Coded Modulation Assisted Genetic Algorithm Based Multiuser Detection for CDMA Systems

S.X. Ng, K. Yen and [1]L. Hanzo

Dept. of ECS, University of Southampton, SO17 1BJ, UK.
Tel: +44-23-8059 3125, Fax: +44-23-8059 4508
Email: [1]lh@ecs.soton.ac.uk, http://www-mobile.ecs.soton.ac.uk

Abstract— **In this contribution we propose a novel Coded Modulation assisted Genetic Algorithm based Multiuser Detection (CM-GA-MUD) scheme for synchronous CDMA systems. The performance of the proposed scheme was investigated using Quadrature-Phase-Shift-Keying (QPSK), when communicating over AWGN and narrowband Rayleigh fading channels. When compared with the optimum MUD scheme, the GA-MUD subsystem is capable of reducing the computational complexity significantly. On the other hand, the CM subsystem is capable of obtaining considerable coding gains despite being fed with sub-optimal information provided by the GA-MUD output.**

I. Introduction

The optimal CDMA Multiuser Detector (MUD) [1] based on the Maximum-Likelihood (ML) detection rule performs an exhaustive search of all the possible combinations of the users' transmitted bit or symbol sequences and then selects the most likely combination as the detected bit or symbol sequence. Since an exhaustive search is conducted, the computational complexity of the detector increases exponentially with the number of users as well as with the number of levels in the modulation scheme employed. Since a CDMA system is required to support a large number of users, the optimum ML multiuser detector is impractical to implement due to its excessive complexity. This complexity constraint led to numerous so-called suboptimal multiuser detection [2] proposals.

Genetic Algorithms (GAs) have been used for efficiently solving combinatorial optimisation problems in many applications [3]. Recently, GA assisted Multiuser Detector (MUD) has been studied using Binary-Phase-Shift-Keying (BPSK) modulation in the context of a CDMA system [4]. In an afford to increase the system's performance with the aid of channel coding, but without increasing the required bandwidth, in this contribution we will investigate the performance of the Coded Modulation (CM) assisted Genetic Algorithm Based Multiuser Detection (CM-GA-MUD) using QPSK modulation.

Trellis Coded Modulation (TCM) [5, 6], which is based on combining the functions of coding and modulation, is a bandwidth efficient scheme that has been widely recognised as an excellent error control technique suitable for applications in mobile communications. Turbo Trellis Coded Modulation (TTCM) [6, 7] is a more recent channel coding scheme, which has a structure similar to that of the family of power efficient binary turbo codes [6, 8], but employs TCM codes as component codes. In our TCM and TTCM schemes, random symbol interleavers were utilised for both the turbo interleaver and the channel interleaver. Another powerful Coded Modulation (CM) scheme utilising bit-based channel interleaving in conjunction with Gray signal labelling, which is referred to as Bit-Interleaved Coded Modulation (BICM), was proposed in [6, 9]. It combines conventional non-systematic convolutional codes with several independent bit interleavers. The number of parallel bit-interleavers used equals the number of coded bits in a symbol [9]. Recently, iteratively decoded BICM using Set Partitioning (SP) based signal labelling, referred to as BICM-ID has also been proposed [6, 10].

The rest of this treatise is organised as follows. Our system overview is presented in Section II, the GA algorithm is explained in Section II-A and the CM principle is summarised in Section II-B. Our simulation parameters and results are discussed in Section III and finally our conclusions are offered in Section IV.

II. System Overview

Each user invokes a CM encoder, which provides a block of M QPSK modulated symbols before spreading. We consider a synchronous CDMA uplink as illustrated in Figure 1, where K users simultaneously transmit data packets of equal length using QPSK modulation to a single receiver. The transmitted signal of the kth user can be expressed in an equivalent lowpass representation as :

$$\hat{s}_k(t) = \sqrt{\xi_k} \sum_{m=0}^{M-1} b_k^{(m)} a_k(t - mT_b), \quad \forall k = 1, \ldots, K \qquad (1)$$

where ξ_k is the kth user's signal energy per transmitted symbol, $b_k^{(m)} = e^{j\theta_k}$; $\theta_k \in \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\}$ denotes the mth CM-coded data symbol of the kth user for the QPSK signals of $\{0(00), 1(01), 2(10), 3(11)\}$, $a_k(t)$ is the kth user's signature sequence, T_b is the data symbol duration and M is the number of data symbols transmitted in a packet. The superscript (m) can be omitted, since no dispersion-induced interference is inflicted by symbols outside a single symbol duration T_b in narrowband channel.

