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Discrete-input continuous-output memoryless channel capacity of cooperative hierarchical modulation

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Abstract: Hierarchical modulation (HM) is a layered modulation scheme, which is widely employed by the telecommunication industry. The higher flexibility and lower complexity of the HM scheme has its dramatic benefits for wireless communications, hence the achievable performance of cooperation-aided coded HM has drawn substantial research interests. In this study, a triple-layer HM-aided four-node cooperative communication system is proposed, and its discrete-input continuous-output memoryless channel capacity is derived, which is used for finding the optimal position of the relay nodes as well as to design appropriate HM constellations. The authors' simulation results show that if a rate-1/2 'perfect' channel code is assumed, the four-node network becomes capable of conveying a coded HM-64QAM signal in three time slots at an average signal-to-noise ratio of −0.71 dB.

1 Introduction

Hierarchical modulation (HM) is widely employed in the telecommunication industry, which may be beneficially invoked for upgrading diverse telecommunication services. The system using HM is capable of superimposing diverse new services layer by layer, while maintaining backward compatibility [1, 2] with the original base-line system. In this way, the original devices may still be supported by the upgraded broadcast system, whilst delivering new additional services. The performance of multi-layer HM schemes was characterised for example in [3–5]. The layered structure of HM ensures that the most important information can indeed be flawlessly received, while the less important layers may be discarded without undue degradation, in case of network congestion or when the signal-to-noise ratio (SNR) is low. This unequal-error-protection (UEP) capability of HM has drawn substantial research interests [6–9]. More specifically, the authors of [10–12] invoked a HM scheme for providing UEP for image encoding, where the information bits are mapped to specific protection layers according to their error-sensitivity-based priority. Moreover, HM has also been combined with sophisticated channel coding schemes in [11, 12] for the sake of protecting the most important information. The simulation results of [11, 12] have shown that receiving the information having the highest priority requires a lower receive SNR than that of the conventional modulation schemes at a given target bit-error-ratio (BER) performance. However, the improved performance of the high-priority layers achieved by the UEP scheme inevitably leads to a degraded performance for the lower-priority layer. Hence, the SNR required for receiving the whole hierarchical modulated signal may be higher than that of the conventional modulation schemes, especially in the absence of relaying.

When evaluating the performance of a system, typically, the BER performance is the salient metric [1–4, 9]. The Gaussian Q-function and the Euclidean distances among the constellation points may be used for quantifying the lower or upper bound of the system. However, when considering the spatial diversity benefits of cooperative communications or the time-diversity gain of channel coding schemes, the Q-function is not applicable. First, the relay node (RN) would lend a path gain to the entire system. Second, as a benefit of channel coding, the BER performance will no longer be solely decided by the Euclidean distance of the constellation maps. Hence, characterising cooperative communication systems relying on HM and channel coding becomes more of a challenge.

Against this background, we proposed a cooperative communication system, which is assisted by a pair of decode-and-forward (DF) RNs intrinsically amalgamated with triple-layer HM. The design goal of our system is to reduce the power consumption of the entire cooperative network, while guaranteeing that all benefits of HM are retained, and additionally ensuring that all the associated transmission links have the same performance. To calculate the lower performance bound of the system, the coding scheme we employed is assumed to be a 'perfect' code. More specifically, the relevant discrete-input continuous-output memoryless channel (DCMC) capacity is used for analysing the cooperative system, relying on the optimised RN positions.

The main contributions of this paper are as follows:

- A triple-layer HM-64QAM scheme is designed for DF cooperative communications.
- The DCMC capacity is derived for characterising the achievable performance (the minimum received SNR required for achieving the target capacity) of the individual HM layers, when assuming that a 'perfect' channel coding scheme is employed.
- On the basis of our DCMC capacity analysis, a coded triple-layer HM scheme-based cooperative communication system is proposed. More explicitly, both the HM constellations and the positions of the RNs are taken into consideration, when deriving the lowest possible power consumption of the entire system.

The organisation of the paper is as follows. Section 2 introduces the system model. The specific HM-64QAM mapping rule designed for cooperative communication is detailed in Section 3. In Section 4, both the DCMC capacity and our optimisation strategy are characterised, while our performance results are discussed in Section 5. Our conclusions and future research ideas are provided in Section 6.

2 System model

Our HM-aided DF RN-based cooperative communication system is illustrated in Fig. 1, where the link is considered to be an uncorrelated Rayleigh fading channel, and the entire system benefits from 100% channel state information. During the first transmission time slot (TS), the source node (SN) will broadcast a

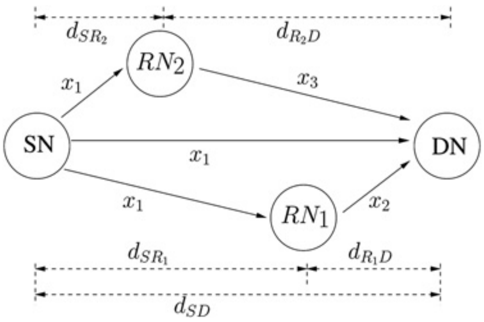

Fig. 1 *Model of a twin-relay cooperative system*

sequence of HM symbols $\{x_1\}$ to both RN_1 and RN_2 as well as to destination node (DN). In the following two TSs, RN_1 will transmit a signal frame $\{x_2\}$ to the DN and another signal frame $\{x_3\}$ will be sent to the DN by the RN_2. Again, the entire system would require three TSs for conveying the triple-layer HM-64QAM-based signal frame $\{x_1\}$ to DN.

