Coherent Versus Non-Coherent Quantum-Assisted Solutions in Wireless Systems

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Abstract

Each mobile phone transmits its own uplink information to the base station, which results in their superposition. Therefore, the base station has to determine which symbol each of the users has transmitted with (coherent), or without (non-coherent) the knowledge of the channels' estimates. In both scenarios, an optimization problem has to be addressed. Conventional, low-complexity solutions experience degraded performance when the number of receive antenna elements at the base station is lower than the number of mobile terminals. The optimal, full-search-based equivalent multi-level symbol detector offers the best bit error ratio performance, but at a potentially excessive complexity. Quantum search algorithms may be invoked for achieving near-optimal performance at low complexity.

Introduction

Meet Shanman, who is a wireless communications engineer. He has been tasked with finding the lowest-complexity optimum-performance receiver for a multi-user wireless system. Every person is requesting data continuously, be it speech, video or accessing the Internet. Shanman has decided to initially focus on the users' signal detection and the channel estimation processes, while bearing in mind other important tasks, such as channel coding and synchronization.

How should he approach the channel estimation problem? Should he ask the users to frequently send pilot symbols so that the base station reliably estimates the channels? Indeed, that would attain good performance, but the system's throughput would decrease. Should he instead ask the users to send fewer pilot symbols and also invoke a decision directed channel estimation (DDCE) process, where the symbols perfectly detected by the multi-user detector (MUD) can be assumed to be pilot signals to help the channel estimation process? That would indeed succeed in increasing the throughput, but it would also increase the complexity of the channel estimation. Somewhat bewildered by the plethora of design options, he wonders how he should solve the optimization problem imposed by the channel estimation in these coherent wireless systems? What if he decided to opt for a non-coherent wireless system by dispensing with channel estimation altogether, hence avoiding its complexity and increasing the throughput this way? As seductive as it is, the performance would degrade and indeed, an ingeniously engineered low-complexity signal detector would have to be used for detecting the transmitted symbols.

Somewhat bemused, he continues pondering how he should deal with the MUD problem? Should he create a system where every user is served individually, while everyone else's signal is treated as interference? Well, that would indeed require a low-complexity detector, but what about its performance? After all, don't we need everybody's data at the BS? Shanman exclaims, there must be something better we can do. After further deliberations, he finds that there is a function that gives the probabilities of the transmitted symbols. The base station knows the set of legitimate signals that each user transmits, albeit the channel states may or may not be known, depending on whether the receiver is coherent or non-coherent, but again, the superimposed signals are obviously available. Hence, Shanman sets out to find which specific combination of composite multi-user symbols offers the highest probability, which is clearly the best he can do performance-wise, while treating all the users fairly. But how should we solve this optimization problem related to multi-user detection without performing a full search, which checks the probability of all the multi-level symbol combinations one by one?

While reflecting on these dilemmas, it dawns on Shanman that regardless of which schemes he finally chooses for his receiver, he will eventually have to deal with optimization problems. While browsing the web, he stumbled upon the Exquisite Ultra Reliable Evolution & Key Algorithms (EUREKA) exhibition, a place where new bio-inspired optimization tools are demonstrated to be able to mysteriously sense and pinpoint meritorious, but not necessarily optimal, solutions of a search problem, while invoking a limited number of search steps. He decided to visit the exhibition. When he arrived, he viewed a few solutions.

Ants in a Chip: Put out some food far away from their nest and wait until one of the ants roaming in a quest for food finds it. By depositing pheromone to mark the way for its successors, soon the entire colony will converge on the food. Shanman is elated: wow, each legitimate symbol combination corresponds to a different path, and a more probable symbol combination corresponds to a shorter path, right? The ants will eventually find the shortest path, which is related to the specific symbol combination that maximizes the probability function.

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Particle Swarm: Similar to the ant colony, the particles initially fly around randomly, but eventually gather around the most likely symbol combination.

Chromosomes in a Chip: Following a genetic algorithm, see them magically mutate from one symbol combination to the next and finally take the form of the most probable combination.

Shrinking Sphere: Place a sphere over a carefully selected subset of legitimate symbols and see it shrink before your eyes, eventually encapsulating only the most likely symbol combination.

Excited by this small sample of available solutions, Shanman asked one of the exhibitors whether these optimization tools will always succeed in finding the particular multi-level symbol combination that maximizes the correct decision probability. He was pleased to hear that in regular scenarios, these algorithms will perform well, but in rank-deficient scenarios, where the number of users is higher than the number of receive antenna elements at the base station, they may erroneously converge to a sub-optimal solution. The glint in Shanman's eyes subsides, he leaves with a brochure that reads:

Pre-order the Qute Brute: The quantum dog that simultaneously sniffs all the symbol combinations at the same time and exuberantly barks at the most probable multi-level symbol.

"Will he ever fail?," asks Shanman.

"Rarely," the exhibitor replies.

Bridging Wireless Communications and Quantum Computing

According to Moore's law, outlined in 1965, the number of transistors in a chip doubles every year. It is inevitable that in a few years the size of a transistor will be below the atomic scale, essentially moving from nanotechnology to quantum technology. In 1985, Richard Feynman proposed the encoding of information in the polarization of photons and in the spin of electrons, instead of just relying on their charge and number, respectively. In the wireless communications camp, Claude Shannon focused his research on a system's channel capacity, without giving cognizance to the complexity required to approach the capacity.