Each user's signal $\hat{s}_k(t)$ is assumed to propagate over a narrowband slowly Rayleigh fading channel, as shown in Figure 1 and the fading envelope of each path is statistically independent for all users. The complex lowpass channel impulse response (CIR) for the link between the kth user's transmitter and the base station's receiver, as shown in Figure 1, can be written as :

$$h_k(t) = \alpha_k(t) e^{j\phi_k(t)} \delta(t), \quad \forall k = 1, \ldots, K \qquad (2)$$

where the amplitude $\alpha_k(t)$ is a Rayleigh distributed random variable and the phase $\phi_k(t)$ is uniformly distributed between $[0, 2\pi)$.

Hence, when the kth user's spread spectrum signal $\hat{s}_k(t)$ given by Equation 1 propagates through a slowly Rayleigh fading channel having an impulse response given by Equation 2, the resulting output signal $s_k(t)$ defined over a single symbol duration can be written as :

$$s_k(t) = \sqrt{\xi_k} \alpha_k b_k a_k(t) e^{j\phi_k}. \quad \forall k = 1, \ldots, K \qquad (3)$$

Upon combining Equation 3 for all K users, the received signal at the receiver, which is denoted by $r(t)$ in Figure 1, can be

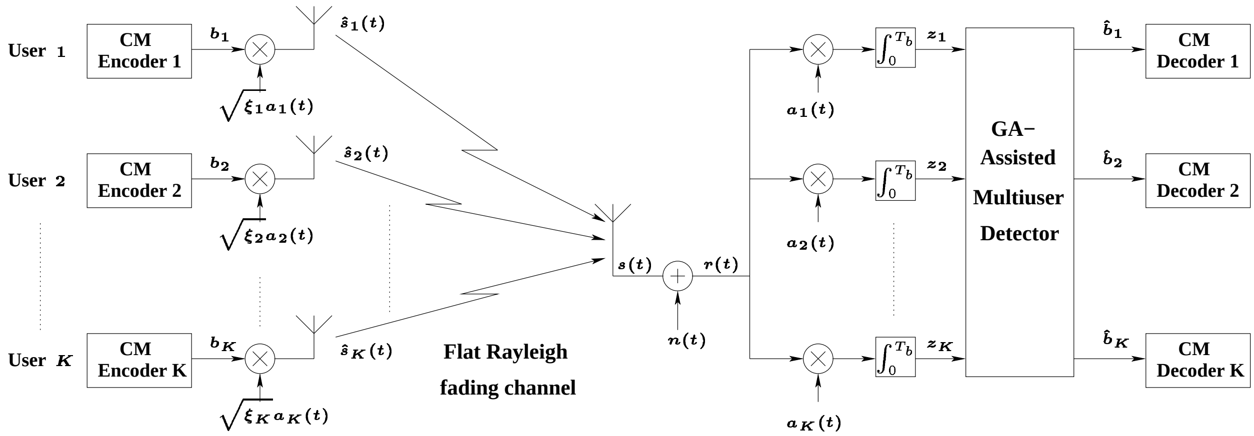

Fig. 1. Block diagram of the K-user synchronous CDMA uplink model in a flat Rayleigh fading channel.

written as :

$$r(t) = \sum_{k=1}^{K} s_k(t) + n(t), \tag{4}$$

where $n(t)$ is the zero-mean complex Additive White Gaussian Noise (AWGN) with independent real and imaginary components, each having a double-sided power spectral density of $\sigma^2 = N_0/2$ W/Hz.