When considering the reduced path loss introduced by the RN, in order to simplify the system model and the related discussions, we employ the simplified path-loss model of [13] and set the path-loss exponent to 3, which is usually used in the simulation of urban areas. Then, the reduced path loss of the two source to relay (SR) links is

$$G_{\mathrm{SR}_k} = \left(\frac{d_{\mathrm{SD}}}{d_{\mathrm{SR}_k}}\right)^3, \quad (1)$$

and similarly, the reduced path loss of the two relay to destination (RD) links are

$$G_{R_kD} = \left(\frac{d_{\mathrm{SD}}}{d_{R_kD}}\right)^3. \quad (2)$$

The path loss between the SN and DN is normalised to be 0 dB. If we set the transmit power to p_t, the receive power to p_r and the additive white Gaussian noise (AWGN) noise power to N_0, the transmit SNR [$\mathrm{SNR_t}$ (The definition of transmit SNR ($\mathrm{SNR_t}$) was proposed in [14], which is convenient for simplifying the discussions, although this is not a physically measurable quantity, because it relates the power at the transmitter to the noise at the receivers.)] will be p_t/N_0 and the receive SNR ($\mathrm{SNR_r}$) will be p_r/N_0. Since we assume that $\mathrm{SNR_t}$ and $\mathrm{SNR_r}$ share the same noise power N_0, $\mathrm{SNR_t}$ and $\mathrm{SNR_r}$ may indicate the strength of the transmit power and of the receive power. Note that by adding the path loss between the SN–DN link and considering a realistic noise power, our system model may be directly converted into a realistic wireless communication system model [15]. If the transmissions between the SN and DN take place on a frame-by-frame basis over an uncorrelated Rayleigh fading channel, the average received SNR at DN ($\overline{\mathrm{SNR_r^{DN}}}$) may be formulated as

$$\overline{\mathrm{SNR_r^{DN}}} = E(|h|^2\mathrm{SNR_t}) = E(|h|^2)\mathrm{SNR_t^{SN}}, \quad (3)$$

where the $\mathrm{SNR_t^{SN}}$ may be expressed as

$$\mathrm{SNR_t^{SN}} = \frac{E(|x|^2)}{N_0} = \frac{1}{N_0}, \quad (4)$$

where $E(|x|^2) = 1$. Furthermore, since the uncorrelated Rayleigh fading parameter h is generated by the complex-valued Gaussian distribution with a zero mean and a unit variance, when the number of uncorrelated Rayleigh fading components we generated is large, we have [16, 17]

$$E(|h|^2) = \int \exp(-|h|^2) \simeq 1. \quad (5)$$

Note that the distribution of $|h|^2$ obeys $f(|h|^2) = \exp(-|h|^2)$, as detailed in [17]. Hence, for a large frame size of N symbols, we may assume that $\mathrm{SNR_r^{DN}}$ is equal to $\mathrm{SNR_t^{SN}}$, or equivalently that we have $\overline{\mathrm{SNR_r^{DN}}} = \mathrm{SNR_t^{SN}}$.

The block diagram of our cooperative communication system is illustrated in Fig. 2. Three rate-1/2 encoders are employed by the SN, and the outputs c_1, c_2, c_3 of the three encoders may be merged together to form HM-64QAM signals. We stipulate that c_1 is in the base layer (L_1) which is the layer with the highest level of protection, c_2 is in the second layer (L_2) and c_3 is in the third layer (L_3), which has the lowest protection level. The $\mathrm{SNR_t^{SN}}$ is assumed to be only sufficient for the DN to receive L_1, where L_2 and L_3 will be forwarded to the DN separately by the two RNs. Therefore, the entire system may require three TSs to complete the transmissions. Note that in order to successfully receive L_2 and L_3 at the two RNs, the reduced path loss achieved by the two RNs should be sufficiently high.

3 Triple-layer HM modulation

Our triple-layer HM-64QAM constellations seen in Fig. 3 was originally introduced in [18] and detailed in [17]. Our system

