M-ary Coded Modulation Assisted Genetic Algorithm Based Multiuser Detection for CDMA Systems

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Abstract—**In this contribution we propose a novel M-ary Coded Modulation assisted Genetic Algorithm based Multiuser Detection (CM-GA-MUD) scheme for synchronous CDMA systems. The performance of the proposed scheme was investigated using Quadrature-Phase-Shift-Keying (QPSK), 8-level PSK (8PSK) and 16-level Quadrature Amplitude Modulation (16QAM) when communicating over AWGN and narrowband Rayleigh fading channels. When compared with the optimum MUD scheme, the GA-MUD subsystem is capable of reducing the computational complexity significantly. On the other hand, the CM subsystem is capable of obtaining considerable coding gains despite being fed with sub-optimal information provided by the GA-MUD output.**

I. Introduction

The optimal CDMA Multiuser Detector (MUD) [1] based on the Maximum-Likelihood (ML) detection rule performs an exhaustive search of all the possible combinations of the users' transmitted bit or symbol sequences and then selects the most likely combination as the detected bit or symbol sequence. Since an exhaustive search is conducted, the computational complexity of the detector increases exponentially with the number of users as well as with the number of levels in the modulation scheme employed. Since a CDMA system is required to support a large number of users, the optimum ML multiuser detector is impractical to implement due to its excessive complexity. This complexity constraint led to numerous so-called suboptimal multiuser detection [2] proposals.

Genetic Algorithms (GAs) have been used for efficiently solving combinatorial optimisation problems in many applications [3]. Recently, GA assisted Multiuser Detector (MUD) has been studied using Binary-Phase-Shift-Keying (BPSK) modulation in the context of a CDMA system [4–6]. In an afford to increase the system's performance with the aid of channel coding, but without increasing the required bandwidth, in this contribution we will investigate the performance of the Coded Modulation (CM) assisted Genetic Algorithm Based Multiuser Detection (CM-GA-MUD) using M-ary modulation modes. More specifically, Quadrature-Phase-Shift-Keying (QPSK), 8-level PSK (8PSK) and 16-level Quadrature Amplitude Modulation (16QAM) were employed.

Trellis Coded Modulation (TCM) [7, 8], which is based on combining the functions of coding and modulation, is a bandwidth efficient scheme that has been widely recognised as an excellent error control technique suitable for applications in mobile communications. Turbo Trellis Coded Modulation (TTCM) [8, 9] is a more recent channel coding scheme, which has a structure similar to that of the family of power efficient binary turbo codes [8, 10], but employs TCM codes as component codes. In our TCM and TTCM schemes, random symbol interleavers were utilised for both the turbo interleaver and the channel interleaver. Another powerful Coded Modulation (CM) scheme utilising bit-based channel interleaving in conjunction with Gray signal labelling, which is referred to as Bit-Interleaved Coded Modulation (BICM), was proposed in [8, 11]. It combines conventional non-systematic convolutional codes with several independent bit interleavers. The number of parallel bit-interleavers used equals the number of coded bits in a symbol [11]. Recently, iteratively decoded BICM using Set Partitioning (SP) based signal labelling, referred to as BICM-ID has also been proposed [8, 12].

The rest of this treatise is organised as follows. Our system overview is presented in Section II, the GA algorithm is explained in Section II-A and the CM principle is summarised in Section II-B. Our simulation parameters and results are discussed in Section III and finally our conclusions are offered in Section IV.

