

# Space Time Variable Length Coding for Wireless Multimedia Communications

Meixia Tao

Department of Electrical and Computer Engineering  
National University of Singapore  
Singapore 117576  
Email: mxtao@nus.edu.sg

Soon Xin Ng

School of Electronics and Computer Science  
University of Southampton  
SO17 1BJ, United Kingdom  
Email: sxn@ecs.soton.ac.uk

**Abstract**—A binary space time variable length coding (STVLC) scheme is proposed for transmission of digital signals with non-uniform occurrence probability distributions in delay-sensitive wireless systems. The proposed STVLC scheme is a unified source coding, channel coding, and spatial diversity/multiplexing design. Specifically, two-dimensional variable length binary codewords in space-time domain are designed and modulated using binary phase shift keying (BPSK), and then transmitted diagonally across the space-time grid using multiple antennas. As the number of bits per codeword is varying by the nature of variable length coding, the number of activated transmit antennas, which equals the number of bits per variable length codeword, also varies. We show that our unified design approach effectively exploits the coding gain, power saving, diversity and multiplexing gain, and thus achieves good source symbol error performance.

## I. INTRODUCTION

The future generations of wireless communication systems are required to provide reliable transmissions at high data rates in order to offer a variety of multimedia services. Motivated by the channel capacity potential of multi-input multi-output (MIMO) channels studied in [1], [2], significant research efforts have been invested in the design of space time transmission techniques, such as the Bell-lab layered space-time architecture (BLAST) [3] and space-time codes (STCs) [4]. The idea of transmitting a space-time codeword diagonally across the space-time grid first appeared in the diagonal BLAST scheme (DBLAST) [3]. More explicitly, DBLAST was designed for attaining multiplexing gain. In [5], the diagonal transmission strategy was employed for attaining full transmit diversity gain. Compared with other coding methods (e.g. hand-design and computer search), the diagonal block space-time coding [5] is systematic and enables efficient code design with high coding gain at minimum trellis states for an arbitrary number of transmit antennas. Moreover, the diagonal block framework in [5] provides a flexible tradeoff between spatial diversity and multiplexing.

At the time of writing, all space-time coding schemes were designed assuming equal-probable and memoryless sources where Shannon's separate source and channel coding approach is applicable. Yet, this is seldom the case for image and video sources. In addition, Shannon's separation theorem is based upon the assumption of allowing infinitely long codewords.

In practise, such long codewords are undesired due to real-time delay constraints. Hence a joint source/channel coding approach is more practical, especially, in a delay-sensitive wireless multimedia communication system [6].

Variable length codes (VLCs) [6] belong to the family of low-complexity source compression schemes, which were adopted by most image coding standards [7], [8] in their entropy coding stage. In order to exploit the residual redundancy of VLCs, numerous trellis-based VLC decoding algorithms have been proposed, such as the joint source/channel coding scheme of [9], [10]. Recently joint detection of STC and VLC schemes were studied in [11], where a near-capacity performance was achieved. Note that all VLC related schemes have to convey explicit side information regarding the total number of VLC encoded bits or symbols per transmission frame.

In this paper, we propose a unified design of VLC and STC, where two-dimensional (2D) binary VLCs are designed and transmitted using the diagonal block space time structure introduced in [5]. Since one source symbol is transmitted during one symbol period, the transmission frame length depends on the number of source symbols rather than the number of VLC encoded bits. As a result, the proposed Space Time Variable Length Coding (STVLC) scheme does not have the synchronization problem of the ordinary time-domain VLCs.

Relevant work on the joint design of source coding and space-time coding can be found in [12] and [13]. These studies considered analog source and used the end-to-end distortion as performance measure. In this paper, however, we assume discrete source or analog source that is already quantized into discrete format prior to our STVLC encoder. Our objective is thus to minimize the error probability of source symbols.

The paper is organized as follows. In Section II, we present the STVLC design, while in Section III the trellis decoder of STVLC is described. The simulation results are discussed in Section IV and finally the conclusion is drawn in Section V.