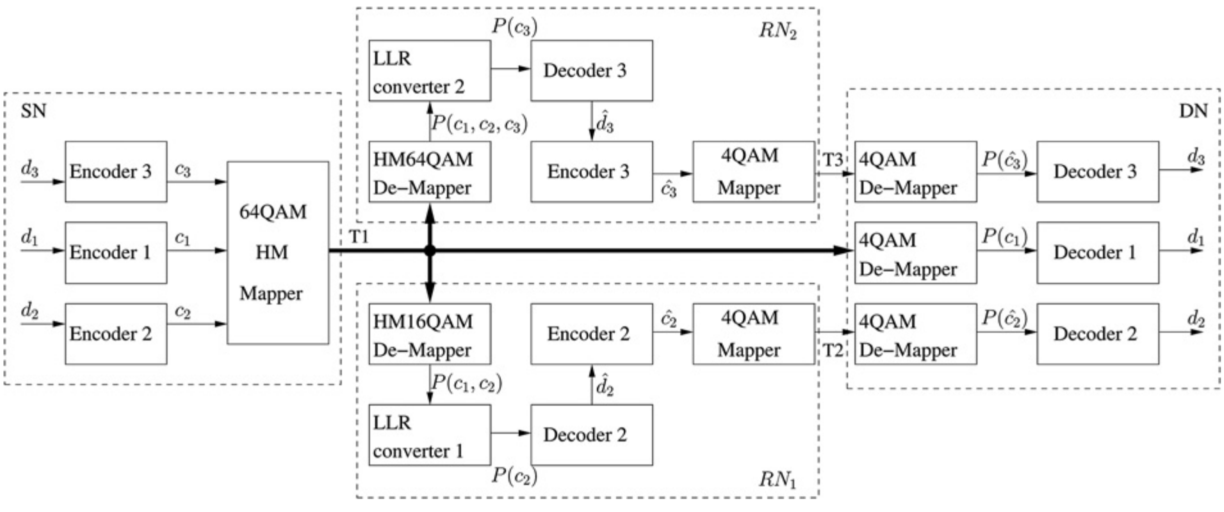

Fig. 2 *System diagram of a twin-relay-aided cooperative system*

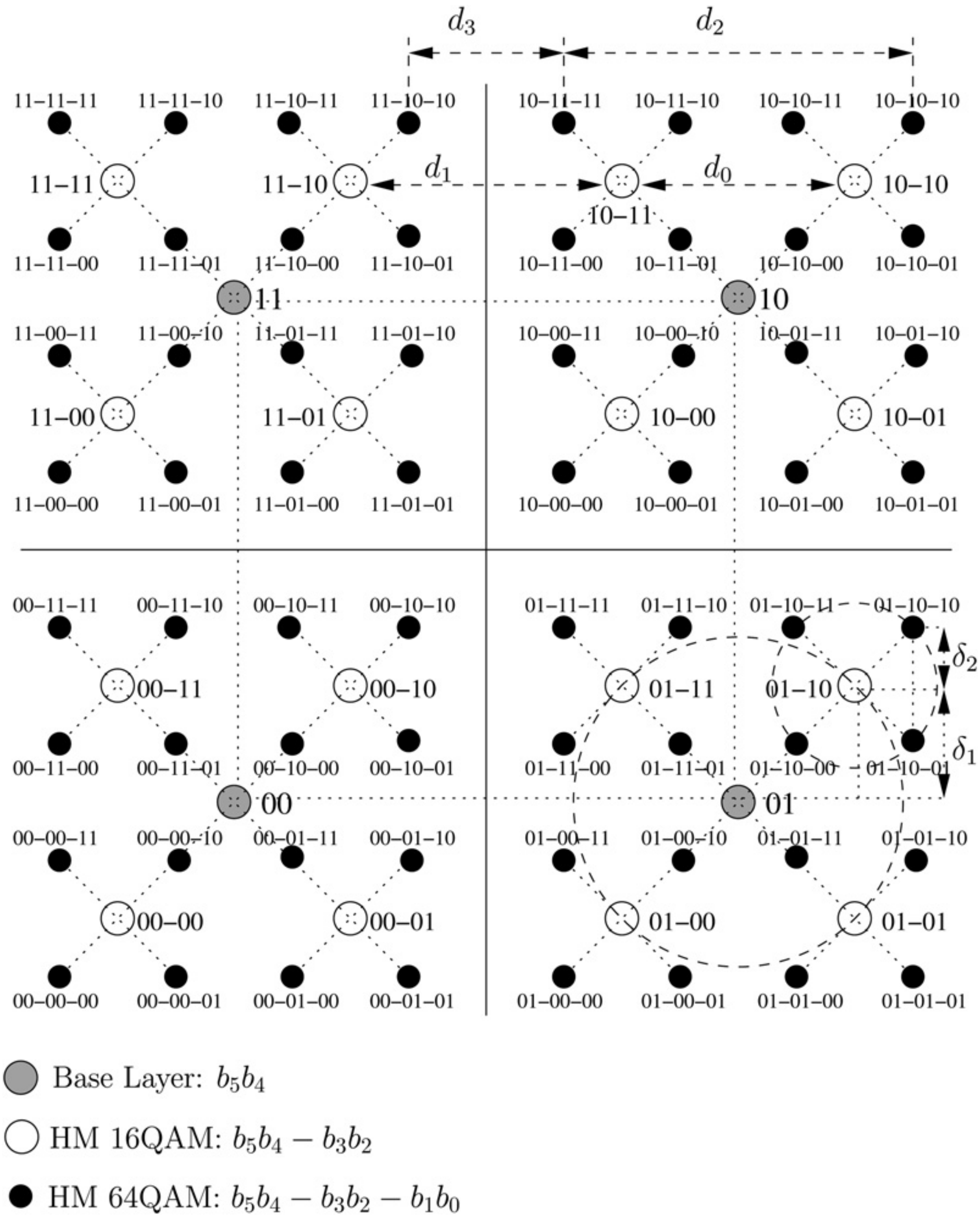

Fig. 3 *Constellation map of the HM scheme, where* $R_1 = d_1/d_0$, $R_2 = d_3/d_2$

proposed in [17] is assisted by turbo-trellis coded modulation, where set-partition-based bit-to-symbol mapping would give a better performance compared with Gray mapping. We also use set-partitioning-based mapping in this paper. Note, however, that the type of mapping used does not affect the DCMC capacity, as shown in Fig. 3.

We denote the six bits of a HM-64QAM symbol as $(b_5b_4b_3b_2b_1b_0)$, where L_1 is occupied by (b_5b_4), while (b_3b_2) belong to L_2 and (b_1b_0) are contained in L_3. The generation rule of the triple-layer HM-64QAM symbols may be expressed as

$$S_{\text{HM-64QAM}} = \beta\left[S_{\text{4QAM}} \pm \sqrt{2}\delta_1\, e^{\pm(\pi/4)j} \pm \sqrt{2}\delta_2\, e^{\pm(\pi/4)j}\right]. \quad (6)$$

A HM ratio pair of $(R_1 = d_1/d_0,\ R_2 = d_3/d_2)$ is defined along with the control parameters β, δ_1 and δ_2. The relationships among these parameters are

$$\beta = 1/\sqrt{1 + 2\delta_1^2 + 2\delta_2^2}, \quad (7)$$

$$\delta_1 = \frac{1}{\sqrt{2}(1 + R_1)}, \quad (8)$$

$$\delta_2 = \frac{R_1 - R_2}{\sqrt{2}(1 + R_1)(1 + R_2)}. \quad (9)$$

The HM ratio pair is used for controlling the formation of the constellation map and the restrictions imposed on the HM ratio pair are

$$\begin{cases} 0 < R_2 < R_1 & \text{if } R_1 < 1 \\ \frac{1}{2}(R_1 - 1) < R_2 < R_1 & \text{if } R_1 > 1. \end{cases} \quad (10)$$

The related derivations of δ_1, δ_2 as well as the restrictions of R_1 and R_2 are detailed in [17]. In the simulations, different HM ratio pairs will be tested and we will optimise the average SNR_t $(\overline{\text{SNR}_t})$ of the system based both on the HM ratio pair (R_1, R_2) and on the positions of the two RNs.