Wireless communication systems and quantum computing may be amalgamated for creating near-capacity, low-complexity wireless systems. Shanman deals with classical communication systems, where some processes, such as the MUD considered here, may be replaced by quantum computing algorithms, such as the Qute Brute, which represents the family of Quantum Search Algorithms (QSA). In other words, the scope of this article is *quantum-assisted communications* [1, 2], where specific classical processes are replaced by quantum computers in an otherwise classical transceiver. The communication protocols, as well as most of the components in a mobile handset and in base stations, will remain intact, while some processes will be replaced by quantum processes, empowered by a quantum chip. *Quantum-domain based communications*, where quantum bits, or *qubits* [3], are not only used for computing, but are also transmitted over wireless channels is beyond the scope of this article, but we suggest

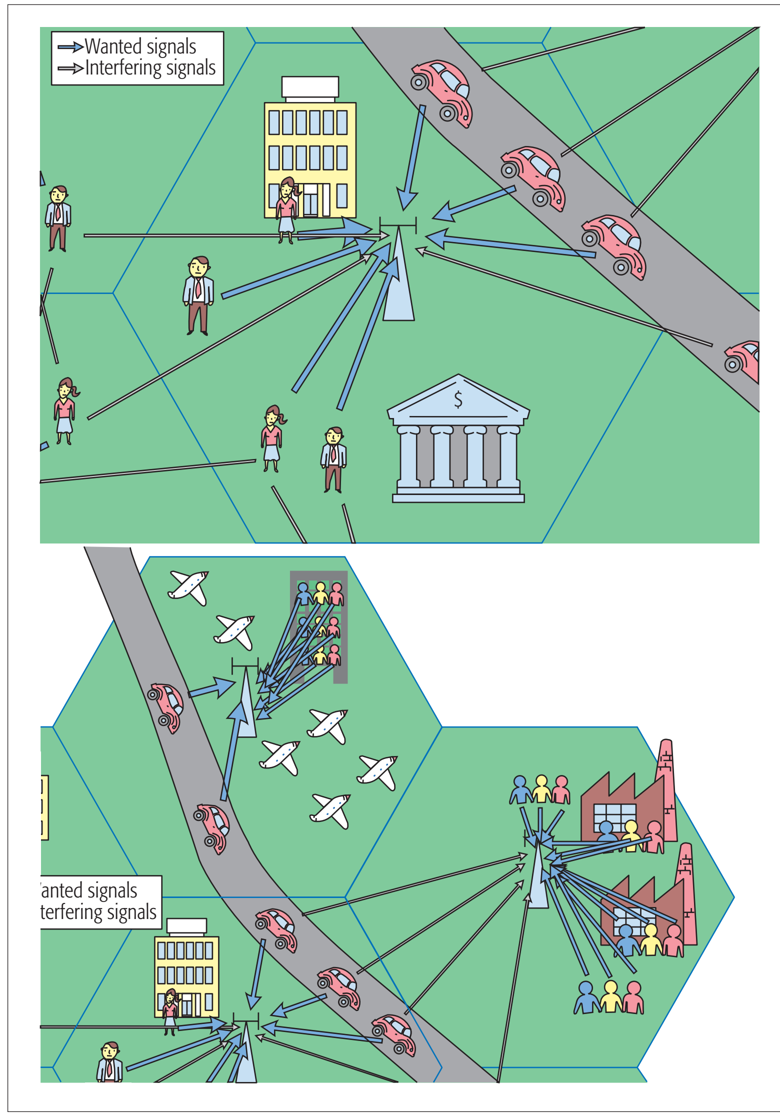

FIGURE 1. Which symbol did each of the users transmit? The symbol detector at the base station has been assigned the task to decide which symbol was transmitted by each user. Signals of users from adjacent cells may be either considered as interference, or desired signals, for improving the overall network's performance.

that motivated readers familiarize themselves with the magazine article in [4].

As demonstrated in Fig. 1, when multiple users transmit to the same base station in a synchronous non-orthogonal multiple access system, where the signals arrive simultaneously at the base station, the multi-level symbol detector has to detect which specific symbol was transmitted by each user, relying on the received signal, the received power, the set of legitimate symbols and, in coherent systems, the channel states [5]. When the symbol detector is required to treat each user equitably, its complexity is increased, but the overall performance is also improved. The complexity of the MUD may make its implementation infeasible in crowded areas such as airports, train stations, industrial areas or city centers, as illustrated in Fig. 1.

Let us now focus our discussions on the three uplink system models depicted in Fig. 2. All the users supported in all investigated systems employ convolutional channel coding on their informa-

