Space-Time Block Coded IQ-interleaved Joint Coding and Modulation for AWGN and Rayleigh Fading Channels

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Abstract – **Space-Time Block Coded (STBC) In-phase/Quadrature-phase (IQ)-interleaved Trellis Coded Modulation (TCM) and Turbo TCM (TTCM) schemes are proposed, which are capable of quadrupling the achievable diversity order of the conventional symbol-interleaved TCM and TTCM schemes, when two transmit antennas are employed. The increased diversity order of the proposed schemes provides significant additional coding gains, when communicating over non-dispersive Rayleigh fading channels, which is achieved without compromising the coding gain attainable over Gaussian channels. Bit-Interleaved Coded Modulation (BICM) as well as Iteratively Decoded BICM (BICM-ID) are also incorporated into the proposed system and their performance is compared to that of TCM and TTCM.**

1. INTRODUCTION

Trellis Coded Modulation (TCM) [1, 2] was originally designed for transmission over Additive White Gaussian Noise (AWGN) channels, where it is capable of achieving a coding gain without bandwidth expansion. Turbo TCM (TTCM) [2, 3] is a more recently proposed bandwidth efficient transmission scheme, which has a structure similar to that of the family of binary turbo codes, distinguishing itself by employing TCM schemes as component codes. Both the TCM and TTCM schemes employed set partitioning based signal labelling, in order to increase the minimum Euclidean distance between the encoded information bits. Symbol interleavers were utilised both for the turbo interleaver and for the channel interleaver, for the sake of achieving time diversity when communicating over Rayleigh fading channels.

It was shown in [4] that the maximisation of the minimum Hamming distance measured in terms of the number of different symbols between any two transmitted symbol sequences is the key design criterion for TCM schemes contrived for flat Rayleigh fading channels, in particular when communicating at high Signal-to-Noise Ratios (SNR). In an effort to increase the achievable time diversity, a multidimensional TCM scheme utilising a symbol interleaver and two encoders was proposed in [5], where the individual encoders generate the In-phase (I) and Quadrature-phase (Q) components of the complex transmitted signal, respectively. Another TCM scheme using constellation rotation was proposed in [6], which utilised two separate channel interleavers for interleaving the I and Q components of the complex transmitted signals, but assumed the absence of I/Q *cross-coupling*, when communicating over fading channels exhibiting a complex-valued Channel Impulse Response (CIR).

Another powerful Coded Modulation (CM) scheme utilising bit-based channel interleaving in conjunction with Gray signal labelling, which is referred to as Bit-Interleaved Coded Modulation (BICM), was proposed in [7]. It combines conventional non-systematic convolutional codes with several independent bit interleavers. The number of parallel bit-interleavers used equals the number of channel coded bits in a symbol [2, 7]. Recently, iteratively decoded BICM using Set Partitioning (SP) based signal labelling, referred to as BICM-ID has also been proposed [8].

In numerous practical situations the wireless channels are neither highly time selective nor significantly frequency selective. This motivated numerous researchers to investigate space diversity techniques with the aim of improving the system's performance. Classic receiver diversity [9] has been widely used at the base stations of both the GSM and IS-136 systems. As an additional performance enhancement, recently the family of transmit diversity techniques [2, 10] has been studied extensively for employment at the base station, since it is more practical to have multiple transmit antennas at the base station, than at the mobile station. Space-Time Trellis Coding (STTC) pioneered by Tarokh *et. al.* [11] incorporates jointly designed channel coding, modulation, transmit diversity and optional receiver diversity [2]. In an attempt to reduce the associated decoding complexity, Alamouti proposed Space-Time Block Coding [12] (STBC) employing two transmit antennas. Alamouti's scheme was later generalised to an arbitrary number of transmit antennas [13].

In order to improve the performance of the existing state-of-the-art systems, in this contribution we proposed a novel system which amalgamates STBC [12] with IQ-interleaved TCM, TTCM, BICM and BICM-ID schemes using no constellation rotation.