## II. SYSTEM OVERVIEW

Let us consider a point-to-point MIMO system equipped with  $N_t$  transmit antennas and  $N_r$  receive antennas, the same model adopted in [5]. The signal to be transmitted on antenna

$m$ , for  $1 \leq m \leq N_t$ , at discrete time index  $t$  is denoted as  $x_m[t]$ . The received signal on antenna  $n$ , for  $1 \leq n \leq N_r$ , at time  $t$  is denoted by  $r_n[t]$  and modelled as:

$$r_n[t] = \sqrt{E_s} \sum_{m=1}^{N_t} h_{n,m}[t] x_m[t] + w_n[t], \quad (1)$$

where  $E_s$  represents the average energy of the signal constellation,  $h_{n,m}$  denotes the channel coefficient between transmit antenna  $m$  and receive antenna  $n$  at time  $t$  and  $w_n[t]$  is the additive complex white Gaussian noise with zero mean and a variance of  $N_0/2$ . The amplitude of the modulation constellation points is scaled by a factor of  $\sqrt{E_s}$ , so that the average energy of the constellation points is unity and the expected Signal-to-Noise Ratio (SNR) per receive antenna is given by  $\gamma = N_t E_s / N_0$  [14]. By considering a transmission frame length of  $T$  symbol periods, a space-time codeword can be defined as

$$\mathbf{X} = \begin{bmatrix} x_1[1] & x_1[2] & \dots & x_1[T] \\ x_2[1] & x_2[2] & \dots & x_2[T] \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_t}[1] & x_{N_t}[2] & \dots & x_{N_t}[T] \end{bmatrix}, \quad (2)$$

The Pair-Wise Error Probability (PWE) of erroneously detecting codeword  $\mathbf{E}$  for  $\mathbf{C}$  is upper bounded at high signal-to-noise ratios (SNR) by [4]:

$$p(\mathbf{C} \rightarrow \mathbf{E}) \leq \left( \frac{E_s}{4N_0} \right)^{-E_H \cdot N_r} (E_p)^{-N_r}, \quad (3)$$

where  $E_H$  is the *effective Hamming distance* which quantifies the transmit diversity order and  $E_p$  is the *effective product distance* which quantifies the coding advantages, of a space time code. The PWE upper bound expression in (3) is suitable for both quasi-static fading and rapid fading channels.

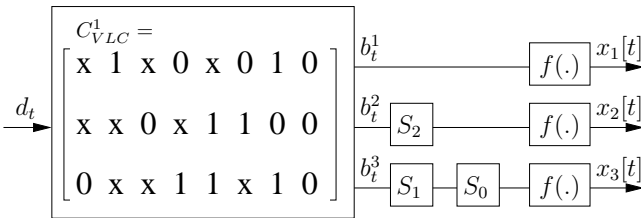


Fig. 1. Block diagram of the STVLC encoder employing code  $C_{VLC}^1$ .

The 2D VLC codebook can be represented by a  $N_t \times N_s$  matrix, where  $N_s$  is the size of source alphabet. Explicitly, the columns and rows of the 2D VLC represents the time domain and space domain, respectively. Fig. 1 illustrates an exemplary block diagram of the proposed STVLC encoder. In Fig. 1, the codebook is denoted as  $C_{VLC}^1$ , in which  $N_s = 8$ , and notation “x” stands for “no transmission”. At each time  $t$  the source symbol  $d_t$ , where  $d_t \in \{0, 1, \dots, N_s - 1\}$ , is source-encoded to a spatial-domain codeword  $\mathbf{b}_t = [b_t^1 \ b_t^2 \ \dots \ b_t^{N_t}]^T$ , which is the  $d_t$ -th column of the matrix  $C_{VLC}^1$ . Then the codeword is transmitted diagonally across the space-time grid using  $N_t$

transmit antennas and with shift registers, denoted as  $S_i$  for  $i = 0, 1$  and 2 in Fig. 1. Note that the symbols of a VLC can have a more flexible arrangement when they are in a 2D domain, compared to the conventional one-dimensional (1D) VLC, where only the time domain is used. Hence, a higher number of combinations of the VLCs can be attained in the 2D domain. In the proposed scheme, the variable nature of the code is transpired in the spatial domain rather than the time domain. The length of the space-domain codeword associated with source symbol  $d$  is given by  $L(d) = N_t - L_X(d)$ , where  $L_X(d)$  is the number of “no transmission” symbol “x” in the codeword of source symbol  $d$ . Let us consider a non-uniform source, where the probabilities of occurrence for the source symbols are given by  $P(d) = \{0.2888, 0.2166, 0.1625, 0.1218, 0.0914, 0.0514, 0.0386, 0.0289\}$ , respectively. Hence, the longest VLC codeword length is given by  $L_{max} = N_t = 3$  and the average VLC codeword length is  $L_{ave} = \sum_{d=0}^{N_t-1} P(d)L(d) = 1.4$ , for this 2D VLC  $C_{VLC}^1$ .