Invoking Equation 3 describing the transmitted signal of each user, the sum of the transmitted signals of all users can be expressed in vectorial notation as :

$$\begin{aligned} s(t) &= \sum_{k=1}^{K} s_k(t) \\ &= \boldsymbol{aC\xi b}, \end{aligned} \tag{5}$$

where

$$\begin{aligned} \boldsymbol{a} &= [a_1(t), \ldots, a_K(t)] \\ \boldsymbol{C} &= \text{diag}\left[\alpha_1 e^{j\phi_1}, \ldots, \alpha_K e^{j\phi_K}\right] \\ \boldsymbol{\xi} &= \text{diag}\left[\sqrt{\xi_1}, \ldots, \sqrt{\xi_K}\right] \\ \boldsymbol{b} &= [b_1, \ldots, b_K]^T. \end{aligned} \tag{6}$$

Hence the received signal of Equation 4 can be written as :

$$r(t) = s(t) + n(t). \tag{7}$$

Based on Equations 5 and 7, the output vector $\boldsymbol{Z}$ of the bank of matched filters portrayed in Figure 1 can be formulated as :

$$\begin{aligned} \boldsymbol{Z} &= [z_1, \ldots, z_K]^T \\ &= \boldsymbol{RC\xi b} + \boldsymbol{n}, \end{aligned} \tag{8}$$

where R is the $K \times K$ dimensional user signature sequence cross-correlation matrix and

$$\boldsymbol{n} = [n_1, \ldots, n_K]^T$$

is a zero-mean Gaussian noise vector having a covariance matrix $\boldsymbol{R}_n = 0.5N_0\boldsymbol{R}$. Based on this discrete-time model, we will next derive the optimum multiuser detector based on the maximum likelihood criterion for the synchronous CDMA system considered [1].

The joint optimum decision rule for the QPSK-modulated K-user CDMA system based on the synchronous system model can be derived from that of the BPSK-modulated system [11], which is expressed in vectorial notation as:

$$\Omega(\boldsymbol{b}) = 2\Re\left[\boldsymbol{b}^H\boldsymbol{\xi}\boldsymbol{C}^*\boldsymbol{Z}\right] - \boldsymbol{b}^H\boldsymbol{\xi}\boldsymbol{C}^*\boldsymbol{RC\xi b}, \tag{9}$$

where $(.)^H$ is the complex conjugate transpose of the matrix (.) and $(.)^*$ is the complex conjugate of the matrix (.). More specifically, for BPSK modulation the term $\boldsymbol{b}^H$ in Equation 9 is substituted by $\boldsymbol{b}^T$, which is the transpose of the matrix $\boldsymbol{b}$, since only the real component is considered in the context of BPSK modulation.

The decision rule for the optimum CDMA multiuser detection scheme based on the maximum likelihood criterion is to choose the specific symbol combination $\boldsymbol{b}$, which maximises the correlation metric of Equation 9, yielding:

$$\hat{\boldsymbol{b}} = \arg\left\{\max_{\boldsymbol{b}}\left[\Omega(\boldsymbol{b})\right]\right\}. \tag{10}$$

Finally, based on the decision vector $\hat{\text{b}}$ output by the GA-MUD, the CM decoder of user k will be invoked for generating the final estimate of the information of user k.

The maximisation of Equation 9 is a combinatorial optimisation problem. Specifically, Equation 9 has to be evaluated for each of the 2^{2K} possible combinations of the QPSK modulated symbols of the K users, in order to find the vector $\boldsymbol{b}$ that maximises the correlation metric of Equation 9. Explicitly, since there are 2^{2K} different possible QPSK vectors $\boldsymbol{b}$, the optimum multiuser detection has a complexity that increases exponentially with the number of users K.

A. The GA-assisted Multiuser Detector Subsystem

The flowchart depicting the structure of the genetic algorithm adopted for our GA-assisted multiuser detection technique is shown in Figure 2. Firstly, an initial population consisting of P number of so-called *individuals* is created in the 'Initialisation' block, where P is known as the population size. Each individual represents a legitimate K-dimensional vector of QPSK symbols constituting the solution of the given optimisation problem. In other words, an individual can be considered as a K-dimensional vector consisting of the QPSK decision variables to be optimised.