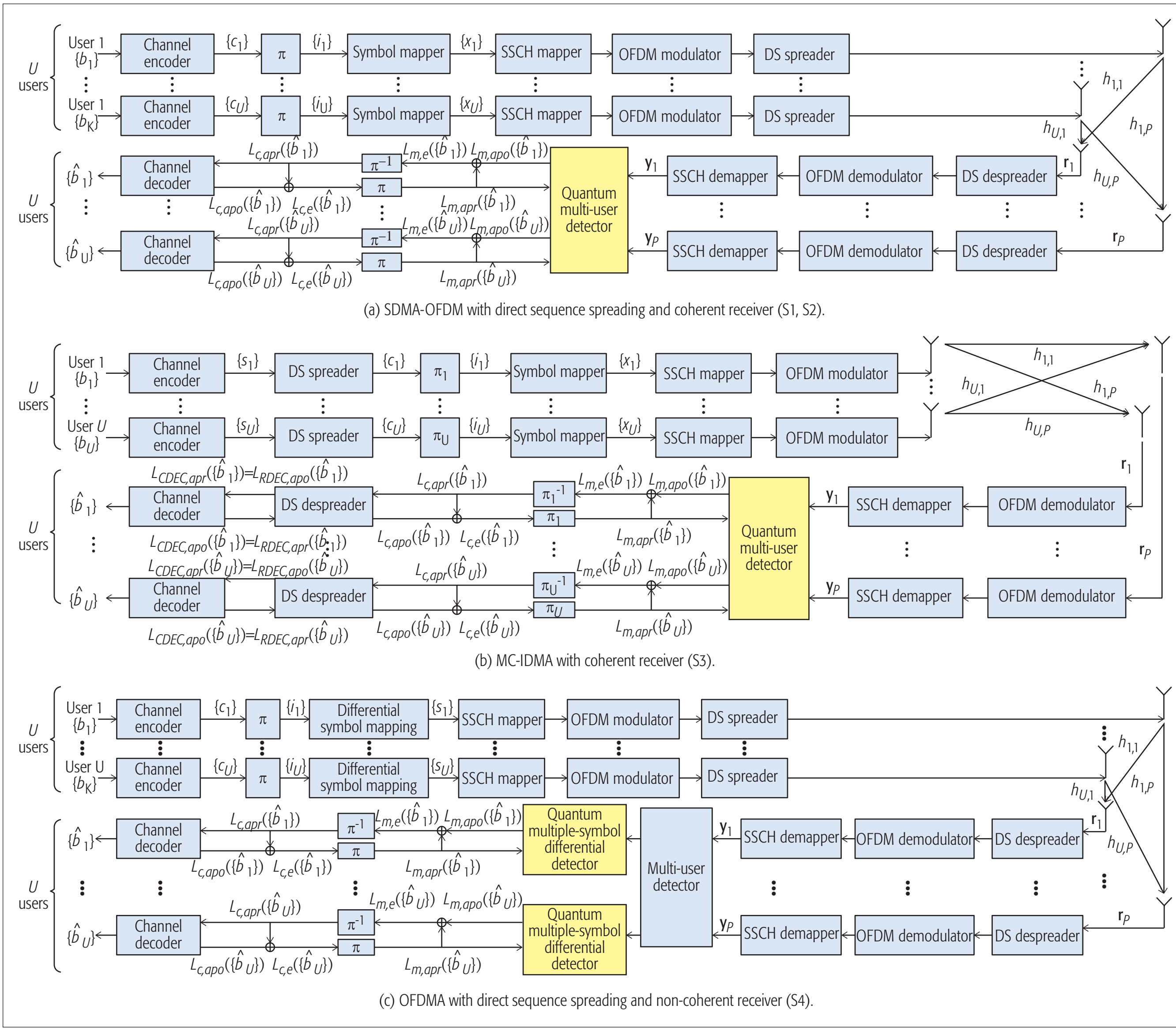

FIGURE 2. The three system models that will be investigated, including two coherent receivers in (a) and (b), and a non-coherent receiver in (c). The abbreviations S1, S2, S3 and S4 will be used for referring to the different example scenarios that will be analyzed in this paper.

tion bit stream $\{b_u\}$, as represented by the "channel encoder/decoder" blocks. Moreover, they eventually interleave their encoded bits $\{c_u\}$ using an interleaving sequence "π" and map the interleaved bits $\{i_u\}$ to symbols $\{x_u\}$, using a coherent or a differential symbol mapper [5]. In a coherent symbol mapper, each symbol is selected based on its own associated bits, while differential modulation maps a user's bits that correspond to a single symbol, with respect to the symbol transmitted during the previous time slot. The "Slow SubCarrier Hopping (SSCH) Mapper/Demapper" and the "Orthogonal Frequency Division Multiplexing (OFDM) Modulator/Demodulator" blocks in Fig. 2 state that all considered uplink systems will rely on multiple carriers for conveying information. In Figs. 2a, and 2b, all the channel estimates are assumed to be available, and coherent symbol mapping takes place. On the other hand, Fig. 2c operates without any knowledge concerning the channel estimates, and differential symbol mapping is adopted. The multiple access scheme used by each system determines its specific methodology of separating the users and detecting their symbols.

In a multi-carrier multiple access system, different orthogonal subcarriers may be allocated to users. Therefore, two users who transmit on a different subcarrier are separated in the frequency domain and they will not interfere, while users who transmit on the same subcarrier will contribute to the multi-user interference exhibited at the receiver. A way to reduce the multi-user interference is to allocate orthogonal spreading sequences to the users. Two users will not interfere if they have been allocated a different orthogonal spreading sequence, even if they transmit on the same subcarrier. However, if two users have been allocated both the same subcarrier and the same spreading sequence, they will interfere. In that case their symbols may be jointly detected at the base station using an MUD, which exploits their spatial signature. Therefore, when a multiple access scheme employs a non-coherent receiver, in which systems the channel states are not available at the base station, two users should be separated either in the frequency domain or code domain. Thus, the MUD of a non-coherent receiver may be represented by simple correlation filters, matching the spreading code of each user. The time domain may also be used for separating the users, by allowing them to use the same subcarrier and spreading code, but only during different time slots. In our investigations, we assume that all users transmit on each time slot, hence the users may be separated in the frequency or code domains.

The Spatial Division Multiple Access scheme combined with OFDM (SDMA-OFDM) and direct sequence spreading in Fig. 2a allows all users to transmit on all available subcarriers. Moreover, each user has been assigned an orthogonal spreading code, resulting in forming groups of users on each subcarrier, where each user in a group uses the same orthogonal spreading code. A pair of users who belong to different groups transmitting on the same subcarrier is separated in the code domain. Therefore, an MUD may be employed at the receiver for detecting the symbols of the users within each group on each subcarrier who have been allocated the same spreading code, based on the users' channel impulse responses.

The Multi-Carrier Interleave Division Multiple Access (MC-IDMA), employed in Fig. 2b, also allows all users to transmit on all available subcarriers. Furthermore, it separates the users by exploiting their unique, user-specific bit-interleaving sequence. Hence it may also be viewed as a chip-interleaved Code Division Multiple Access (CDMA) system. Please note that each user has been assigned a unique interleaving sequence in Fig. 2b, encapsulated by π_u, in contrast to the models in Figs. 2a and 2c. Additionally, a short spreading code is used in the MC-IDMA system after the encoding procedure, as illustrated in Fig. 2b, for increasing the length of the interleaving sequence and hence decreasing the cross-correlation among the users supported. Nevertheless, since the unique interleaving sequences are not orthogonal to each other, the symbols transmitted by each user are detected with the aid of an MUD, by exploiting their channel state information (CSI).

Even though we may separate all users either in the frequency domain or in the code domain in the non-coherent receiver of Fig. 2c, using orthogonal subcarriers and orthogonal spreading sequences, the symbols of each user still have to be detected without any knowledge of the channel states. In order to achieve this, differentially encoded modulation [5, 6] is used in Fig. 2c. At the receiver, the sophisticated "Multiple-Symbol Differential Detector" (MSDD) may be employed for detecting each user's transmitted symbols.