2. SYSTEM OVERVIEW

The block diagram of the Space-Time (ST) based IQ-interleaved (ST-IQ) TCM/TTCM system is shown in Figure 1. Specifically, we employ two transmitters and one receiver in the ST scheme. Furthermore we invoke two independent IQ interleavers in the TCM/TTCM arrangement, which are denoted as π_I and π_Q in the block diagram of Figure 1. We denote the IQ-interleaved modulated signal by $\tilde{s} = \tilde{s}_I + j\tilde{s}_Q$, which is transmitted over the flat Rayleigh fading channel having a complex fading coefficient of $h = \alpha e^{j\theta}$ with the aid of two STBC transmitters. As seen in Figure 1, during the first symbol period of the STBC transmission, the signals $x_1 = \tilde{s}_1$ and $x_2 = \tilde{s}_2$ are transmitted, while during the second symbol period, the signals $-x_2^*$ and x_1^* are emitted from the transmit antennas 1 and 2, respectively. We assume that the fading envelope and phase of the CIR are constant across the two STBC time slots. The signal is also contaminated by the zero-mean AWGN n exhibiting a variance of $\sigma^2 = N_0/2$, where N_0 is the single-sided noise power spectral density.

It can be shown that the two signals received during the two consecutive symbol periods can be represented in matrix form as $\mathbf{r} =$

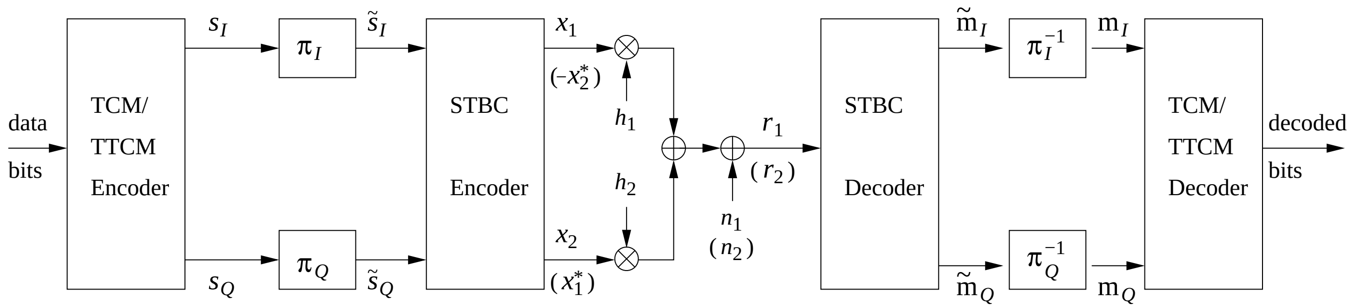

Figure 1: Block diagram of the ST-based IQ-interleaved system. The notations π and π^{-1} denote the interleaver and deinterleaver, while (.) denotes the STBC signals during the second symbol period.

$\mathbf{A} \cdot \mathbf{x} + \mathbf{n}$:

$$\begin{pmatrix} r_1 \\ r_2^* \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2^* \end{pmatrix}, \qquad (1)$$

where $\mathbf{A}$ is termed the system matrix and x^* denotes the complex conjugate of symbol x. Note that the I (or Q) component of the received signal r_i, namely $r_{i,I}$ (or $r_{i,Q}$) where $i \in \{1, 2\}$, is dependent on both the I and Q components of x_1 and x_2, namely on $x_{1,I}$, $x_{1,Q}$, $x_{2,I}$ and $x_{2,Q}$. More explicitly, we have: $r_{1,I} = (h_{1,I}x_{1,I} - h_{1,Q}x_{1,Q} + h_{2,I}x_{2,I} - h_{2,Q}x_{2,Q} + n_{1,I})$, owing to the cross-coupling effect imposed by the complex CIR. It is however desirable to decouple them, so that we can compute the I (or Q) branch metrics m_I (or m_Q) in Figure 1 for a particular x_i independently, as a function of only $x_{1,I}$ and $x_{2,I}$ (or $x_{1,Q}$ and $x_{2,Q}$). Surprisingly, this may be achieved without carrying out any explicit decoupling operation for the STBC based IQ-interleaved TCM/TTCM scheme. More specifically, the signals have been decoupled during the STBC decoding operation, when the received vector $\mathbf{r}$ is multiplied with the conjugate transpose of $\mathbf{A}$, namely with $\mathbf{A^H}$, yielding $\hat{\mathbf{x}} = \mathbf{A^H} \cdot \mathbf{r}$:

$$\begin{pmatrix} \hat{x}_1 \\ \hat{x}_2 \end{pmatrix} = \left(\alpha_1^2 + \alpha_2^2\right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \hat{n}_1 \\ \hat{n}_2 \end{pmatrix}, \qquad (2)$$

where $\hat{n}_1$ and $\hat{n}_2$ contain the resultant noise. Explicitly, the signal $\hat{x}_1 = (\alpha_1^2 + \alpha_2^2)x_1 + \hat{n}_1$ is the decoupled version of r_1, where $\hat{x}_{1,I}$ (or $\hat{x}_{1,Q}$) is independent of $x_{1,Q}$ and $x_{2,Q}$ (or $x_{1,I}$ and $x_{2,I}$).