II. System Overview

In our study, each user invokes a CM encoder, which provides a block of N modulated symbols before spreading. We consider a synchronous CDMA uplink as illustrated in Figure 1, where K users simultaneously transmit data packets of equal length using M-ary modulation to a single receiver. The transmitted signal of the k^{th} user can be expressed in an equivalent lowpass representation as :

$$\hat{s}_k(t) = \sum_{n=0}^{N-1} b_k^n a_k(t - nT_b), \quad \forall k = 1, \ldots, K \tag{1}$$

where $a_k(t)$ is the k^{th} user's signature sequence, T_b is the data symbol duration, N is the number of data symbols transmitted in a packet and $b_k^n = \sqrt{\xi_k^n} e^{j\theta_k^n}$ represents the n^{th} coded-modulated M-ary symbol of the k^{th} user, where ξ_k^n and θ_k^n are the k^{th} user's signal energy and phase of the n^{th} transmitted symbol, respectively. More explicitly, b_k^n denotes a complex CM-coded symbol in the range of $0, 1, \ldots, M-1$, where M is the number of possible constellation points in the M-ary modulation, which is equals to 4, 8 and 16 for QPSK, 8PSK and 16QAM. The superscript n can be omitted, since no dispersion-induced interference is inflicted by symbols outside a single symbol duration T_b in narrowband channel.

Each user's signal $\hat{s}_k(t)$ is assumed to propagate over a narrowband slowly Rayleigh fading channel, as shown in Figure 1 and the fading envelope of each path is statistically independent for all users. The complex lowpass channel impulse response (CIR) for the link between the kth user's transmitter and the base station's receiver, as shown in Figure 1, can be written as :

$$h_k(t) = \alpha_k(t) e^{j\phi_k(t)} \delta(t), \quad \forall k = 1, \ldots, K \tag{2}$$

where the amplitude $\alpha_k(t)$ is a Rayleigh distributed random variable and the phase $\phi_k(t)$ is uniformly distributed between $[0, 2\pi)$.

The joint optimum decision rule for the M-ary modulated K-user CDMA system based on the synchronous system model

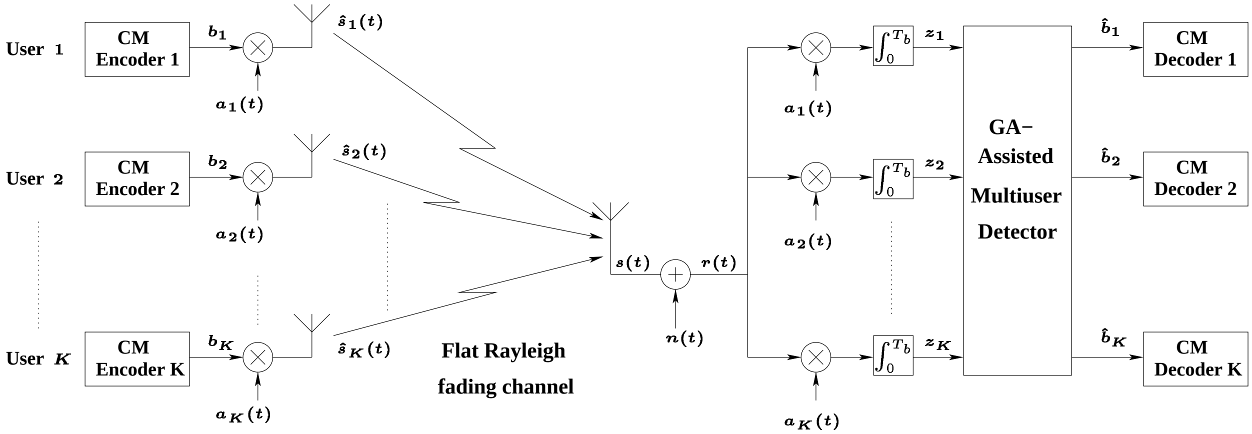

Fig. 1. Block diagram of the K-user synchronous CDMA uplink model in a flat Rayleigh fading channel.

can be derived from that of the BPSK-modulated system [13], which is expressed in vectorial notation as:

$$\Omega(\boldsymbol{b}) = 2\Re\left[\boldsymbol{b}^H \boldsymbol{C}^* \boldsymbol{Z}\right] - \boldsymbol{b}^H \boldsymbol{C}^* \boldsymbol{R} \boldsymbol{C} \boldsymbol{b}, \tag{3}$$

where

$$\begin{aligned} \boldsymbol{C} &= \text{diag}\left[\alpha_1 e^{j\phi_1}, \ldots, \alpha_K e^{j\phi_K}\right] \\ \boldsymbol{b} &= [b_1, \ldots, b_K]^T, \\ \boldsymbol{Z} &= \textit{ouput vector of the matched filters.} \end{aligned}$$