In all systems, iterations exchanging soft-information may be invoked between the symbol detector and the channel decoders, before a final decision is performed on the estimated bits. Regardless of the selected receiver structure, the symbol detector has to solve an optimization problem, which is ultimately related to finding the most likely symbols transmitted by all the users, by maximizing the probability at the channel output. Let us describe that probability as the cost function

$$f(\mathbf{x}) = P(\mathbf{y}\,|\,\mathbf{x}) \cdot P(\mathbf{x}), \tag{1}$$

where $\mathbf{x}$ is a legitimate multi-level symbol vector and $\mathbf{y}$ is the vector that includes the received signals at the P receive antennas of the base station. Please note that the conditional probability $P(\mathbf{y}\,|\,\mathbf{x})$ in Eq. 1 is different in coherent and non-coherent receivers, while a symbol's *a priori* probability $P(\mathbf{x})$ remains the same. In iterative receivers, the CF of Eq. 1 is used for calculating the log-likelihood ratios of the bits $L(\{\hat{b}\})$, which are forwarded to the channel decoders after they are deinterleaved, based on Fig. 2. Let us represent $f(\mathbf{x})$ with $f(x)$, where x now represents the index of a specific symbol combination $\mathbf{x}$ drawn from the set of all legitimate, potentially transmitted multi-level symbols. For example, the 2-user BPSK symbol $\mathbf{x} = [-1, -1]^T$ may be represented by $x = 11_2 = 3_{10}$. The specific symbol $x_{\max}$, that maximizes the cost function $f(x_{\max}) \geq f(x)$ is the solution to the symbol detection optimization problem. Each evaluation of the cost function $f(x)$ will count as a single cost function evaluation (CFE), which will be our complexity metric.

When U users are supported with the aid of the same frequency and code resources, a coherent receiver and M-ary modulation, the optimal maximum *a posteriori* probability (MAP) MUD requires M^U CFEs, one for each legitimate multi-user symbol. In non-coherent receivers, the MAP MSDD searches for $x_{\max}$ jointly in N_w-symbol windows in the time domain for each user independently. Since the first symbol is a reference symbol, transmitted in order for the transmitter and the receiver to have the same reference basis [5], when an M-ary modulation scheme is adopted, the MSDD requires M^{N_w-1} CFEs for searching among all legitimate multi-level symbol combinations for each user.

As presented in Fig. 2, with the aid of quantum computing, a quantum-assisted MUD (QMUD) and a quantum-assisted MSDD (QMSDD) may be employed in coherent and non-coherent receivers, respectively, for reducing the complexity of the symbol detection process, while achieving near-optimal performance.

In a multi-carrier multiple access system, different orthogonal subcarriers may be allocated to users. Therefore, two users who transmit on a different subcarrier are separated in the frequency domain and they will not interfere, while users who transmit on the same subcarrier will contribute to the multi-user interference exhibited at the receiver.

Quantum Search Algorithms for Symbol Detection

Let us define the search problem by assuming the hypothetical scenario, where we know the maximum value of the cost function $\delta = f(x_{\max})$, but we do not know the specific symbol $x_{\max}$ that yields this value. The optimal classic search algorithm would be the full search, which would evaluate $f(x)$ for all legitimate symbols, until one is found to correspond to δ. Assuming that only a single x corresponds to $f(x) = \delta$, the minimum complexity of the full search, in a legitimate symbol set of size N, is equal to 1 CFE, which corresponds to the scenario, where the first evaluated symbol x immediately yielded $f(x) = \delta$. On the other hand, the maximum complexity of the full search is equal to N, encountered in the scenario, where the last evaluated symbol has a cost function value of δ. On average, $N/2$ CFEs are

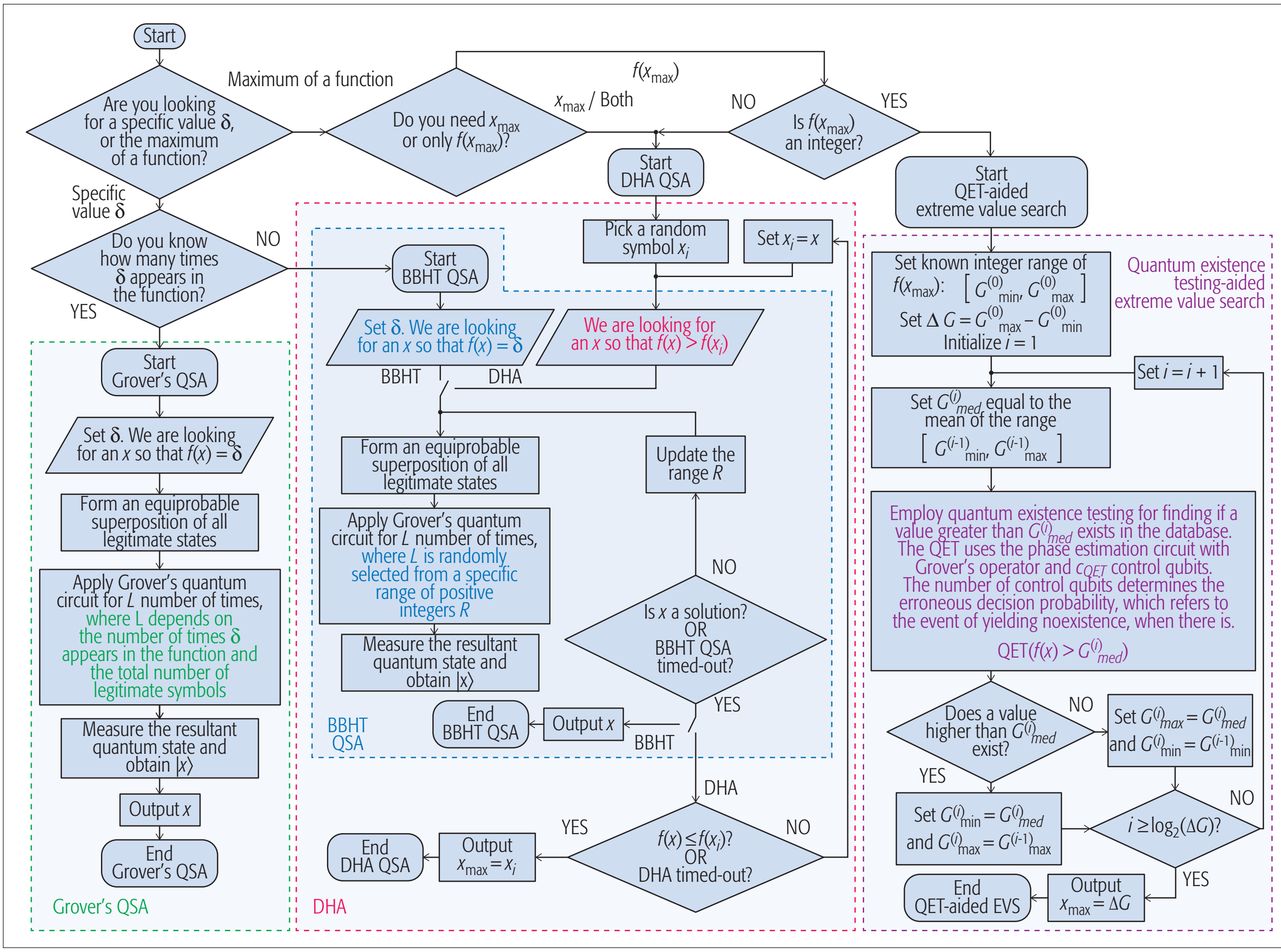

FIGURE 3. The selection of the appropriate quantum search algorithm depending on the search problem and their operation. The "maximum of a function" may be replaced by the "minimum of a function," in conjunction with replacing the associated inequalities.