Hence, it can be readily shown that the associated IQ branch metrics of the STBC coded signal $x_1 = x_{1,I} + jx_{1,Q}$ can be derived from $\hat{x}_1 = \hat{x}_{1,I} + j\hat{x}_{1,Q}$ as:

$$\tilde{m}_I(x_{1,I}|\hat{x}_{1,I}, D_I) = -\frac{\left|\hat{x}_{1,I} - D_I x_{1,I}\right|^2}{2\sigma^2 D_I} \qquad (3)$$

and

$$\tilde{m}_Q(x_{1,Q}|\hat{x}_{1,Q}, D_Q) = -\frac{\left|\hat{x}_{1,Q} - D_Q x_{1,Q}\right|^2}{2\sigma^2 D_Q}, \qquad (4)$$

where we have $D_I = D_Q = D = \left(\alpha_1^2 + \alpha_2^2\right)$. The corresponding branch metric of x_2 is computed similarly. The effect of the associated second order transmit diversity attained may be observed in the context of the term $(\alpha_1^2 + \alpha_2^2)$. As for the single-transmitter scheme, the resultant IQ branch metric $\tilde{m}_I$ and $\tilde{m}_Q$ engendered by the transmitted signal x can be computed from Equations 3 and 4 by using $D = \alpha^2$. More specifically, the corresponding received signal is $r = h \cdot x + n$ and the I/Q-decoupled signal is $\hat{x} = h^* \cdot r = \alpha^2 x + h^* \cdot n$ for the single-transmitter scheme.

Note that $\tilde{m}_I$ and $\tilde{m}_Q$ share the same channel-envelope related $D = \alpha^2$ value for the same transmitted signal of $x(=\tilde{s})$, but after the IQ deinterleavers of π_I^{-1} and π_Q^{-1} seen in Figure 1, m_I and m_Q will be associated with different D values. The branch metric of the TCM/TTCM-coded signal s is computed from:

$$m(s) = m_I(x_I = s_I) + m_Q(x_Q = s_Q). \qquad (5)$$

Since there are two independent IQ coordinates for a complex TCM/TTCM symbol, and since they are independently interleaved by the interleavers π_I and π_Q in Figure 1, m_I and m_Q provide independent diversity for a particular symbol. More explicitly, **since we have $D_I \neq D_Q$, the IQ-interleaved TCM/TTCM scheme is expected to double the achievable diversity order compared to its symbol-interleaved counterpart.** Therefore the achievable Hamming distance of the proposed IQ-interleaved TCM/TTCM scheme is based on the number of different I and Q coordinates between the different transmitted messages, rather than on the number of different symbols, which was the case in the context of conventional symbol-interleaved TCM/TTCM.

We have also amalgamated the proposed ST-IQ scheme of Figure 1 with BICM and BICM-ID schemes [2]. More specifically, in addition to their internal bit-interleavers [2] two extra random interleavers were invoked for interleaving the I and Q components of their bit-interleaved complex symbol $s = s_I + js_Q$ for yielding $\tilde{s} = \tilde{s}_I + j\tilde{s}_Q$, as it was illustrated in Figure 1.

3. SIMULATION RESULTS

We evaluated the performance of the proposed schemes using 16-level Quadrature Amplitude Modulation (16QAM) in the context of the non-iterative 64-state TCM and BICM schemes, as well as in conjunction with the iterative 8-state TTCM arrangement using four decoding iterations and along with an 8-state BICM-ID arrangement using eight decoding iterations. The rationale of using 64 and 8 states, respectively, was that the TCM/BICM and TTCM/BICM-ID schemes considered here exhibit a similar decoding complexity expressed in terms of the total number of trellis states. Explicitly, since there are two 8-state TTCM decoders, which are invoked in four iterations, we encounter a total of $2 \cdot 8 \cdot 4 = 64$ TTCM trellis states. By contrast, only a single 8-state BICM-ID decoder is required, which is invoked in eight iterations, involving a total of $8 \cdot 8 = 64$ BICM-ID trellis states. The effective throughput was 3 Bits Per Symbol (BPS) for all the 16QAM based CM schemes. The generator polynomials expressed in octal format for TCM and TTCM are $\begin{bmatrix} 101 & 16 & 64 & 0 \end{bmatrix}$ from [1] and $\begin{bmatrix} 11 & 2 & 4 & 10 \end{bmatrix}$ from [3], respectively. BICM and BICM-ID employ Paaske's non-systematic convolutional codes [14] and their