4 DCMC capacity-based system analysis

On the basis of the capacity bounds of the full-duplex relay channels proposed in [19], the general upper bound on the continuous-input continuous-output memoryless channel capacity of a half-duplex relaying system has been investigated in [20]. By contrast, the DCMC capacity was detailed in [21], which is more pertinent for the design of channel-coded modulation. The DCMC capacity bounds of a practical half-duplex relaying system were further investigated in [22, 23]. In this paper, the DCMC capacity will be used to calculate the bound of our HM-aided cooperative communication system, as well as the minimum receive SNR_r required for receiving L_1, L_2, L_3 from the triple-layer HM-64QAM symbols, when assuming that a 'perfect' channel code is employed. The input to the DCMC channel is $X = \{x_0, x_1, \ldots, x_{M-1}\}$, where M is the constellation size and x_i is the complex-valued modulated symbol. The corresponding output symbols are $Y = \{y_0, y_1, \ldots, y_{M-1}\}$. The transition probability of receiving y given that x_k is transmitted is expressed as [21]

$$p(y|x_k) = \frac{1}{\pi N_0}\exp\left(\frac{-|y - hx_k|^2}{N_0}\right), \tag{11}$$

where we have

$$p(y) = \sum_{k=0}^{M-1} p(y|x_k)p(x_k). \tag{12}$$

The mutual information of receiving y when x_k is transmitted is given by $\log_2[p(y|x_k)/p(y)]$, hence the average mutual information of getting the output Y with the input X may be derived as [21, 24]

$$I(X;Y) = \sum_{i=0}^{M-1}\int_{-\infty}^{+\infty} p(y|x_i)p(x_i)\log_2\left(\frac{p(y|x_i)}{\sum_{k=0}^{M-1} p(y|x_k)p(x_k)}\right)\mathrm{d}y. \tag{13}$$

So the DCMC capacity [Some authors refer to this modulation-dependent DCMC capacity as the achievable rate.] C can be formulated as

$$C_{\text{DCMC}}^{\text{ML}} = \max_{p(x_i)} I(X;Y), \tag{14}$$

where ML stands for maximum likelihood, $I(X;Y)$ is maximised when we have $p(x_i) = 1/M(i \in [0{:}M-1])$ and (14) may be simplified as [21]

$$C_{\text{DCMC}}^{\text{ML}} = \log_2(M) - \frac{1}{M}\sum_{i=0}^{M-1} E\left[\log_2\sum_{k=0}^{M-1}\exp(\Phi_{i,k})|x_i\right], \tag{15}$$

where the unit of C is bits per symbol (bps). $E[A|x_i]$ is the expectation of A conditioned on x_i, whereas the term $\Phi_{i,k}$ may expressed similar to that in [21]

$$\Phi_{i,k} = \frac{-\left|\sqrt{G}h(x_i - x_k) + n\right|^2 + |n|^2}{N_0}. \tag{16}$$

where h is the fading coefficient, G is the path gain and n is the AWGN at the receiver.

4.1 Channel capacity of the SN–DN link

The detection rules of the triple-layer HM-64QAM symbols are detailed in [17]. Note that the assumption of the system is that $\text{SNR}_\text{t}^{\text{SN}}$ is set to be relatively low so that the DN may only be capable of receiving L_1. Hence, even though the signal broadcast by the SN during the first TS is a HM-64QAM symbol, the DN may treat it as those 4QAM symbols, where the soft information derived by the De-Mapper at the DN may be expressed as

$$p\left(y_{\text{SD}}|L_{1,q}^{(i)}\right) = p\left(y_{\text{SD}}|x_q^{(i)}\right) = \frac{1}{\pi N_0}\exp\left(-\frac{\left|y_{\text{SD}} - \sqrt{G_{\text{SD}}}h_{\text{SD}}x_q^{(i)}\right|^2}{N_0}\right)$$
$$x_q^{(i)} \in \{\beta\,\mathrm{e}^{\mathrm{j}\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}3\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}-3\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}-\pi/4}\}, \tag{17}$$

where we have $i \in \{0, 1, 2, 3\}$ and $q \in \{1, 2, \ldots, \eta\}$, q is the order of the symbol in the received signal frame, while η is the block size of the soft decoder. The $(\eta \times 4)$-element soft information matrix $p(y_{\text{SD}}|L_{1,q}^{(i)})$ may then be sent to decoder 1 for detecting the information contained in L_1. Therefore, the layer L_1 may be received by the DN, while the information in L_2 and L_3 may be discarded because of having an insufficient receive SNR_r. Upon substituting (17) into (13), the DCMC capacity of only receiving L_1 from the coded HM-64QAM signal at DN may be expressed as

$$C_{\text{HM-64QAM}}^{L_1} = 2 - \frac{1}{4}\sum_{i=0}^{3} E\left[\log_2\sum_{k=0}^{3}\exp(\Phi_{i,k})|x^{(i)}\right], \tag{18}$$

where we have $x^{(i)} \in \{\beta\,\mathrm{e}^{\mathrm{j}\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}3\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}-3\pi/4}, \beta\,\mathrm{e}^{\mathrm{j}-\pi/4}\}$ and β is the normalisation parameter of the HM-64QAM symbols based on the current HM ratio, as exemplified in Section 3.