required, and the average complexity of the full search is $O(N)$.

In quantum computing, Qute Brute is boisterously running and barking based on Grover's quantum search algorithm [7]. Explicitly, Grover's QSA may be employed for finding x_{max}, assuming that we know the value δ we are looking for, as well as both the size of the database N and the number of times S that δ appears at the function's output, as illustrated in Fig. 3. Grover's QSA succeeds with ~100 percent probability in finding a solution to the search problem, with as few as $O(\sqrt{N})$ CFEs. In order to achieve this, Grover's QSA evaluates $f(x)$ in parallel for *all* legitimate symbol combinations at the same time, but at the cost of a single CFE. After a predetermined number of purely parallel evaluations of the cost function, we obtain the symbol we were looking for. This seems like science fiction, or even "cheating." Indeed, it can only be achieved with the aid of a quantum computer.

A classical bit may be found in the states 0 or 1. By contrast, a qubit $|q\rangle$ may be found in the state $|q\rangle = \alpha|0\rangle + \beta|1\rangle$, where we have $\alpha, \beta \in \mathbb{C}$, essentially meaning that a qubit may be simultaneously in both the 0 and 1 states, if neither α nor β are equal to 0. We can interpret this by considering a box, in which we have a spinning coin in a superposition of the Head and Tail states. If we *measure* or *observe* $|q\rangle$ on an orthonormal basis, such as the one formed by $|0\rangle$ and $|1\rangle$, we will obtain one of the states of the measurement basis; either $|0\rangle$ with a probability of $|\alpha|^2$ or $|1\rangle$ with a probability of $|\beta|^2$. Upon observing the above coin by lifting the lid of the box, $|q\rangle$ collapses to the classic domain. If we superimpose two qubits, we would have $|q_1\rangle|q_2\rangle = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\alpha_2|11\rangle$. Among a range of other tested technologies, a qubit may be implemented as the spin of an electron, the polarization of a photon, or even the spin of a nucleus for memory storage purposes.

Let us now return to Qute Brute's operation. Grover's QSA may employ more than one qubits ($\log_2(N)$ to be precise) for creating an equiprobable superposition of N states. Each of the N states corresponds to a unique symbol of the legitimate symbol set. S of these N states (again, S has to be known to us) correspond to an $f(x)$ value of δ. Each time all these qubits pass through a *quantum gate*, the operation this gate performs on the input is applied to all the superimposed quantum states *simultaneously*! Grover found a specific quantum operation gate sequence that eventually transforms an initially equiprobable superposition of all legitimate states or symbols to an equiprobable superposition of only the S *solution* states we are looking for. The probability of observing a quantum state that does not return δ as the output of the function $f(x)$ becomes approximately 0. Therefore, after observing the resultant qubits, we will obtain an x_{max} value associated with $f(x_{max}) = \delta$, at ∼100 percent probability.

But what if we do not know the exact number of times that δ appears at the function's output? What if we do not even know the maximum value δ of $f(x)$, which is the case in the optimization problems found in wireless communications? Qute Brute may be transformed. Dürr and Høyer [8] proposed a variant of Grover's QSA, namely the Dürr-Høyer Algorithm (DHA), which succeeds in finding the specific symbol x_{max} that maximizes $f(x)$ with ∼100 percent probability, without any knowledge of the value of $f(x_{max})$. Starting from a legitimate symbol combination, either randomly or deterministically [9] selected, the DHA will perform a quantum search for finding any symbol with a higher function value than that offered by the initially selected symbol. Since it is not guaranteed that the newly found symbol is x_{max}, another quantum search is initiated for finding another symbol with an even higher function output value. This process of initiating quantum searches is repeated until the quantum search is concluded without observing a symbol with a higher function value. Then, it is concluded that the most recently acquired symbol with the highest function value found as yet is x_{max}. But again, how do we know the number of symbols that are "better" than the initial symbol of a quantum search in a practical optimization problem, so that we are able to invoke Grover's QSA? Frankly, we cannot, hence the original Grover's QSA cannot be employed in practice.

Fortunately, Boyer *et al*. [10] proposed the Boyer-Brassard-Høyer-Tapp (BBHT) quantum search algorithm, which is another variance of Grover's QSA that might find a solution symbol x_s even if we do not know how many solutions exist in our search problem. Since we do not know the exact number of symbols that result in a higher function output value than the initial reference symbol, we are unable to determine how many times we should use Grover's quantum gates. Boyer *et al*. proved that if we employ a specific trial and error method, we will eventually observe a solution symbol after a number of failed attempts, with a success probability of ∼100 percent at a complexity of $O(\sqrt{N})$ CFEs.

Therefore, Qute Brute, following the DHA methodology, which in turn employs the BBHT algorithm, in his quest of finding x_{max}, starts from an initial symbol and starts barking at other symbols after applying Grover's quantum circuit a pseudorandom number of times. Some of these barks will correspond to a lower function output value, but eventually one of them will have a higher function output value, provided that one exists. Qute Brute will then start barking again and again, until a predetermined, specific number of CFEs have been completed without yielding a better result. Then we may conclude that x_{max} had already been found and it is the final output of the previous, last-but-one quantum search.