More specifically, $(.)^H$ is the complex conjugate transpose of the matrix (.) and $(.)^*$ is the complex conjugate of the matrix (.). For BPSK modulation the term $\boldsymbol{b}^H$ in Equation 3 is substituted by $\boldsymbol{b}^T$, which is the transpose of the matrix $\boldsymbol{b}$, since only the real component is considered in the context of BPSK modulation.

The decision rule for the optimum CDMA multiuser detection scheme based on the maximum likelihood criterion is to choose the specific symbol combination $\boldsymbol{b}$, which maximises the correlation metric of Equation 3, yielding:

$$\hat{\boldsymbol{b}} = \arg\left\{\max_{\boldsymbol{b}} \left[\Omega(\boldsymbol{b})\right]\right\}. \tag{4}$$

Here, the optimum decision vector $\hat{\boldsymbol{b}}$ represents the *hard decision* values for a specific K-symbol combination of the K users during a symbol period. Based on the *hard decision* vector component $\hat{b_k}$ of vector $\hat{\boldsymbol{b}}$, the log-likelihood channel metrics for the CM decoder can be computed for all the M possible M-ary modulated symbols as follows:

$$P_m(\hat{b_k}|b_{k,m}) = -\frac{|\hat{b_k} - b_{k,m}|^2}{2\sigma^2}, \tag{5}$$

where $m \in \{0, \ldots, M-1\}$, $\sigma^2 = \frac{N_0}{2}$ is the noise's variance and N_0 is the noise's Power Spectral Density (PSD). Note that $\hat{b_k}$ is a *hard decision* value, where $\hat{b_k}$ equals to one of the legitimate M-ary modulated symbols, hence $P_m(\hat{b_k} = b_{k,m}|b_{k,m}) = 0$. Therefore, the performance of the *hard decision*-based CM scheme is not as high as that of the *soft decision*-based arrangement.

The maximisation of Equation 3 is a combinatorial optimisation problem. Specifically, Equation 3 has to be evaluated for each of the M^K possible combinations of the M-ary modulated symbols for the K users, in order to find the vector $\boldsymbol{b}$ that maximises the correlation metric of Equation 3. Explicitly, since there are M^K different possible vectors $\boldsymbol{b}$, the optimum multiuser detection has a complexity that increases exponentially with the number of users K and the modulation mode employed M.

In this contribution, we aim at reducing the complexity of the optimum MUD, which performs M^K full search, by employing the sub-optimum GA-based MUD, which performs only partial search. Hence, the sub-optimum decision vector $\hat{\text{b}}$ output by the GA-MUD is input to the CM decoder for generating the final estimate of the information.

A. The GA-assisted Multiuser Detector Subsystem

The configuration of the GA employed in our system is shown in Table I. For a detailed description of the GA-MUD, the interested readers are referred to the literature [4, 8]. A brief description of the GA-MUD is given below.

An initial population consisting of P number of so-called *individuals* is created, where P is known as the population size. Each individual represents a legitimate K-dimensional vector of M-ary modulated symbols constituting the solution of the given optimisation problem. At the beginning, we employed the hard decisions offered by the matched filter outputs $\boldsymbol{Z}$ which were denoted as:

$$\hat{\boldsymbol{b}}_{MF} = \left[\hat{b}_{1,MF}, \hat{b}_{2,MF}, \ldots, \hat{b}_{K,MF}\right], \tag{6}$$

where $\hat{b}_{l,MF}$ for $l = 1, \ldots, K$ is given by:

$$\hat{b}_{l,MF} = arg\left\{\min_{b} \left|z_l - \alpha_l e^{j\phi_l} b\right|\right\}. \tag{7}$$

In Equation 7 where the multiplication by $\alpha_l e^{j\phi_l}$ is necessary for coherent detection, because the phase rotation introduced by the channel has to be taken into account. A different randomly 'mutated' version [4,14] of the hard decision vector $\hat{\boldsymbol{b}}_{MF}$ of Equation 6 was assigned to each of the individuals in the initial population, where the same probability of mutation, namely p_m was adopted for all individuals. The process of incest prevention [15] is invoked, which will not allow identical individuals to mate.