In order to aid our GA-assisted search at the beginning, we employed the hard decisions offered by the matched filter outputs $\boldsymbol{Z}$ of Equation 8, which were denoted as:

$$\hat{\boldsymbol{b}}_{MF} = \left[\hat{b}_{1,MF}, \hat{b}_{2,MF}, \ldots, \hat{b}_{K,MF}\right], \tag{11}$$

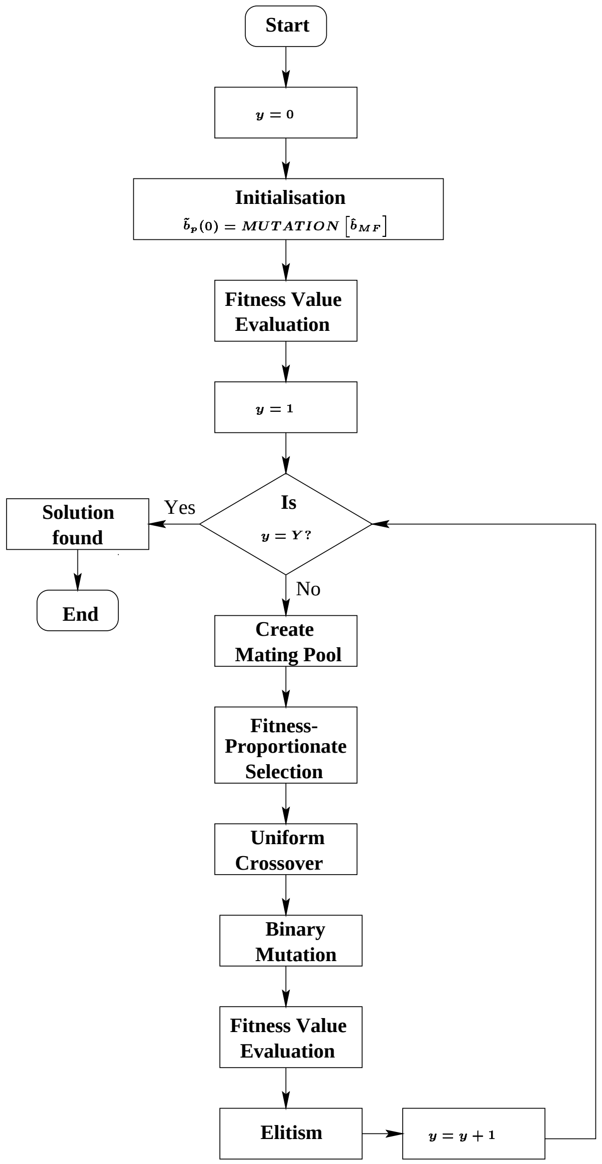

Fig. 2. A flowchart depicting the structure of the genetic algorithm adopted for our GA-assisted multiuser detection technique.

where $\hat{b}_{l,MF}$ for $l = 1, \ldots, K$ is given by:

$$\hat{b}_{l,MF} = arg\left\{\min_{b}\left|z_l - \alpha_l e^{j\phi_l}\sqrt{\xi_l}b\right|\right\}. \quad (12)$$

In Equation 12 where the multiplication by $\alpha_l e^{j\phi_l}$ is necessary for coherent detection, because the phase rotation introduced by the channel has to be taken into account.

A different randomly 'mutated' version [4, 12] of the hard decision vector $\hat{\boldsymbol{b}}_{MF}$ of Equation 11 was assigned to each of the individuals in the initial population, where the same probability of mutation, namely p_m was adopted for all individuals. Note that we cannot assign the same hard decision vector $\hat{\boldsymbol{b}}_{MF}$ to all the individuals, since the process of incest prevention [13] is invoked, which will not allow identical individuals to mate.

The so-called fitness value [3] associated with each individual in the population is evaluated by substituting the candidate solution represented by the individual under consideration into the objective function, as indicated by the 'Evaluation' block of Figure 2. Individuals having the T number of highest fitness values are then placed in a so-called *mating pool* [3, 4] where $2 \leq T \leq P$. Using a kind of natural selection scheme [3] together with the genetically-inspired operators of *crossover* [12] and *mutation* [12], the individuals in the mating pool are then evolved to a new population.