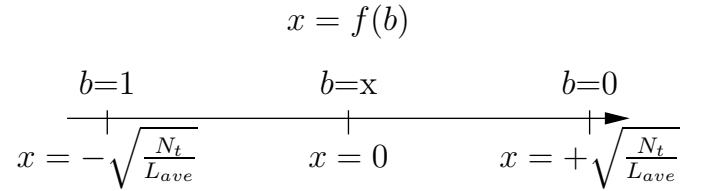


Fig. 2. The mapper at each transmit antennas of the STVLC scheme in Fig. 1.

The signal mapper function  $f(\cdot)$  at each transmit antennas of the STVLC scheme in Fig. 1 is shown in Fig. 2. Note that the transmission energy for  $b = x$  is zero, i.e.  $x = f(x) = 0$ . Hence, power saving can be achieved when the number of activated transmit antennas is less than  $N_t$ . More specifically, the average number of activated transmit antennas equals the average codeword length of the 2D VLCs. Hence a power saving of  $10 \log_{10}(N_t/L_{ave}) = 3.3$  dB is attained in the example STVLC- $C_{VLC}^1$ . Therefore, the variable length structure of the 2D VLC is converted into power saving. For the sake of normalizing the resultant transmitted energy from the multiple transmit antennas to unity, we need to re-assign this power saving into the BPSK modulation constellation. Hence, the transmission energy for bits 0 and 1 is given by  $+\sqrt{N_t/L_{ave}}$  and  $-\sqrt{N_t/L_{ave}}$ , respectively, which is dependent on the total number of transmit antennas  $N_t$  as well as the average VLC codeword length  $L_{ave}$ .

Since the VLC codewords are transmitted diagonally, the transmitted signal at the  $m$ th antenna at time  $t$  is given by  $x_m[t] = f(b_{t-m+1}^m)$ . Hence, the space-time codeword matrix of (2) can be written as:

$$\mathbf{X} = \begin{bmatrix} \dots & f(b_t^1) & f(b_{t+1}^1) & \dots & f(b_{t+N_t-1}^1) & \dots \\ \dots & f(b_{t-1}^2) & f(b_t^2) & \dots & f(b_{t+N_t-2}^2) & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \dots & f(b_{t-N_t+1}^{N_t}) & f(b_{t-N_t+2}^{N_t}) & \dots & f(b_t^{N_t}) & \dots \end{bmatrix}$$

It can be shown based on [5] that the minimum  $E_H$  and minimum  $E_P$  of STVLC scheme are given by

$$E_{H \min} = d_{free}^{VLC}, \quad (4)$$

where  $d_{free}^{VLC}$  is the minimum Hamming distance of the 2D VLC code employed, with no transmission “x” included as an extra symbol, and

$$E_{P \min} = \min_{0 \leq d < \tilde{d} \leq N_s - 1} \prod_{m \in \xi} \left| f(b^m) - f(\tilde{b}^m) \right|^2, \quad (5)$$

where  $\xi$  is the set for index  $m$  with  $b^m \neq \tilde{b}^m$  for  $1 \leq m \leq N_t$ , and  $[b^1 \dots b^m]$  and  $[\tilde{b}^1 \dots \tilde{b}^m]$  are the two VLC codewords associated with source symbols  $d$  and  $\tilde{d}$ , respectively. In the example of Fig 1, we have  $d_{free}^{VLC} = 2$ . Therefore, a transmit diversity order of  $E_{H \min} = 2$  can be achieved at high SNRs. Furthermore, the corresponding minimum product distance is given by  $E_{P \min} = (3/1.4)^2 = 4.59$ , which is higher than that of the BPSK modulation of 4.

