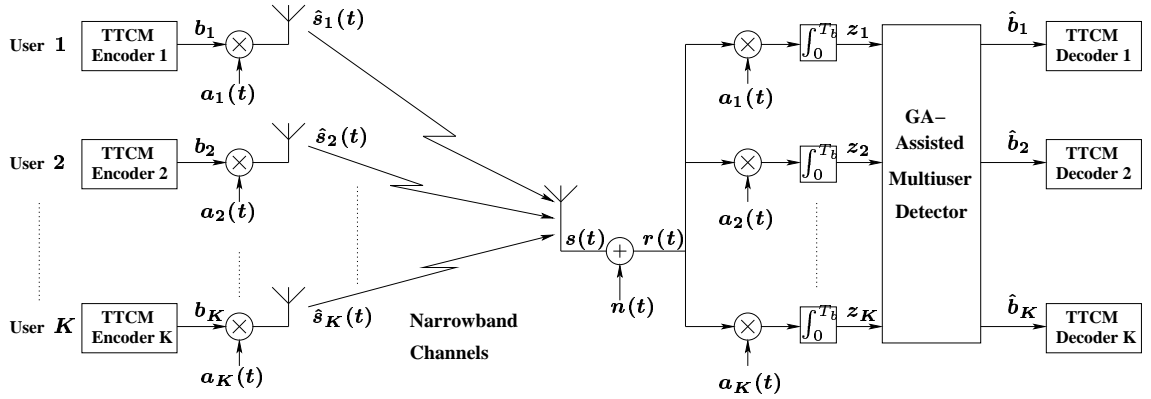


Figure 1: Block diagram of the  $K$ -user synchronous CDMA uplink model.

Table 1: The configuration of the TTCM-GA-MUD employed in our system.

Setup/Parameter	Method/Value
Selection method	Fitness-proportionate
Crossover operation	Uniform crossover
Mutation operation	Standard binary mutation
Elitism, Incest Prevention	Yes
Population size $P$	$KM$
Mating pool size $T$	$T \leq P$
Probability of mutation $p_m$	0.1
Termination generation $Y$	$\frac{1}{2}KM$
M-ary modulation $M$	4(QPSK), 16(16QAM)
Spreading factor	31 chips
TTCM interleaver length	1000 symbols

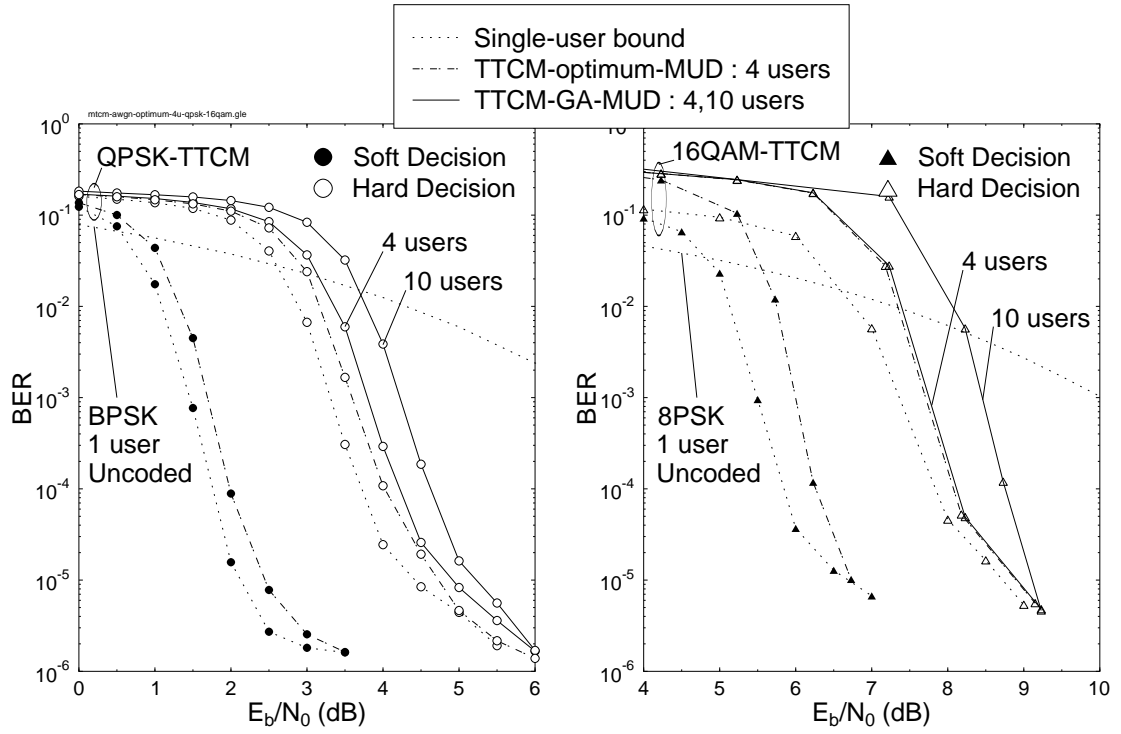


Figure 2: BER versus  $E_b/N_0$  performance of the TTCM-GA-MUD and TTCM-optimium-MUD schemes, when communicating over non-dispersive AWGN channels; left: effective throughput of 1 bit/symbol; right: effective throughput of 3 bit/symbol.