Our objective function, or synonymously, fitness value is defined by the correlation metric of Equation 9. Here, the legitimate solutions are the 2^{2K} possible combinations of the K-symbol vector $\boldsymbol{b}$, where there are 2 bits in each of the QPSK symbols. Hence, each individual will take the form of a K-symbol vector corresponding to the K users' QPSK symbols during a single symbol interval. We will denote the pth individual here as $\tilde{\boldsymbol{b}}_p(y) = \left[\tilde{b}_{p,1}(y), \ldots, \tilde{b}_{p,K}(y)\right]$, where y denotes the yth generation. Our goal is to find the specific individual that corresponds to the highest fitness value in the sense of Equation9. In order to ensure that the fitness values are positive for all combinations of $\boldsymbol{b}$ for the so-called *fitness-proportionate* selection scheme [3], we modify the correlation metric of Equation 9 according to [14]:

$$\exp\left\{\Omega\left(\boldsymbol{b}\right)\right\} = \exp\left\{2\Re\left[\boldsymbol{b}^H\boldsymbol{\xi}\boldsymbol{C}^*\boldsymbol{Z}\right] - \boldsymbol{b}^H\boldsymbol{\xi}\boldsymbol{C}^*\boldsymbol{R}\boldsymbol{C}\boldsymbol{\xi}\boldsymbol{b}\right\}. \quad (13)$$

The associated probability of fitness-proportionate selection selection p_i of the ith individual is defined as [3]:

$$p_i = \frac{f_i}{\sum_j^T f_j}, \quad (14)$$

where f_i is the fitness value associated with the ith individual. Once a pair of parents is selected, the crossover and mutation operations are then applied to this pair of parents.

The *crossover* [12] operation is a process in which arbitrary decision variables are exchanged between a pair of selected parents, 'mimicking the biological recombination process between two single-chromosome organisms'. Hence, the crossover operation creates two new individuals, known as *offspring* in GA parlance [3], which have a high probability of having better fitness values than their parents. In order to generate P number of new offspring, $P/2$ number of crossover operations are required. A new pair of parents is selected from the mating pool for each crossover operation. The newly created offspring will form the basis of the new population. During the *mutation* operation [12], each decision variable in the offspring is perturbed, i.e. corrupted, with a probability of p_m, by either a predetermined or a random value. This allows new areas in the search space to be explored. The mutation probability of a decision variable is usually low, in the region of 0.1-0.01 [3]. This value is often reduced throughout the search, when the optimisation is likely to approach the final solution. In this contribution, *uniform crossover* [4] and *binary mutation* [12] were employed.

In order to ensure that high-merit individuals are not lost from one generation to the next, the best or a few of the best individuals are copied into the forthcoming generation, replacing the worst offspring of the new population. This technique is known as *elitism* [12]. In our application, we will terminate the GA-assisted search at the Yth generation and the individual associated with the highest fitness value at this point will be the detected solution. The configuration of the GA employed in our system is shown in Table I.

B. The Coded Modulation Subsystem

Due to the lack of space, here we specify only the generator polynomials of the CM schemes used in this section. For a detailed description of the various CM schemes the interested readers are referred to the literature [6]. Specifically, [5,6,15,16] are recommended for TCM, TTCM is discussed in [6,7], BICM is considered in [6,9,17] and BICM-ID in [6,10,17,18]

TABLE I

THE CONFIGURATION OF THE GA EMPLOYED IN OUR SYSTEM.

Setup/Parameter	**Method/Value**
Individual initialisation method	Mutation of $\hat{\boldsymbol{b}}_{MF}$ of Equation 11
Selection method	Fitness-proportionate
Crossover operation	Uniform crossover
Mutation operation	Standard binary mutation
Elitism	Yes
Incest Prevention	Yes
Population size P	40
Mating pool size T	$T \leq P$ depends on the no. of non-identical individuals
Probability of mutation p_m	0.1
Termination generation Y	20

TABLE II

THE GENERATOR POLYNOMIAL, H^i, OF THE TCM AND TTCM CONSTITUENT CODES IN OCTAL FORMAT.