In general, a 2D VLC having the greatest  $d_{free}^{VLC}$  would yield the highest performance gain. In the binary case, the greatest  $d_{free}^{VLC}$  results in the most elements in the  $\xi$  set of (5) and, accordingly, the greatest  $E_{P \min}$ . Therefore, the binary 2D VLC designed with the greatest  $d_{free}^{VLC}$  for a given  $L_{max}$  and  $N_s$  combination is the best 2D VLC code. It was found through exhaustive search that the 2D VLC code  $C_{VLC}^1$  employed in Fig. 1 has the greatest  $d_{free}^{VLC}$  for a given  $L_{max} = 3$  and  $N_s = 8$ .

### III. TRELLIS-BASED JOINT DECODER

The proposed STVLC scheme may be described by a trellis structure and thus an efficient trellis-based joint source/channel decoding algorithm can be constructed.

The trellis diagram of the STVLC scheme employing  $C_{VLC}^1$  is computed based on the STVLC encoder in Fig. 1, and is shown in Fig. 3. Note that each of the three shift registers, which are denoted as  $S_0$ ,  $S_1$  and  $S_2$  in Fig. 1, can have three possible contents,  $\{x, 0, 1\}$ . Hence a total of  $3^3 = 27$  combinations are possible for the three shift registers in Fig. 1. However, not all combinations are legitimate due to the constraint imposed by the 2D VLC code. As can be seen from the resulting trellis diagram in Fig. 3, there are only 24 trellis states. Each of the trellis state represents one legitimate shift register combination. As shown in Fig. 3, the trellis state labeled 0 represents the  $\{S_2 = x, S_1 = 0, S_0 = 0\}$  combination and the trellis state labeled 23 represents the  $\{S_2 = 0, S_1 = 0, S_0 = x\}$  combination. As we can see from Fig. 3, there are always eight diverging trellis branches from each states due to the eight possible source symbols. However, the number of converging trellis paths may vary from one state to another due to the variable length structure of the space-time codewords. Explicitly, there are six and nine trellis paths converging to state 1 and state 2, respectively, in the trellis structure of Fig. 3.

From the trellis diagram obtained in Fig. 3, the Viterbi or MAP algorithm may be used for the trellis-based decoding.

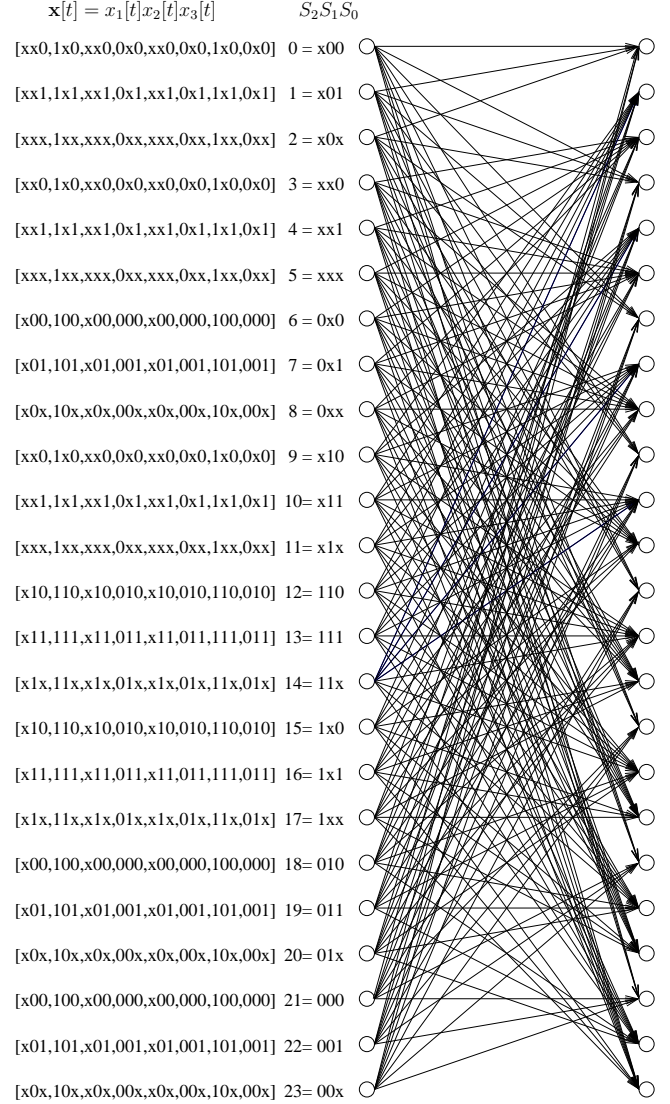


Fig. 3. Trellis diagram of STVLC using code  $C_{VLC}^1$ .