Our objective function, or synonymously, fitness value is defined by the correlation metric of Equation 3. Here, the legitimate solutions are the M^K possible combinations of the K-symbol vector $\boldsymbol{b}$, where there are $log_2(M)$ bits in each of the M-ary symbols. Hence, each individual will take the form of a K-symbol vector corresponding to the K users' M-ary symbols during a single symbol interval. We will denote the p^{th} individual here as $\tilde{\boldsymbol{b}}_p(y) = \left[\tilde{b}_{p,1}(y), \ldots, \tilde{b}_{p,K}(y)\right]$, where y denotes the y^{th} generation. In order to ensure that the fitness values

TABLE I

THE CONFIGURATION OF THE GA EMPLOYED IN OUR SYSTEM.

Setup/Parameter	**Method/Value**
Individual initialisation method	Mutation of $\hat{\boldsymbol{b}}_{MF}$ of Equation 6
Selection method	Fitness-proportionate
Crossover operation	Uniform crossover
Mutation operation	Standard binary mutation
Elitism	Yes
Incest Prevention	Yes
Population size P	40, 80, 160
Mating pool size T	$T \leq P$ depends on the no. of non-identical individuals
Probability of mutation p_m	0.1
Termination generation Y	20, 40, 80

are positive for all combinations of $\boldsymbol{b}$ for the so-called *fitness-proportionate* selection scheme [3], we modify the correlation metric of Equation 3 according to [16]:

$$\exp\{\Omega(\boldsymbol{b})\} = \exp\left\{2\Re\left[\boldsymbol{b}^H \boldsymbol{C}^* \boldsymbol{Z}\right] - \boldsymbol{b}^H \boldsymbol{C}^* \boldsymbol{R} \boldsymbol{C} \boldsymbol{b}\right\}. \quad (8)$$

The associated probability of fitness-proportionate selection selection p_i of the i^{th} individual is defined as [3]:

$$p_i = \frac{f_i}{\sum_j^T f_j}, \quad (9)$$

where f_i is the fitness value associated with the ith individual. Once a pair of parents is selected, the *uniform crossover* [4,17] and *binary mutation* [14] operations are then applied to this pair of parents. In a *uniform crossover* operation [17], a so-called *crossover mask* is invoked. The crossover mask is a vector consisting of randomly generated 1s and 0s of equal probability, having a length equal to that of the individuals. Bits or M-ary symbols are exchanged between the selected pair of parents at locations corresponding to a 1 in the crossover mask. While it was shown in [18] that the uniform crossover operation has a higher probability of destroying a schema, it is also capable of creating new schemata. In a *binary mutation* operation [14], there are only two possible values for each binary decision variable hosted by an individual. Hence, when mutation is invoked for a particular bit, the value of the bit is toggled to the other possible value. For example, a bit of logical '1' is changed to a logical '0' and vice versa.

In order to ensure that high-merit individuals are not lost from one generation to the next, the best or a few of the best individuals are copied into the forthcoming generation, replacing the worst offspring of the new population. This technique is known as *elitism* [14]. In our application, we will terminate the GA-assisted search at the Y^{th} generation and the individual associated with the highest fitness value at this point will be the detected solution.