Code Rate	Coding	State	H^0	H^1
1/2	TTCM	8	13	06
(QPSK)	TCM	64	117	26

Table II shows the generator polynomials of the TCM and TTCM codes, which are presented in octal format. These are Recursive Systematic Convolutional (RSC) codes and the encoder attaches only one parity bit to the information bits. More specifically, in the context of QPSK modulation the number of useful information Bits Per Symbol (BPS) is 1 and the coding rate is $R = \frac{1}{2}$. Table III shows the BICM and BICM-ID codes' generator polynomials in octal format, which were obtained from page 331 of [19]. These are non-systematic convolutional codes, which also produce one parity bit only. Hence, the code rates of these codes are similar to those of the TCM and TTCM codes, seen in Table II.

TABLE III

THE GENERATOR POLYNOMIAL, g^i, OF THE CONVOLUTIONAL CODES EMPLOYED IN THE BICM ENCODER IN OCTAL FORMAT.

Code Rate	Coding	State	g^0	g^1
1/2	BICM-ID	16	23	35
(QPSK)	BICM	64	113	171

Soft decision trellis decoding utilising the Log-Maximum A Posteriori (Log-MAP) algorithm [20] was invoked for decoding. The Log-MAP algorithm is a numerically stable version of the MAP algorithm operating in the log-domain, in order to reduce its complexity and to mitigate the numerical problems associated with the MAP algorithm [21].

The complexity of the CM schemes is compared in terms of the number of decoding states and the number of decoding iterations. For a TCM or BICM code of memory M, the corresponding complexity is proportional to the number of decoding states $S = 2^M$. Since TTCM schemes invoke two component TCM codes, a TTCM code employing t iterations and using an S-state component code exhibits a complexity proportional to $2 \cdot t \cdot S$. As for BICM-ID schemes, only one decoder is used, but the demodulator is invoked in each decoding iteration. However, the complexity of the demodulator is assumed to be insignificant compared to that of the CM decoder. Hence, a BICM-ID code invoking t iterations using an S-state code exhibits a complexity proportional to $t \cdot S$. The codes shown in Tables II and III exhibit similar complexity, where both TTCM and BICM-ID utilise four decoding iterations.

III. Simulation Results And Discussions

Our performance metric is the average Bit Error Ratio (BER) evaluated over the course of several GA generations. The detection time of the GA is governed by the number of generations Y required, in order to obtain a reliable decision. The computational complexity of the GA, quantified in the context of the total number of objective function evaluations, is related to $P \times Y$. Since our GA-assisted multiuser detector is based on optimising the modified correlation metric of Equation 13, the computational complexity is deemed to be acceptable, if there is a significant amount of reduction in comparison to the optimum multiuser detector, which requires 2^{2K} objective function evaluations, in order to reach the optimum decision.

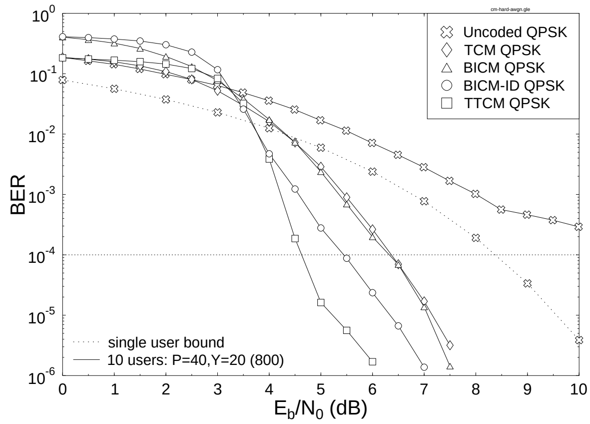

Fig. 3. BER versus E_b/N_0 performance of the various CM-GA-MUD schemes for transmissions over the AWGN channels utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