4.2 Channel capacity of the SN–RN$_1$ link

For decoding the information contained in layer L_2 of the HM-64QAM symbols, RN_1 would detect the signal frame $\{x_1\}$ as HM-16QAM symbols, which is formulated as

$$p\left(y_{\text{SR}_1}|x_q^{(i)}\right) = \frac{1}{\pi N_0}\exp\left(-\frac{\left|y_{\text{SR}_1} - \sqrt{G_{\text{SR}_1}}h_{\text{SR}_1}x_q^{(i)}\right|^2}{N_0}\right)$$
$$x_q^{(i)} \in \left\{\beta\left[S_{4\text{QAM}} \pm \sqrt{2}\delta_1\,\mathrm{e}^{\pm(\pi/4)\mathrm{j}}\right]\right\}, \tag{19}$$

where $i \in \{0, 1, \ldots, 15\}$, and then the log-likelihood ratio (LLR) converter 1 (as shown in Fig. 2) will calculate the soft information of L_2 from the HM-64QAM symbols we received, which may be expressed as

$$p\left(y_{\text{SR}_1}\left|L_{2,q}^{(l)}\right.\right) = p\left(y_{\text{SR}_1}|x_q^{(l)}\right) + p\left(y_{\text{SR}_1}|x_q^{(l+4)}\right) + p\left(y_{\text{SR}_1}|x_q^{(l+8)}\right) + p\left(y_{\text{SR}_1}|x_q^{(l+12)}\right). \tag{20}$$

In (20), we defined $L_2^{(0)}$ as the pair of bits (00) in L_2, $L_2^{(1)}$ as (01), $L_2^{(2)}$ (10) and finally $L_2^{(3)}$ for (11). Then, RN_1 may decode the layer L_2 according to the $(\eta \times 4)$-element soft information matrix $p(y_{\text{SR}_1}|L_{2,q}^{(l)})$. Since RN_1 demaps the HM-64QAM symbol as a 16QAM signal, the DCMC capacity that we calculate based on the output of the HM-16QAM De-Mapper block (as seen in Fig. 2) is the joint capacity of the two independent layers, namely of L_1 and L_2. The DCMC capacity of receiving L_1 and L_2 of HM-64QAM may be expressed as

$$C_{\text{HM-64QAM}}^{L_1,L_2} = 4 - \frac{1}{16}\sum_{i=0}^{15} E\left[\log_2\sum_{k=0}^{15}\exp(\Phi_{i,k})|x^{(i)}\right], \tag{21}$$

where we have

$$x^{(i)} \in \left\{\beta\left[S_{4\text{QAM}} \pm \sqrt{2}\delta_1\,\mathrm{e}^{\pm(\pi/4)\mathrm{j}}\right]\right\}.$$

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We assume that the pair of bits contained in L_1 is (b_5b_4) of Fig. 3, while (b_3b_2) belong to L_2. Then, based on the chain rule of mutual information [24, 25], we arrive at

$$I(b_5, b_4, b_3, b_2; y) = I(b_5, b_4; y) + I(b_3, b_2; y|b_5, b_4). \quad (22)$$

Therefore, it can be stated that

$$I(b_3, b_2; y|b_5, b_4) = I(b_5, b_4, b_3, b_2; y) - I(b_5, b_4; y), \quad (23)$$

where we have $C^{L_1,L_2}_{\text{HM-64QAM}} = \max\{I(b_5, b_4, b_3, b_2; y)\}$, which is the DCMC capacity of receiving both L_1 and L_2. Furthermore, we have $C^{L_1}_{\text{HM-64QAM}} = \max\{I(b_5, b_4; y)\}$, which is the DCMC capacity of receiving L_1 from the triple-layer HM-64QAM signal. When considering $I(b_3, b_2;y|b_5, b_4)$, we found that the reception of L_2 will not be totally independent of the information contained in L_1. The DCMC capacity of receiving L_2 will only be approached when L_1 is perfectly received. Hence we may define the DCMC capacity of receiving L_2 to be $C^{L_2}_{\text{HM-64QAM}} = \max\{I(b_3, b_2; y|b_5,b_4)\}$, where the DCMC capacity of the layer L_2 is given by

$$C^{L_2}_{\text{HM-64QAM}} = C^{L_1,L_2}_{\text{HM-64QAM}} - C^{L_1}_{\text{HM-64QAM}}. \quad (24)$$

4.3 Channel capacity of the SN–RN$_2$ link

The RN$_2$ will retransmit the information of L_3 of the HM signal and the output of the HM-64QAM De-Mapper block in Fig. 2 will be a ($\eta \times 64$)-element soft information matrices

$$p\left(y_{\text{SR}_2}\middle|x_q^{(i)}\right) = \frac{1}{\pi N_0}\exp\left(-\frac{\left|y_{\text{SR}_2} - \sqrt{G_{\text{SR}_2}}h_{\text{SR}_2}x_q^{(i)}\right|^2}{N_0}\right)$$
$$x_q^{(i)} \in \left\{\tilde{\beta}\left[S_{4\text{QAM}} \pm \sqrt{2}\delta_1 e^{\pm(\pi/4)j} \pm \sqrt{2}\delta_2 e^{\pm(\pi/4)j}\right]\right\}, \quad (25)$$

where $i \in \{0, 1, \ldots, 63\}$. Let $L_3^{(0)}$ denote the pair of bits (00) in L_3, $L_3^{(1)}$ represent (01), $L_2^{(3)}$ (10) and finally $L_3^{(3)}$ for (11). Then the LLR converter 2 of Fig. 2 may produce the soft information matrix for decoding L_3

$$p\left(y_{\text{SR}_2}|L_{3,n}^{(l)}\right) = \sum_{k=0}^{15} p\left(y_{\text{SR}_1}|x_n^{(i=4k+l)}\right). \quad (26)$$