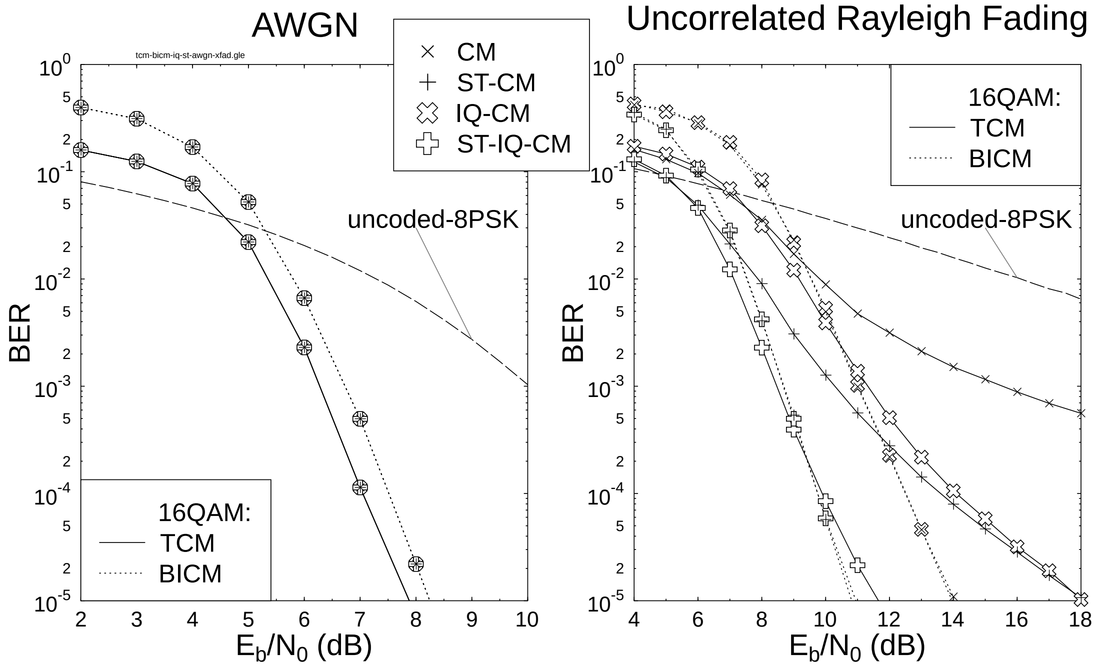

Figure 2: BER versus E_b/N_0 performance of the 16QAM based **TCM** and **BICM** schemes, when communicating over AWGN and uncorrelated flat Rayleigh fading channels. The legend is described at Footnote 1. A codeword length of 1000 symbols was used and the performance of the uncoded 8PSK scheme is also plotted for benchmarking the CM schemes having an effective throughput of **3 BPS**.

generator polynomials shown in octal format are : $\begin{bmatrix} 6 & 1 & 0 & 7 \\ 3 & 4 & 1 & 6 \\ 2 & 3 & 7 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & 4 & 4 & 4 \\ 0 & 6 & 2 & 4 \\ 0 & 2 & 5 & 5 \end{bmatrix}$, respectively.

In Figure 2, we portray the Bit Error Ratio (BER) versus signal to noise ratio per bit, namely E_b/N_0, performance of the 16QAM based TCM and BICM schemes, when communicating over AWGN as well as over uncorrelated flat Rayleigh fading channels[1]. A codeword length of 1000 symbols was used and the BER performance curve of the uncoded 8-level Phase-Shift-Keying (8PSK) scheme is also plotted for benchmarking the schemes having an effective throughput of 3 BPS. As illustrated in Figure 2, all the TCM/BICM schemes associated with the conventional CM, ST-CM, IQ-CM and ST-IQ-CM arrangements exhibit a similar performance in AWGN channels. This is because no space diversity or time diversity is attainable over Gaussian channels despite using multiple transmitters and interleaving. On the other hand, the TCM scheme performs approximately 0.5 dB better, than BICM scheme, when communicating over AWGN channels, since it has a higher Euclidean distance than that of BICM, which is the decisive criterion in the context of AWGN channels.