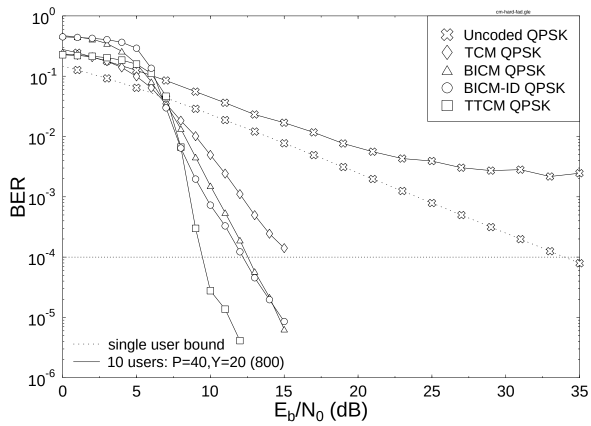

Fig. 4. BER versus E_b/N_0 performance of the various CM-GA-MUD schemes for transmissions over the uncorrelated Rayleigh fading channels utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

The BER versus Signal to Noise Ratio (SNR) per bit, namely E_b/N_0, performance of the CM-GA-MUD schemes is shown in Figures 3 and 4 for transmissions over AWGN channels and uncorrelated Rayleigh fading channels, respectively. The simulation parameters were summarised in Table I, II and III. A 'codeword length' of 1000 symbols and a spreading factor of 31 chips were employed. As determined by the 'codeword length', the turbo interleaver of TTCM and the internal bit interleavers of BICM and BICM-ID had a memory of 1000 symbol duration.

The employment of an uncorrelated Rayleigh fading channel implies ideal channel interleaving, which has an infinitely long interleaver depth.

It is widely recognised that a QPSK signal consists of two orthogonal BPSK signals in a single user scenario and that the associated BERs of BPSK and QPSK are identical in terms of E_b/N_0. Hence the single user bounds for QPSK modulation shown in Figure 3 for AWGN channels and Figure 4 for uncorrelated Rayleigh fading channels, are identical to that of the BPSK modulation. However, the orthogonality of the in-phase and quadrature-phase BPSK signals is corrupted by the MAI when a QPSK signal is transmitted in a CDMA system. Hence the BER of QPSK signal is not identical to that of BPSK signals in the context of a MAI-limited CDMA environment. Therefore, the uncoded QPSK performance of a $K = 10$-user CDMA system is worse than that of the single user bounds illustrated in Figures 3 and 4.

Note that the computational complexity of the GA-MUD is $\frac{2^{2K}}{P \times Y} = 1310.72$ times lower, than that of the optimum MUD, when supporting $K = 10$ users employing QPSK modulation in this study. The penalty for this complexity reduction is the BER error floor experienced by the GA-MUD schemes at high SNRs, as shown in the Figures 3 and 4. However, this disadvantage is eliminated, when the CM schemes are utilised. In particular, the TTCM assisted GA-MUD constitutes the best candidate, followed by the BICM-ID assisted GA-MUD, as evidenced in Figures 3 and 4 for transmissions over the AWGN and uncorrelated Rayleigh fading channels encountered. More specifically, for a throughput of 1 BPS and a target BER of 10^{-4}, the $K = 10$-user TTCM-GA-MUD assisted CDMA system is capable of providing SNR gains of about 4 and 25 dBs in AWGN and perfectly interleaved narrowband Rayleigh fading channels, respectively, against the single-user bounds of the uncoded BPSK scheme.

IV. Conclusion

In this contribution, TCM, TTCM, BICM and BICM-ID assisted GA-based MUD schemes were proposed and evaluated in performance terms when communicating over the AWGN and narrowband Rayleigh fading channels encountered. It was shown that the GA-MUD is capable of significantly reducing the computational complexity of the optimum-MUD, but experiences an error floor at high SNRs due to invoking an insufficiently large population size and a low number of generations. However, with the advent of the bandwidth efficient CM schemes proposed, this problem is eliminated. When comparing the four CM schemes at the same decoding complexity, TTCM was found to be the best candidate for assisting the operation of the GA-MUD system.

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