B. The Coded Modulation Subsystem

Due to the lack of space, here we specify only the generator polynomials of the CM schemes used in this section. For a detailed description of the various CM schemes the interested readers are referred to the literature [8]. Specifically, [7,8,19,20] are recommended for TCM, TTCM is discussed in [8,9], BICM is considered in [8,11,21] and BICM-ID in [8,12,21,22]

Table II shows the generator polynomials of the TCM and TTCM codes, which are presented in octal format. These are Recursive Systematic Convolutional (RSC) codes and the encoder attaches only one parity bit to the information bits. More specifically, in the context of M-ary modulation the number of useful information Bits Per Symbol (BPS) is $log_2(M) - 1$ and the coding rate is $R = \frac{BPS}{BPS+1}$. Table III shows the BICM and BICM-ID codes' generator polynomials in octal format, which were obtained from page 331 of [23]. These are non-systematic convolutional codes, which also produce one parity bit only. Hence, the code rates of these codes are similar to those of the TCM and TTCM codes, seen in Table II.

TABLE II

THE GENERATOR POLYNOMIAL, H^i, OF THE TCM AND TTCM CONSTITUENT CODES IN OCTAL FORMAT.

Code Rate	Coding	State	H^0	H^1	H^2	H^3
1/2(QPSK)	TCM	64	117	26	-	-
	TTCM	8	13	06	-	-
2/3(8PSK)	TTCM	8	11	02	04	-
3/4(16QAM)	TTCM	8	11	02	04	10

TABLE III

THE GENERATOR POLYNOMIAL, g^i, OF THE CONVOLUTIONAL CODES EMPLOYED IN THE BICM ENCODER IN OCTAL FORMAT.

Code Rate	Coding	State	g^0	g^1
1/2	BICM-ID	16	23	35
(QPSK)	BICM	64	133	171

Soft decision trellis decoding utilising the Log-Maximum A Posteriori (Log-MAP) algorithm [24] was invoked for decoding. While the complexity of the CM schemes is compared in terms of the number of decoding states and the number of decoding iterations. More specifically, the decoding complexity for TCM, BICM, TTCM and BICM-ID are $S = 2^\nu$, $S = 2^\nu$, $2 \cdot t \cdot S$ and $t \cdot S$ respectively [21], where ν is the component code's memory, S is the number of decoding states and t is the number of iterations. The codes shown in Tables II and III exhibit similar complexity when QPSK is employed, where both TTCM and BICM-ID utilise four decoding iterations.

III. Simulation Results And Discussions

Our performance metric is the average Bit Error Ratio (BER) evaluated over the course of several GA generations. The detection time of the GA is governed by the number of generations Y required, in order to obtain a reliable decision. The computational complexity of the GA, quantified in the context of the total number of objective function evaluations, is related to $P \times Y$. Since our GA-assisted multiuser detector is based on optimising the modified correlation metric of Equation 8, the computational complexity is deemed to be acceptable, if there is a significant amount of reduction in comparison to the optimum multiuser detector, which requires M^K objective function evaluations, in order to reach the optimum decision.

The BER versus Signal to Noise Ratio (SNR) per bit, namely E_b/N_0, performance of the QPSK-based CM-GA-MUD schemes is shown in Figures 2 and 3 for transmissions over AWGN channel and uncorrelated Rayleigh fading channel, respectively. The simulation parameters were summarised in Table I, II and III. A 'codeword length' of 1000 symbols and a spreading factor of 31 chips were employed. As determined by the 'codeword length', the turbo interleaver of TTCM and the internal bit interleavers of BICM and BICM-ID had a memory of 1000 symbol duration. The employment of an uncorrelated Rayleigh fading channel implies ideal channel interleaving, which has an infinitely long interleaver depth.