By contrast, when communicating over uncorrelated flat Rayleigh fading channels, the BER curve of IQ-TCM merged with that of ST-TCM in the high-SNR region of Figure 2, since they both exhibit twice the diversity potential compared to conventional TCM. As seen in Figure 2, with the advent of ST-IQ-TCM a further 6.4 dB gain can be obtained at a BER of 10^{-5} compared to the IQ-TCM and ST-TCM schemes. By contrast, the BICM scheme exhibits only transmit diversity gain but no IQ diversity gain as we observe in Figure 2. This is because the four random bit-interleavers employed in the 16QAM-BICM scheme have already provided IQ diversity inherently. Since in BICM the bit-based minimum Hamming distance is maximised [2], which is the decisive criterion in the context of Rayleigh fading channels, BICM will benefit from a lower bit error probability in Rayleigh fading channels than that of TCM, because TCM maximises the free Euclidean distance within the modulated signal constellation. Note that the performance of the conventional TCM scheme is significantly worse than that of conventional BICM owing to the existence of unprotected bits in the TCM-protected 16QAM symbol. However, the achievable coding gain of the ST-IQ-TCM scheme is only about 0.8 dB less than that of the ST-IQ-BICM scheme at a BER of 10^{-5}.

Let us now study in Figure 3 the BER versus E_b/N_0 performance of the 16QAM based iterative TTCM and BICM-ID schemes, when communicating over AWGN and uncorrelated flat Rayleigh fading channels, again in the context of the conventional CM, ST-CM, IQ-CM and ST-IQ-CM arrangements as described at Footnote 1. A codeword length of 1000 symbols was used and the performance of the uncoded 8PSK scheme was also plotted as a benchmarker. As portrayed in Figure 3, the TTCM scheme exhibits a better performance than BICM-ID in the low SNR region, when communicating over AWGN channels, although their BER curves converge beyond $E_b/N_0 = 5.8$ dB. Again, no space and time diversity gain was achieved when communicating over Gaussian channels.

From Figure 3 we can notice that similarly to the performance trends observed for the TCM schemes of Figure 2, the BER curve of IQ-TTCM merged with that of ST-TTCM in the high-SNR region, when communicating over the uncorrelated flat Rayleigh fading channels, since they both exhibit twice the diversity potential compared to conventional TTCM. However, the achievable diversity/coding gain of TTCM was found lower than that of TCM owing to the fact that TTCM has already achieved part of its attainable diversity gain with the aid of its iterative turbo decoding procedure. Nonetheless, at a BER of 10^{-5}, the performance of ST-IQ TTCM is about 5.1 dB better than that of the conventional TTCM scheme. Note from Figure 3 that

[1] CM: the conventional CM scheme; ST-CM: the ST based conventional CM scheme; IQ-CM: the proposed IQ-interleaved CM scheme but without ST coding; ST-IQ-CM: the proposed ST-based IQ-interleaved CM scheme.