Since the RN$_2$ has to fully demap the HM-64QAM signal, the DCMC capacity of transmitting the HM-64QAM symbols may be expressed as

$$C_{\text{HM-64QAM}} = 6 - \frac{1}{64}\sum_{i=0}^{63} E\left[\log_2 \sum_{k=0}^{63} \exp(\Phi_{i,k})|x^{(i)}\right], \quad (27)$$

where we have

$$x^{(i)} \in \{\beta\left[S_{4\text{QAM}} \pm \sqrt{2}\delta_1 e^{\pm(\pi/4)j} \pm \sqrt{2}\delta_2 e^{\pm(\pi/4)j}\right]\}$$

and β is the normalisation parameter of the HM-64QAM symbols based on the current HM ratio. The pair of bits contained in the layers $L_1 = (b_5b_4)$, $L_2 = (b_3b_2)$ and $L_3 = (b_1b_0)$ may be expressed according to the chain rule of mutual information [24, 25] as

$$I(b_5, b_4, b_3, b_2, b_1, b_0; y) = I(b_5, b_4, b_3, b_2; y) + I(b_1, b_0; y|b_5, b_4, b_3, b_2). \quad (28)$$

We have $C_{\text{HM-64QAM}} = \max\{\text{I}(b_5, b_4, b_3, b_2, b_1, b_0; y)\}$ and $C^{L_1,L_2}_{\text{HM-64QAM}} = \max\{I(b_5, b_4, b_3, b_2; y)\}$, hence the DCMC capacity of receiving L_3 is $C^{L_3}_{\text{HM-64QAM}} = \max\{I(b_1, b_0; y|b_5, b_4, b_3, b_2)\}$, which is given by

$$C^{L_3}_{\text{HM-64QAM}} = C_{\text{HM-64QAM}} - C^{L_1,L_2}_{\text{HM-64QAM}}. \quad (29)$$

4.4 Overall system optimisation

In our simulations, the same rate-1/2 encoder is employed by all three SN encoders. Hence we only focus our attention on the specific SNR values, where the DCMC capacity reaches 1 bps. Multiple values of the two HM ratios R_1 and R_2 had been tested. At a given HM ratio pair (R_1, R_2), the minimum receive SNR required $\text{SNR}_r^{L_1}$ for decoding the L_1 at the DN, $\text{SNR}_r^{L_2}$ of L_2 at the RN$_1$ and $\text{SNR}_r^{L_3}$ of L_3 at RN$_2$ may be computed. The SNR differences among the three layers are

$$\mathcal{G}^{L_1,L_2}_{\text{SNR}} = \text{SNR}_r^{L_2} - \text{SNR}_r^{L_1}(\text{dB}), \quad (30)$$

$$\mathcal{G}^{L_1,L_3}_{\text{SNR}} = \text{SNR}_r^{L_3} - \text{SNR}_r^{L_1}(\text{dB}), \quad (31)$$

where $\mathcal{G}^{L_1,L_j}_{\text{SNR}}$ is the SNR difference between $\text{SNR}_r^{L_1}$ and $\text{SNR}_r^{L_j}$, for $j \in \{2, 3\}$. If we set SNR_t^{SN} to be identical to the SNR required for receiving the information of L_1, namely to $\text{SNR}_t^{\text{SN}} = \text{SNR}_r^{L_1}$, this would guarantee that the BER of decoding L_1 would reach an arbitrarily low value. In this situation, if we want the BER performance of receiving L_2 to become sufficiently low, the channel gain G_{SR_1} of the SN–RN$_1$ link should satisfy

$$10\log_{10} G_{\text{SR}_1} + \text{SNR}_r^{L_1} = \text{SNR}_r^{L_2}. \quad (32)$$

If we use the distance ratio $d_{\text{SR}_1}/d_{\text{SD}}$ to represent the position of the RN, we arrive at

$$\mathcal{G}^{L_1,L_2}_{\text{SNR}} = 10\log_{10}\left(\frac{d_{\text{SD}}}{d_{\text{SR}_1}}\right)^3, \quad (33)$$

where $\mathcal{G}^{L_1,L_2}_{\text{SNR}}$ is given by (30) and hence we have

$$\frac{d_{\text{SR}_1}}{d_{\text{SD}}} = 10^{-\frac{\mathcal{G}^{L_1,L_2}_{\text{SNR}}}{30}}. \quad (34)$$

Once the position of the RN is available, the path gain between the RN and DN link can be formulated as

$$G_{R_1D} = \left(1 - \frac{d_{\text{SR}_1}}{d_{\text{SD}}}\right)^{-3}. \quad (35)$$