When the actual value $f(x_{max})$ is required rather than x_{max}, the quantum existence testing (QET)-aided extreme value search of [11] may also be used. The QET shares the quantum circuit of the quantum counting algorithm [12]. However, while the quantum counting algorithm employs the phase estimation algorithm with Grover's operator for finding the exact number solutions in the database, the QET efficiently searches if at least one solution exists in it. In [11], Imre proposed the use of the QET for finding the maximum value of a database, by continuously halving the problem-specific search range and employing the QET for checking if a solution exists in the upper half. Depending on the QET's result, the search range is updated and the QET is called again, for a predetermined number of times, as illustrated in Fig. 3. The accuracy and the complexity of the algorithm both depend on the number of control qubits c_{QET} employed for estimating the existence of a solution, which in turn relies on the database size and the affordable probability of making an erroneous decision. The algorithm uses $O(\sqrt{N})$ CFEs in terms of the number of Grover operators employed. For a detailed tutorial on quantum search algorithms and their amalgamation with the symbol detection problem in wireless communications, please refer to [13].

Let us demonstrate the operation of the quantum search algorithms with the aid of an MUD example. The left-side plot of Fig. 4 depicts the legitimate set of the $M^U = 8^3 = 512$ symbol combinations that three users may transmit, when 8-ary pulse amplitude modulation (PAM) is used. The full search evaluates all 512 symbols before concluding that x_{max} is found. On the other hand, the DHA starts from a randomly selected symbol, represented by the purple circle, on the right-side plot of Fig. 4 and observes 53 symbols before concluding that x_{max}, depicted by the red square, is found. In more detail, the DHA initially employs the BBHT algorithm for finding a symbol, which corresponds to a higher function output value than that of the purple symbol. After a specific number of failed attempts, which result in obtaining some of the blue symbols in the right-side plot of Fig. 4, a better symbol, represented by the yellow dot at the output of the first BBHT in Fig. 4, is observed. Starting from that newly found symbol, a similar process is initiated by employing the BBHT algorithm again, resulting in observing more blue (worse) symbols, until the next yellow (better) symbol is found. After a total of six BBHT QSA's instances, x_{max} is obtained. At this point we know that a better symbol was found, but not that this symbol is the *best* symbol x_{max}. Therefore, we perform the BBHT algorithm again, hoping to find an even better symbol than x_{max}. Since there is no solution to this search problem, during the seventh BBHT QSA instance the algorithm will only output failed attempts (blue dots) until it times out, allowing us to conclude that x_{max} had already been found during the sixth BBHT QSA instance. The complexity of the DHA in this example is higher than 53 CFEs, since multiple Grover iterations are performed before observing and obtaining a single symbol. More specifically, 158 CFEs were required for obtaining x_{max} with the aid of the DHA, which is 30.86 percent of the brute force algorithm's complexity.

Soft-input soft-output detectors will use all the evaluated symbols presented in the right-side plot of Fig. 4 for creating the bit-based log-likelihood ratios $L_{m,e}(\{\hat{b}\})$ of Fig. 2, which describe both the

Regardless of the selected receiver structure, the symbol detector has to solve an optimization problem, which is ultimately related to finding the most likely symbols transmitted by all the users, by maximizing the probability at the channel output.

This article has been accepted for inclusion in a future issue of this magazine. Content is final as presented, with the exception of pagination.

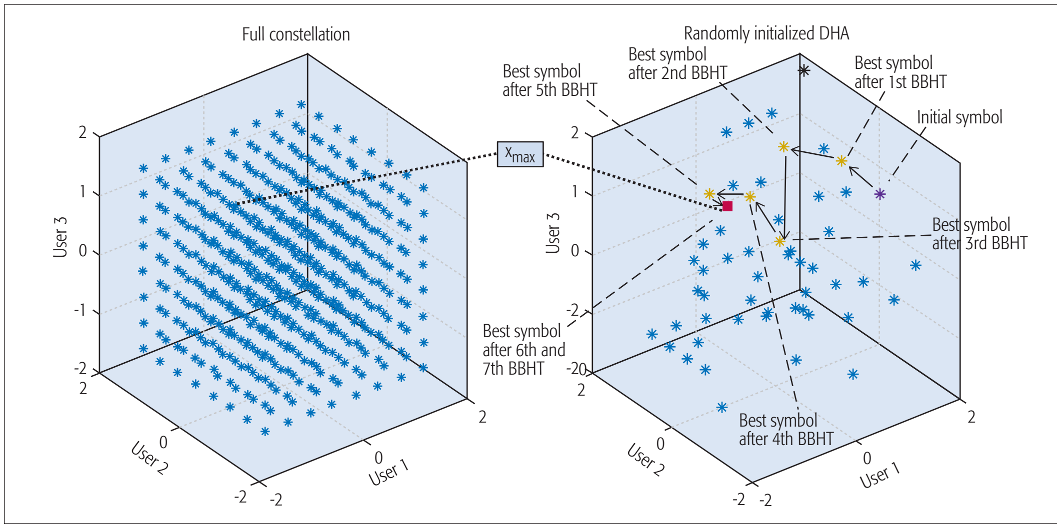

FIGURE 4. Left: The full multi-user symbol constellation when 3 users are supported in the system with a coherent receiver and the 8-PAM scheme is used. The brute force search evaluates all 83 = 512 symbols for finding x_{max}. Right: The symbols evaluated by the DHA with a randomly selected initial symbol (purple dot), before concluding that x_{max} is found (red square). The yellow circles correspond to the symbols that were observed as solutions to the first 5 instances of the BBHT QSA's instances, before the sixth one found x_{max} and the seventh one did not output a better symbol. The blue dots represent observed symbols that correspond to failed attempts for finding a better symbol in a BBHT search.