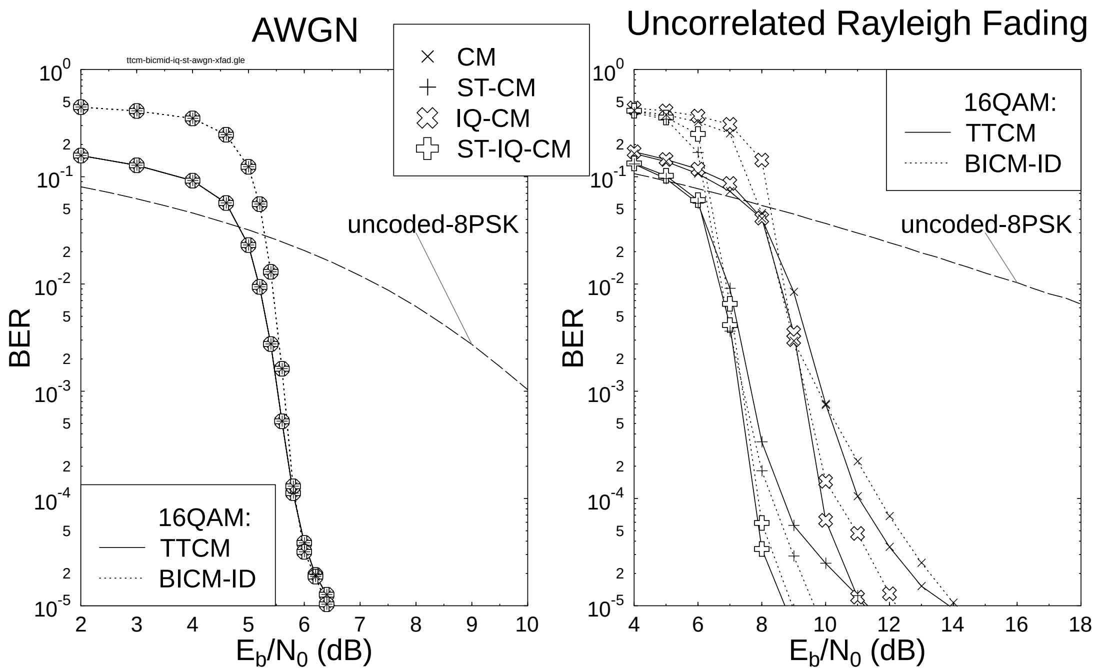

Figure 3: BER versus E_b/N_0 performance of 16QAM based **TTCM** and **BICM-ID** schemes, when communicating over AWGN and uncorrelated flat Rayleigh fading channels. The legend is described at Footnote 1. A codeword length of 1000 symbols was used and the performance of the uncoded 8PSK scheme is also plotted for benchmarking the CM schemes having having an effective throughput of **3 BPS**.

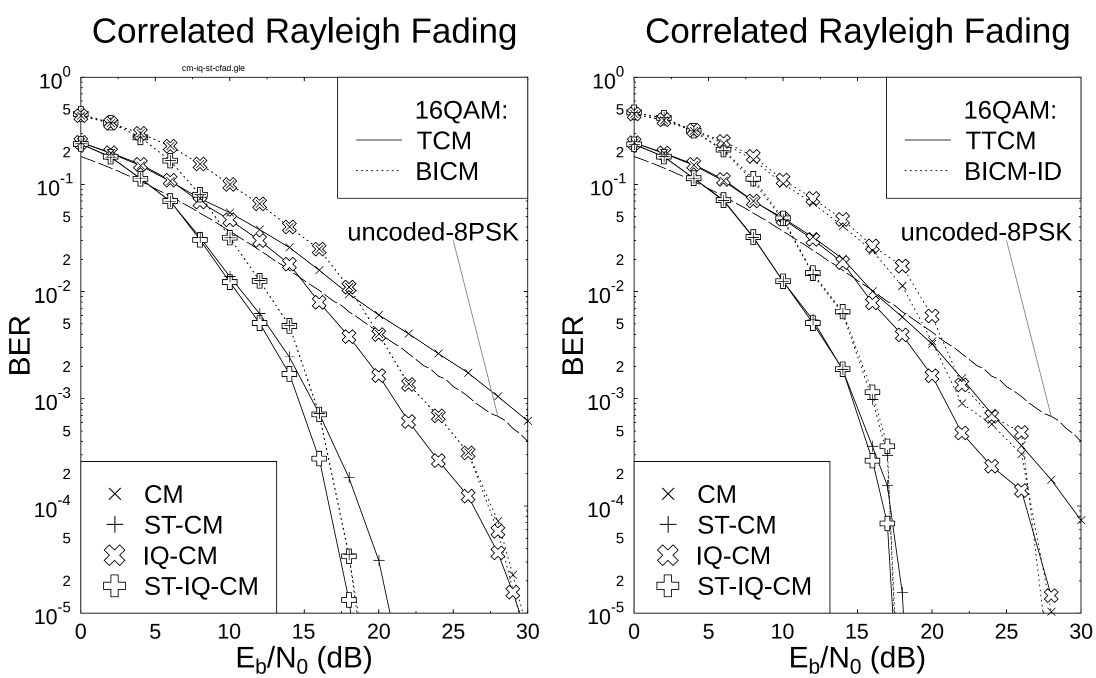

Figure 4: BER versus E_b/N_0 performance of 16QAM based **TCM**, **BICM**, **TTCM** and **BICM-ID** schemes, when communicating over correlated flat Rayleigh fading channels having a normalised Doppler frequency of 3.25×10^{-5}. The legend is described at Footnote 1. A codeword length of 10000 symbols was used and the performance of the uncoded 8PSK scheme is also plotted for benchmarking the CM schemes having an effective throughput of **3 BPS**.