In the capacity analysis, we observe from (15) that a system employing a rate-1/2 channel coding scheme and 4QAM modulation for communication over uncorrelated Rayleigh fading channels requires $\text{SNR}_r = 1.81$ dB to reach a DCMC capacity of 1 bit per symbol. Hence the SNR_t of RN$_1$ ($\text{SNR}_t^{\text{RN}_1}$) has to satisfy

$$\text{SNR}_t^{\text{RN}_1} = 1.81 - 10\log G_{R_1D}\ (\text{dB}). \quad (36)$$

Similarly, the SNR_t of RN$_2$ ($\text{SNR}_t^{\text{RN}_2}$) may be formulated as

$$\text{SNR}_t^{\text{RN}_2} = 1.81 - 10\log G_{R_2D}\ (\text{dB}), \quad (37)$$

while the position of RN$_2$ is related to

$$\frac{d_{\text{SR}_2}}{d_{\text{SD}}} = 10^{-\frac{\mathcal{G}^{L_1,L_3}_{\text{SNR}}}{30}}. \quad (38)$$

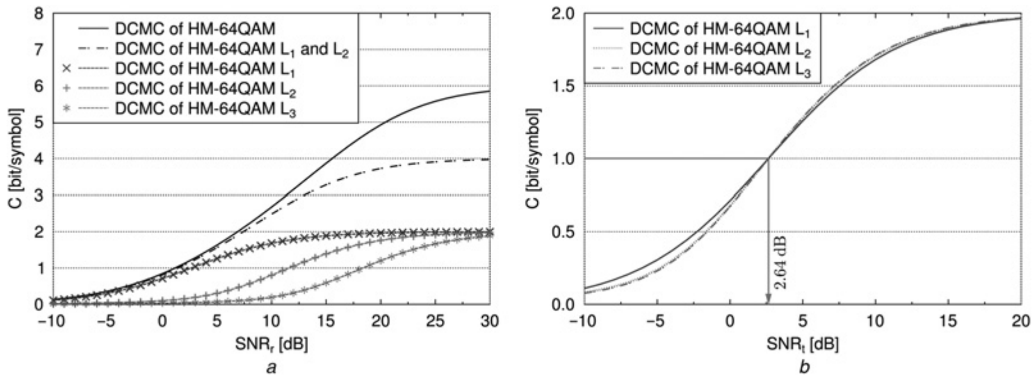

Fig. 4 *DCMC capacity against SNR of our optimised triple-layer HM scheme, where the HM-64QAM ratio pair is ($R_1 = 1.5$, $R_2 = 0.6$). The simulations are based on (17)–(32), and the number of samples used for calculating the DCMC capacity is 100,000. The channel is an uncorrelated Rayleigh fading channel*

a DCMC capacity against SNR_r
b DCMC capacity against SNR_t

Likewise, the SNR_t^{SN} of the SN should guarantee that

$$SNR_t^{SN} = SNR_r^{L_1}. \tag{39}$$

Hence, the average SNR_t of the entire system is given by

$$\overline{SNR_t}(dB) = 10\log_{10}\left(\frac{10^{\left(SNR_t^{SN}/10\right)} + 10^{\left(SNR_t^{RN_1}/10\right)} + 10^{\left(SNR_t^{RN_2}/10\right)}}{3}\right). \tag{40}$$

5 DCMC capacity analysis-based results

According to the analysis of Section 4, we investigated the performance of the cooperative communication system of Fig. 2, when assuming that a 'perfect' channel coding scheme is employed. Fig. 4 shows the DCMC capacity of HM-64QAM, when the HM ratio pair is ($R_1 = 1.5$, $R_2 = 0.6$). From (18), the DCMC capacity of the system transmitting HM-64QAM symbols which are received as 4QAM symbols of L_1 may be derived. By contrast, the DCMC capacity of receiving L_1 and L_2 from the HM-64QAM signal is determined by (21), while (27) formulates the DCMC capacity of receiving the HM-64QAM symbol streams. The DCMC capacity of receiving only L_2 (or L_3) information from the triple-layer HM-64QAM symbol is expressed by (24) [or (29)]

From the results in Fig. 4*a*, the SNR_r values required for receiving the information of L_1, L_2 and L_3 from the HM symbol are 2.64, 11.60 and 18.35 dB. Therefore, with the assistance of the two RNs, SNR_t^{SN} was reduced to 2.64 dB. Now the optimum position of the two RNs and the minimised $\overline{SNR_t}$ of the HM-64QAM system using the ratio pair of ($R_1 = 1.5$, $R_2 = 0.6$) may be determined. From (30) to (40), the optimum position of RN_1 was formulated to be $d_{SR_1}/d_{SD} = 0.50$ and $SNR_t^{RN_1} = -7.30$ dB, while the optimum position of RN_2 is $d_{SR_2}/d_{SD} = 0.30$ and $SNR_t^{RN_2} = -2.82$ dB. Hence, we have $\overline{SNR_t} = -0.71$ dB according to (40). Observe from the DCMC capacity versus SNR_t^{SN} curves of Fig. 4*b* that there is an intersection point among the three capacity curves of L_1, L_2 and L_3. This illustrates that with the aid of a sufficient path gain, the detection of L_2 at RN_1 and the detection of L_3 at RN_2 are achieved together with the detection of L_1 at the DN for the same $\overline{SNR_t}$ value of −0.71 dB. In order to find the optimum HM-64QAM ratio pair, multiple groups of (R_1, R_2) have been investigated in the same way and Fig. 5 was generated.

The resultant three-dimensional (3D)$\overline{SNR_t}$ versus (R_1, R_2) plot is shown in Fig. 5, where we can observe that the optimum HM-64QAM ratio pair for our cooperative system is ($R_1 = 1.5$, $R_2 = 0.6$) and the minimum $\overline{SNR_t}$ of −0.71 dB is considered to be the lower performance bound of our cooperative communication system. Hence, the lower performance bound of our cooperative communication system shown in Fig. 2 is $\overline{SNR}_{tmin} = -0.71$ dB, while the throughput of the system is 1 bps.