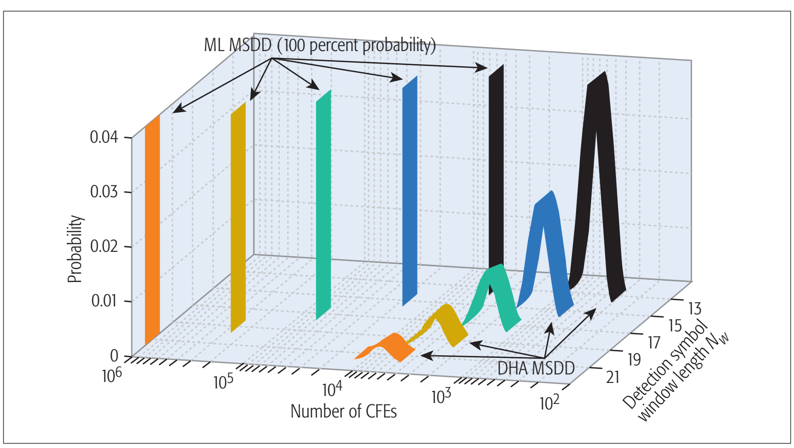

FIGURE 5. Complexity of the hard-input hard-output DHA-aided QMSDD and the brute force equivalent, optimal classical ML MSDD, when N_w = 13, 15, 17, 19, 21-symbol detection windows are employed, while transmitting BPSK symbols in a non-coherent system, where all U users are orthogonally separated either in the frequency or the code domain. The complexity of the ML MSDD is that of the brute force search and is always equal to 2^{N_w-1} CFEs, while the complexity of the DHA follows a Gaussian-like distribution. Bins of 10 CFEs were used for the probability density function of the DHA detector's complexity.

value of each detected bit and how confident we are this bit has that specific value. On the other hand, only x_{max} will be used by hard-input hard-output detectors, representing just the value of each detected bit in the created log-likelihood ratios.

At a non-coherent receiver, for detecting the differentially encoded symbols in groups of N_w = 13 up to N_w = 21-symbol windows, the complexity of the DHA-aided QMSDD is illustrated in Fig. 5. As expected, due to the random nature of the DHA, its complexity is not fixed, in contrast to that of the Maximum Likelihood (ML) MSDD, which performs a full search and has a complexity of M^{N_w-1} = 2^{N_w-1} CFEs with 100 percent probability. Based on Fig. 5, the complexity of the DHA-aided detector is lower than that of the ML detector and for the scenario where N_w = 21-symbol windows are adopted, the complexity reduction may become as high as two orders of magnitude. The same order of complexity would be required at the DHA-aided QMUD of a coherent system supporting U = 20 users.

In the above examples, the initial symbol of the DHA was selected randomly, as stated in Fig. 3. Please note that if the initial symbol of the DHA was selected deterministically instead, by using, for example, the output of the low-complexity minimum mean square error (MMSE) detector, the complexity required by the DHA would be even lower [9].

Qute Brute in Action

Let us now characterize the performance of the soft-input soft-output QMUD and QMSDDs in the systems described in Fig. 2, with the parameters stated in Table 1. All the systems support U = 8 users, who transmit QPSK symbols over the extended typical urban (ETU) channels [6] with the aid of P = 2 antenna elements at the base station. Please note that these are rank-deficient systems, since the number of transmitters is higher than the number of receive antenna elements, having a normalized user load of $U_L = U/P$ = 2. Each user encodes his/her information bits using non-systematic convolutional codes associated with eight trellis states and different rate, depending on the scenario. More precisely, in the coherent SDMA-OFDM systems, a R = 1/4 rate code is employed. On the other hand, a R = 1/2 rate code is used in the MC-IDMA scenario, which in combination with the spreading code in the frequency domain with a spreading factor of SF_{FD} = 2 results in an effective code rate of R/SF_{FD} = 1/4. The SDMA-OFDM system is used in three variations. First, both with and without the use of Walsh-Hadamard orthogonal spreading codes in the time domain, associated with a spreading factor of SF_{TD} = 8, while using coherent receivers. Second, in conjunction with a non-coherent receiver, where the time-domain direct sequence spreading is also applied. As far as the non-coherent receiver of the SDMA-OFDM system is concerned, a reference OFDM symbol is transmitted every T_h = 17 OFDM symbols for assisting the differential detection of each user's symbols. All systems have an interleaver of 32,768 bits per user, Q = 1024 subcarriers and a cyclic prefix of CP = 128, while all users transmit on all the subcarriers. The normalized effective Doppler frequency is F_d = 0.02. Unless stated otherwise, J = 2 iterations are allowed between the detectors and the channel decoders.

Figure 6 depicts the BER performance of the investigated systems. The optimal classic maximum *a posteriori* probability (MAP) detector [5] and the quantum-assisted Dürr-Høyer Algorithm-based Multi-Input Approximation with Forward Knowledge Transfer (DHA-MUA-FKT) detector [6, 14, 15] were employed. We may conclude that in all these rank-deficient scenarios, the quantum-assisted detector performed very close to the optimal classic detector, while imposing a lower complexity.

Allowing a second iteration between the MUD and the decoders in the SDMA-OFDM system (S1) associated with a coherent receiver and operating without direct-sequence spreading, yields a gain of approximately 5 dB at BER =10^{-5}. The performance of the MC-IDMA scenario (S3) is within 0.1 dB of the aforementioned SDMA-OFDM scenario (S1) with J = 2 MUD-DEC iterations, but has a lower decoding complexity, since it includes fewer branches on the decoding trellis diagram [5] of the half-rate non-systematic convolutional code. Both the (S1) and (S3) scenarios assume perfect channel knowledge. In practice, this comes with a high complexity imposed by performing channel estimation, in addition to that required by the MUD.

To circumvent this problem we may employ a non-coherent receiver, such as the one described by the scenario (S4). Still based on Fig. 6, the conventional differential detector associated with N_w = 2 has the lowest complexity of only four CFEs per bit after J = 2 iterations, but due to the high Doppler frequency of our scenarios it experiences a performance degradation of 4.5 dB with respect to the SDMA-OFDM scenario relying on the coherent receiver (S1). In order to improve the performance of the non-coherent system, we may employ an MSDD associated with N_w = 9 symbols in each detection window, including the reference symbol. By using an MSDD, we encounter only a modest performance degradation of approximately 0.3 dB, at a cost of a higher complexity. More specifically, by allocating a similar complexity to the MAP MSDD of the non-coherent scenario (S4) with N_w = 9 as the one required by the MAP MUD of the coherent scenario (S1), a degradation of 0.3 dB is observed. However, no additional complexity is used for channel estimation, but eight times the bandwidth is required in the non-coherent scenario (S4) with respect to the coherent scenario (S1), since spreading codes associated with SF_{TD} = 8 are used for separating the users on each subcarrier. Therefore, a trade-off between performance, complexity and bandwidth is observed.