The log-domain likelihood metric of source symbols can be obtained from

$$m(d^l; \mathbf{r}[t]) = -\frac{1}{N_0} \sum_{n=1}^{N_r} \left| r_n[t] - \sqrt{E_s} \sum_{m=1}^N h_{n,m} x_m[t] \right|^2 \quad (6)$$

where  $d^l$  is the  $l$ th source symbol,  $l \in \{0, \dots, N_s - 1\}$ , and  $\mathbf{r}[t] = [r_1[t] \dots r_{N_r}[t]]$  is the received signal vector at time  $t$ . As the source symbol occurrence probability  $P(d)$  is non-uniform, not all branch transitions in Fig. 3 are equiprobable. Hence, the source symbol occurrence probability  $P(d)$  can be utilized as the *a priori* probability for assisting the the trellis decoder. Hence, the log-domain trellis branch transition metric  $\gamma(d = d^l)$  can be computed as

$$\gamma(d = d^l) = m(d^l; \mathbf{r}[t]) + Pr(d^l), \quad (7)$$

where  $m(d^l; \mathbf{r}[t])$  is from (6) and  $Pr(d^l)$  is the log domain *a priori* probability for symbol  $d^l$ , which is based on  $P(d)$ .

Note that a more complex STVLC scheme having a higher minimum product distance can also be found from exhaustive search by using a trellis with a larger number of states. However, in this contribution we aim at introducing a new joint source and space time coding scheme rather than proposing the STVLC scheme that has the highest coding gain.

#### IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed STVLC scheme when communicating over fast Rayleigh fading channels. The performance is measured in the *source* Symbol Error Rate (SER) versus SNR per source-bit, namely  $E_b/N_0 = \gamma/\eta$ , where  $\gamma$  is the average SNR per receive antennas and  $\eta$  is the transmission efficiency. The transmission efficiency of STVLC is given by  $\eta = \log_2(N_s)$  bit/s/Hz, where  $N_s$  is the size of the source alphabet. For the STVLC- $C_{VLC}^1$  scheme of Fig. 1, we have  $\eta = 3$ . Evidently, when  $N_s > M$ , with  $M$  being the constellation size of modulated symbols and equal to 2 for BPSK modulation, the transmission efficiency of the STVLC scheme is higher than the number of bits per modulated symbol, namely  $\log_2(M)$ . Hence, the binary STVLC scheme is capable of achieving multiplexing gain when  $N_s > 2$ .

A benchmarking scheme was created for the STVLC arrangements. More specifically, for the same non-uniform source alphabet, we employ a Fixed Length Code (FLC), denoted as  $C_{FLC}^0 = \{000, 001, 010, 011, 100, 101, 110, 111\}$ , in the proposed STVLC transmission structure. In  $C_{FLC}^0$ ,  $N_t = 3$  transmit antennas are active at all time and it has  $N_s = 8$ ,  $L_{ave} = 3$ ,  $E_{H\min} = d_{free}^{VLC} = 1$  and  $E_{P\min} = 4$ . The FLC based benchmarker also has the same transmission efficiency of  $\eta = \log_2(8) = 3$  bit/s/Hz, as that of the proposed STVLC. However, the STVLC- $C_{FLC}^0$  benchmarker requires only 8 trellis states compared to 24 trellis states in the STVLC- $C_{VLC}^1$  scheme. Hence the decoding complexity of the proposed STVLC- $C_{VLC}^1$  is three times that of the STVLC- $C_{FLC}^0$  benchmarker.

To further highlight the performance gain of the STVLC scheme, the VBLAST [15] scheme was serially concatenated with a conventional time-domain or 1D VLC  $C_{VLC}^0 = \{00, 11, 010, 101, 0110, 1001, 01110, 10001\}$ , which was designed based on the same  $P(d)$  of  $C_{VLC}^1$  and it has  $N_s = 8$ ,  $L_{ave} = 2.8$  and a minimum free distance of two. The transmission efficiency of VLC-VBLAST- $C_{VLC}^0$  is given by  $\eta = N_t \times \log_2(M) \times \log_2(N_s)/L_{ave} = 3.2$  bit/s/Hz, which is slightly higher than that of STVLC. At the receiver, after the VBLAST detection, a bit-based VLC trellis decoder [9], [10] was used for estimating the transmitted source symbol sequence. Perfect side information reception is assumed for the VLC-VBLAST scheme regarding the number of VLC encoded bits.