As mentioned in Section 4.4, for a single link system assisted by a rate-1/2 'perfect' channel coding scheme using conventional 4QAM mapping, which has the same throughput as our optimised system, the SNR_t required for achieving the DCMC capacity of 1 bps is about 1.81 dB. This means that in order to transmit three independent symbol frames, the system requires three TSs and the $\overline{SNR_t}$ necessitated is 1.81 dB, which is 1.81 + 0.71 = 2.52 dB higher than that of our optimised and idealised cooperative communication system operating at the DCMC capacity. Meanwhile, Fig. 6 makes a comparison between our optimised triple-layer HM-64QAM scheme and the conventional 16QAM scheme. It can be observed that, if a rate-3/4 'perfect' channel coding scheme is employed by the conventional 16QAM, the required receive SNR for receiving the three information bits in the coded 16QAM symbol would be 12.35 dB. While, the required receive SNR for receiving the information bit contained in L_1, L_2 and L_3 of our optimised coded HM-64QAM symbol are 2.64, 11.60 and 18.35 dB, respectively. Therefore, it can be observed that, in order to receive the total three-layer information in our optimised HM scheme, the required receive SNR_r is about 18.35 − 12.35 = 6.0 dB higher than that of the coded conventional 16QAM scheme, which is considered to be the main drawback of the HM scheme. As a remedy, cooperative communications may

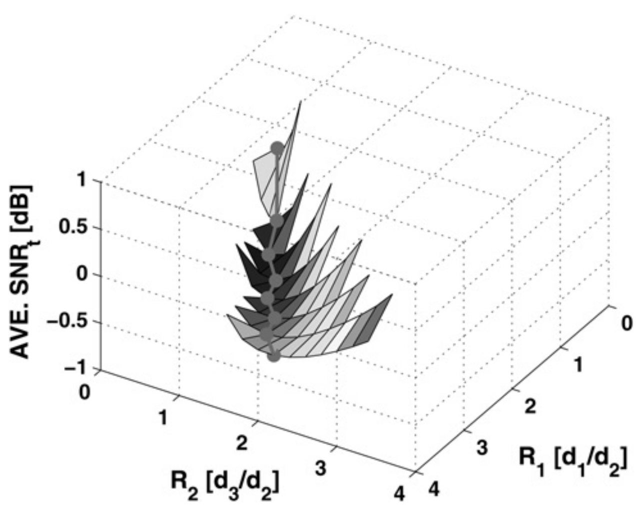

Fig. 5 *3D plot of the DCMC-based $\overline{SNR_t}$ surface of the entire system when using 'perfect' channel codes. The simulations are based on (30)–(40), and the number of samples used for calculating the DCMC capacity is 100, 000. The channel is an uncorrelated Rayleigh fading channel*

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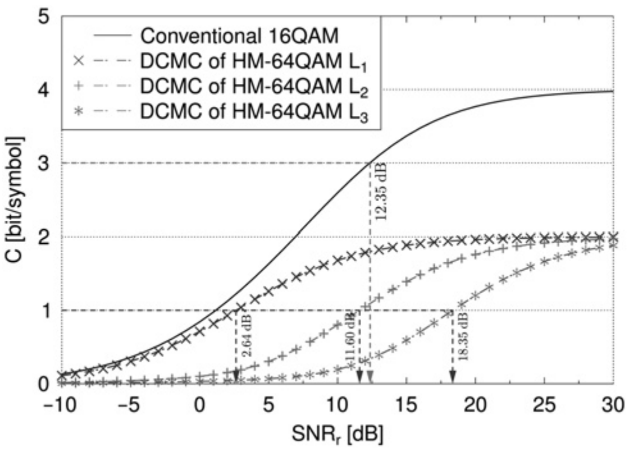

Fig. 6 *DCMC capacity against SNR of both our optimised triple-layer HM scheme and of the conventional 16QAM scheme. The HM-64QAM ratio pair is ($R_1 = 1.5$, $R_2 = 0.6$). The simulations are based on (17)–(29), and the number of samples used for calculating the DCMC capacity is 100, 000. The channel is an uncorrelated Rayleigh fading channel*

be employed and we found that the transmit SNR_t for the SN may be reduced to 2.64 dB. More explicitly, due to the reduced path loss, the two RNs may be able to receive L_2 and L_3 correctly and separately with a reduced transmission power at the SN. Furthermore, each RN only needs to forward a single layer of information to the DN, which further helps to reduce the required SNR_t at the two RNs. Hence, by employing the HM in cooperative communications, we may be able to reduce the SNR_t at each node in the cooperative networks. However, the improved power efficiency will lead to a decreased time efficiency, note that our optimised HM scheme requires three TSs to convey three information bits. By contrast, if the transmit SNR_t is higher enough, it only needs a single TS for the rate-3/4 encoder-aided conventional 16QAM scheme to deliver the same amount of information.

6 Conclusions

A HM-aided cooperative communication system was proposed in this paper. The DCMC capacity of our specifically designed HM schemes has been formulated and the DCMC capacity of each individual layer of our HM-64QAM scheme was also derived. Theoretically, if the system relies on a 'perfect' rate-1/2 channel coding scheme, our communication strategy becomes capable of reducing the $\overline{SNR_t}$ of the entire system investigated in Section 5 to −0.71 dB, while the required SNR_t^{SN} may be set to 2.64 dB. The results showed in this paper are mainly based on simulations. The theoretical analysis of the coded HM in cooperative communication may be considered in future work, although simplifying assumptions may be required for making the problem analytically tractable. On the basis of this DCMC capacity analysis, near-capacity HM may be designed for cooperative communication in our future research.

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