the BICM-ID scheme is capable of exploiting the IQ diversity owing to employing iterative decoding, when communicating over uncorrelated flat Rayleigh fading channels. Although IQ-BICM-ID exhibits a performance, which is about 1 dB worse than that of IQ-TTCM at BER=10^{-5}, nonetheless the coding gain of ST-IQ-BICM-ID is only marginally lower than that of ST-IQ-TTCM. However, unlike in the context of the TCM and TTCM schemes, the BER performance of the IQ-BICM-ID scheme – which benefits from IQ diversity – was lower than the performance of the ST-BICM-ID scheme in the high-SNR region, where the latter exhibits a transmit diversity of order two. This is because the IQ diversity gain of IQ-BICM-ID is a benefit of the iterative decoding, rather than accruing from IQ interleaving alone. This observations may be confirmed in Figure 3.

Note that encountering uncorrelated flat Rayleigh fading channels would imply that the channel interleaving has an infinitely long memory or that the vehicular speed is infinite. However, practical Rayleigh fading channels exhibit correlated fading and the degree of the correlation experienced depends on the associated normalised Doppler frequency. Let us now investigate the performance of the proposed schemes under correlated flat Rayleigh fading channel conditions having a normalised Doppler frequency of 3.25×10^{-5} in Figure 4, where a codeword length of 10000 symbols was used. Note that the BER performance of the uncoded 8PSK benchmarker is the same when communicating over uncorrelated and correlated flat Rayleigh Fading channels. However, the performance of the CM schemes degrades, when the fading exhibits a high degree of correlation. As portrayed at the left of Figure 4, the performance of the conventional TCM scheme becomes worse than that of the uncoded 8PSK benchmarker, when communicating over correlated flat Rayleigh fading channels. However, with the advent of IQ interleaving, the performance of the IQ-TCM scheme improved significantly and it becomes better than that of BICM (or IQ-BICM) under these conditions. On the other hand, ST-IQ-TCM performs better than ST-BICM (or ST-IQ-BICM), although ST-TCM performs worse than ST-BICM, when the slowly fading channel exhibits a normalised Doppler frequency of 3.25×10^{-5}, as evidenced in Figure 4. However, IQ-TCM is outperformed by ST-TCM, when communicating over correlated flat Rayleigh fading channels. This is because, the two STBC transmitter antennas were arranged sufficiently far apart, so that their transmitted signals experience independent channel fading, whereas the IQ-interleaved signals suffer from correlated channel fading.

The performance of the TTCM and BICM-ID schemes communicating over correlated flat Rayleigh fading channels was also shown at the right of Figure 4. Specifically, IQ-BICM-ID (or ST-IQ-BICM-ID) shows no advantage over its conventional BICM-ID (or ST-BICM-ID) counterpart, when communicating over slowly fading channels. By contrast, IQ-TTCM still outperforms conventional TTCM by approximately 2 dBs under these conditions. However, the performance of the ST-IQ-TTCM arrangement is only marginally better than that of its ST-TTCM counterpart.

4. CONCLUSIONS

In this contribution we proposed a set of novel ST-IQ aided CM schemes for transmissions over both AWGN and Rayleigh fading channels. The ST-IQ-TCM, ST-IQ-TTCM and ST-IQ-BICM-ID schemes are capable of providing significant diversity gains over their conventional counterparts. Specifically, in case of the uncorrelated flat Rayleigh fading channel, coding gains of 26.1 dB, 28.2 dB, 26.9 dB and 28.1 dB were achieved over the identical-throughput uncoded 8PSK benchmarker at a BER of 10^{-4} by the ST-IQ-TCM, ST-IQ-TTCM, ST-IQ-BICM and ST-IQ-BICM-ID schemes, respectively. All schemes achieved an effective throughput of 3 BPS without bandwidth expansion. From Figures 2, 3 and 4 ST-IQ-TTCM was found to be the best scheme, when communicating over AWGN as well as uncorrelated and correlated flat Rayleigh fading channels.

For systems requiring the reduced complexity of a single-transmitter scheme, the IQ-interleaved TCM/TTCM scheme is still capable of doubling the achievable diversity potential of conventional symbol-interleaved TCM/TTCM with the aid of a single transmit antenna, although the IQ diversity attainable decreased when the fading channel exhibited a higher correlation.

5. ACKNOWLEDGEMENTS

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