It is widely recognised that a QPSK signal consists of two orthogonal BPSK signals in a single user scenario and that the

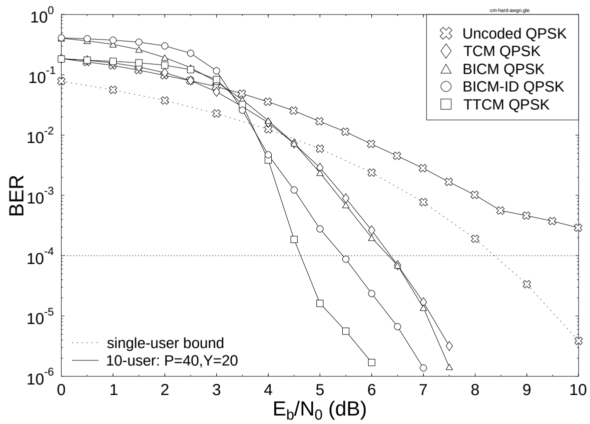

Fig. 2. BER versus E_b/N_0 performance of the various CM-GA-MUD schemes for transmissions over the AWGN channel employing QPSK and utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

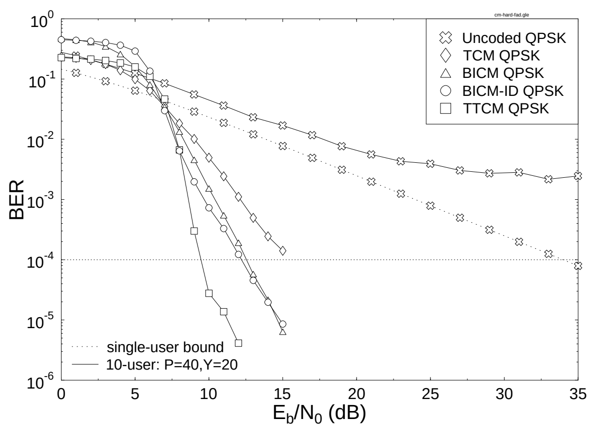

Fig. 3. BER versus E_b/N_0 performance of the various CM-GA-MUD schemes for transmissions over the uncorrelated Rayleigh fading channel employing QPSK and utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

associated BERs of BPSK and QPSK are identical in terms of E_b/N_0. Hence the single user bounds for QPSK modulation shown in Figure 2 for AWGN channel and Figure 3 for uncorrelated Rayleigh fading channel, are identical to that of the BPSK modulation. However, the orthogonality of the in-phase and quadrature-phase BPSK signals is corrupted by the MAI when a QPSK signal is transmitted in a CDMA system. Hence the BER of QPSK signal is not identical to that of BPSK signals in the context of a MAI-limited CDMA environment. Therefore, the uncoded QPSK performance of a $K = 10$-user CDMA system is worse than that of the single user bounds illustrated in Figures 2 and 3.

Note that the computational complexity of the GA-MUD is $\frac{M^K}{P \times Y} = 1310.72$ times lower, than that of the optimum MUD, when supporting $K = 10$ users employing QPSK modulation. The penalty for this complexity reduction is the BER error floor experienced by the GA-MUD schemes at high SNRs, as shown in the Figures 2 and 3. However, this disadvantage is eliminated, when the CM schemes are utilised. In particular, the TTCM assisted GA-MUD constitutes the best candidate, followed by the BICM-ID assisted GA-MUD, as evidenced in Figures 2 and 3 for transmissions over the AWGN and uncorrelated Rayleigh fading channels encountered. More specifically, for a throughput of 1 BPS and a target BER of 10^{-4}, the $K = 10$-user TTCM-GA-MUD assisted CDMA system is capable of providing SNR gains of about 4 and 25 dBs in AWGN and perfectly interleaved narrowband Rayleigh fading channels, respectively, against the single-user bounds of the uncoded BPSK scheme.

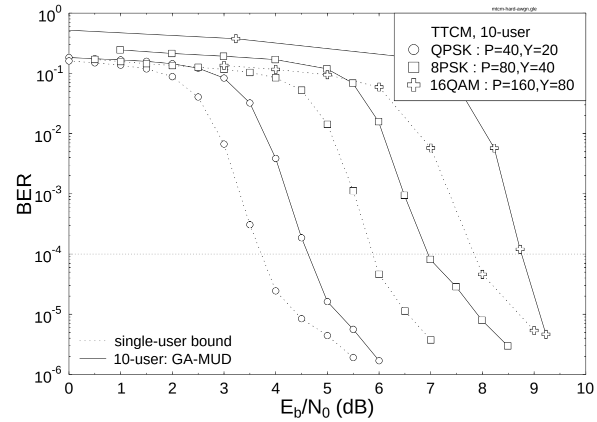

Fig. 4. BER versus E_b/N_0 performance of the TTCM-GA-MUD scheme for transmissions over the AWGN channel employing QPSK, 8PSK and 16QAM and utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