Number of users	$U = 8$
Number of AEs at the BS	$P = 4$
Normalized user-load	$U_L = U/P = 2$
Modulation	QPSK $M = 4$
Channel code	Non Systematic Convolutional Code, 8 trellis states, $R = 1/4$ (SDMA-OFDM: S1, S2 & S4) $R = 1/2$ (MC-IDMA: S3)
Spreading factor of the	$SF_{TD} = 8$ (SDMA-OFDM: S2, S4)
Walsh-Hadamard Codes	$SF_{FD} = 2$ (MC-IDMA: S3)
Channel model	Extended typical urban (ETU)
Mobile velocity	υ = 130 km/h
Carrier frequency	f_c = 2.5 GHz
Sampling frequency	f_s = 15.36 GHz (78 delay taps)
Doppler frequency	f_d = 70 Hz
Number of subcarriers	$Q = 1024$
Cyclic prefix	CP = 128
Detector - DEC	$J = 2$ iterations
Interleaver length	32,768 bits per user
Reference symbols every	T_h = 17 OFDM symbols (non-coh: S4)
Differential detection	N_w = 9 symbols window (non-coh: S4)

TABLE 1. Parameters of the investigated scenarios of Fig. 2.

Using the same spreading codes in the coherent SDMA-OFDM scenario (S2), the same bandwidth as that of the non-coherent scenario (S4) is required. A 3 dB degradation should be observed between uncoded single-user coherent and non-coherent systems associated with conventional differential detectors (N_w = 2) and a low Doppler frequency, only due to the increased AWGN levels experienced at the receiver [5]. The degradation becomes higher when channel coding is used and multiple users are supported by a system at a high Doppler frequency, as exemplified by the 7.5 dB loss of Fig. 6 between the coherent system (S2) with perfect CSI and the non-coherent system (S4) with N_w = 2.

An increased complexity may be invested in the MSDD of the non-coherent system (S4), which should however still be lower than the complexity invested in the channel estimation and MUD of the coherent scenario (S2). For example, when we have N_w = 9 at the non-coherent system (S4), the complexity of the MSDD according to Fig. 6 is much higher than that of the MUD in the coherent system (S2), which is only four CFEs/bit.

However, the complexity required by the channel estimation procedure in practical coherent

The MC-IDMA scheme performs almost equally well as the SDMA-OFDM scheme, but requires lower decoding complexity. Non-coherent receivers avoid the complexity required by the channel estimation process of coherent receivers, but either experience a performance degradation, or more bandwidth is required for achieving a similar performance to that of the equivalent coherent receivers.

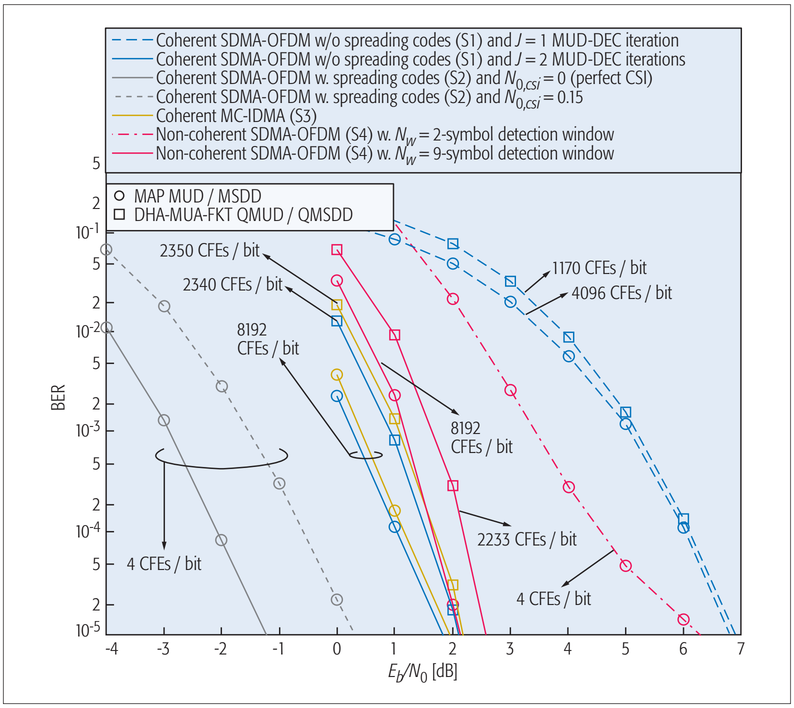

FIGURE 6. BER performance of the four scenarios with their system models presented in Fig. 2 and the parameters summarized in Table 1, when soft-input soft-output detectors are used. The complexity of each system in terms of CFEs/bit at BER = 10^{-5} is also displayed. All systems allow J = 2 MUD-DEC (or MSDD-DEC) iterations, unless stated otherwise in the legend.

receivers, which may require multiple activations of the channel estimation process, the multi-user detection and the decoders, is avoided by the non-coherent receivers.

In Fig. 6 we also present a scenario of the coherent SDMA-OFDM system (S2), where imperfect channel estimation is modelled by contaminating the perfect channel estimates with AWGN sample of zero mean and variance $N_{0,CSI}$ = 0.15. The performance degradation of the non-coherent system (S4) with N_w = 9 compared to the coherent system associated with direct sequence spreading (S2) is now only 1.4 dB.

Conclusions

As quantum computing is gradually becoming a commercial reality (http://www.dwavesys.com), quantum search algorithms may be employed in wireless communication systems for accelarating specific processes, such as multi-user detection or multiple symbol differential detection. In this article, we stated the problem of symbol detection in multi-user scenarios and showed how quantum search algorithms may be used in this context. The flow-chart of Fig. 3 may help in deciding which of the investigated quantum search algorithms is more suitable for a specific problem.

In Fig. 6 we analyzed the performance of four systems, relying on different multiple access schemes, as well as on both coherent and non-coherent receivers. The MC-IDMA scheme performs almost equally as well as the SDMA-OFDM scheme, but requires lower decoding complexity. Non-coherent receivers avoid the complexity required by the channel estimation process of coherent receivers, but either experience a performance degradation, or more bandwidth is required for achieving a performance similar to that of the equivalent coherent receivers.

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