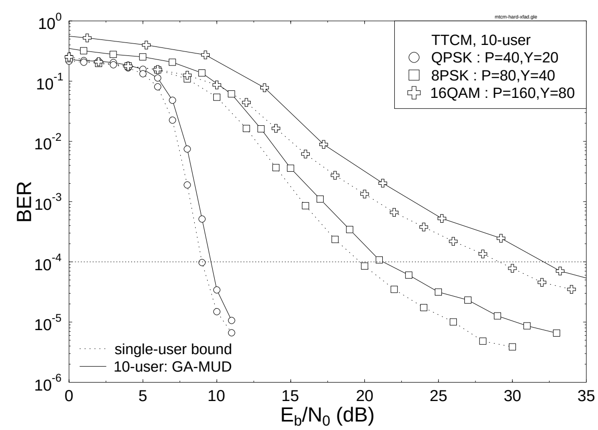

Fig. 5. BER versus E_b/N_0 performance of the TTCM-GA-MUD scheme for transmissions over the uncorrelated Rayleigh fading channel employing QPSK, 8PSK and 16QAM and utilising the simulation parameters of Table I, II and III. A codeword length of 1000 symbols and a spreading factor of 31 chips were employed.

Next, let us study the performance of the TTCM-GA-MUD scheme in conjunction with QPSK, 8PSK and 16QAM in Figures 4 and 5 for transmissions over both AWGN and uncorrelated Rayleigh fading channels, respectively. The *hard decision*-based single user performance bounds for TTCM are also plotted in Figures 4 and 5 as benchmarkers. We found that the performance of the TTCM-GA-MUD scheme, which supports 10 users is comparable to that of the single user TTCM benchmarker, in the 1, 2 and 3 BPS effective throughput modes associated with QPSK, 8PSK and 16QAM, respectively. In the higher-throughput modes, it was achievable by doubling the population size P and the number of generations Y of the TTCM-GA-MUD, every time when the BPS throughput was increased by one, as shown in the legends of Figures 4 and

5. More specifically, **the computational complexity reductions obtained by the GA-MUD compared to that of the optimum MUD when supporting $K = 10$ users are $\frac{M^K}{P \times Y}$=1.3×10^3, 6.7×10^5 and 8.6×10^7 for QPSK, 8PSK and 16QAM, respectively.** Despite these huge complexity reduction gains, the BER penalty for TTCM-GA-MUD is only around 0.5 to 2 dBs at a BER of 10^{-4} compared to the single-user benchmarker, when communicating over the AWGN and uncorrelated Rayleigh fading channels employing QPSK, 8PSK and 16QAM, as evidenced by Figures 4 and 5.

IV. Conclusion

In this contribution, TCM, TTCM, BICM and BICM-ID assisted GA-based MUD schemes were proposed and evaluated in performance terms when communicating over the AWGN and narrowband Rayleigh fading channels encountered. It was shown that the GA-MUD is capable of significantly reducing the computational complexity of the optimum-MUD, but experiences an error floor at high SNRs due to invoking an insufficiently large population size and a low number of generations. However, with the advent of the bandwidth efficient CM schemes proposed, this problem is eliminated. When comparing the four CM schemes at the same decoding complexity, TTCM was found to be the best candidate for assisting the operation of the GA-MUD system.

When higher throughput M-ary modulation schemes were investigated with the advent of TTCM-GA-MUD arrangements, we found that the complexity of TTCM-GA-MUD was dramatically lower than that of the optimum-MUD, with the penalty of only 0.5 to 2 dBs SNR loss compared to that of the TTCM single user bound.

V. Acknowledgements

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