Fig. 4 illustrates the SER versus  $E_b/N_0$  performance of STVLC using both the  $C_{VLC}^1$  and  $C_{FLC}^0$  codes, in comparison to VLC-VBLAST- $C_{VLC}^0$  over fast Rayleigh fading channels. The source alphabet input to the three transmission schemes is the same and has the same occurrence probability distribution. Three transmit antennas and three receive antennas as well

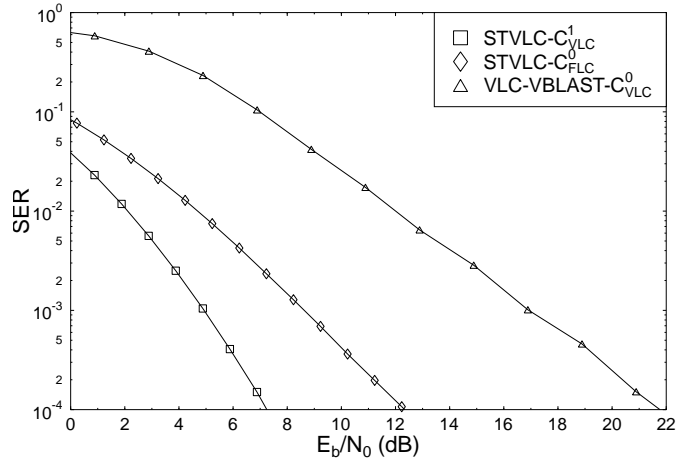


Fig. 4. SER versus  $E_b/N_0$  performance of STVLC and VLC-VBLAST over Rayleigh fading channels.

as BPSK modulation are used for all schemes. As shown in the figure, an overall gain of approximately 5 and 14.5 dB is attained by STVLC- $C_{VLC}^1$ , compared to STVLC- $C_{FLC}^0$  and VLC-VBLAST- $C_{VLC}^0$ , respectively, at SER of  $10^{-4}$ . Note that the VLC-VBLAST provides no transmit diversity gain while the STVLC- $C_{FLC}^0$  exhibits no power saving gain. However, the STVLC- $C_{VLC}^1$  scheme is capable of attaining both transmit-diversity and multiplexing gains as well as a coding gain. Furthermore, the proposed STVLC scheme employs one trellis encoder to encode a source symbol directly into a space-time codeword. By contrast, the VLC-VBLAST scheme employs a conventional VLC to encode/compress the source symbols into bits and then transmits the bits through the multiple transmit antennas. As we can see from Fig. 4 the 8-state STVLC- $C_{FLC}^0$  outperforms the VLC-VBLAST by approximately 9.5 dB, even though both schemes exhibit no transmit diversity gain. This performance difference is partly due to the employment of a maximum-likelihood detection in the STVLC- $C_{FLC}^0$  scheme and the employment of a low complexity receiver with ordered successive interference suppression and cancellation in the VBLAST scheme. A further 5 dB gain was attained by the STVLC- $C_{VLC}^1$  at the cost of three times higher decoding complexity compared to the STVLC- $C_{FLC}^0$  benchmarker.

#### V. CONCLUSIONS

In this paper we introduced a unified source-coding and space-time channel coding design for a delay-sensitive wireless communication system. This scheme is capable of, simultaneously, achieving spatial diversity gain, multiplexing gain, coding gain and power-saving. Hence, further research on joint source and space time coding design is very promising. More specifically, the design of non-binary STVLC is of immediate interest in order to further improve the transmission efficiency with the aid of high-level modulation schemes. Furthermore, the proposed STVLC scheme can also be employed as a constituent code in turbo equalization for attaining further

iteration gains.

## REFERENCES

- [1] G. J. Foschini and M. Gans, "On limits of wireless communication in a fading environment when using multiple antennas," *Wireless Personal Communications*, vol. 6, pp. 311–335, March 1998.
- [2] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunication*, vol. 10, pp. 585–595, Nov–Dec 1999.
- [3] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs Tech. J.*, pp. 41–59, 1996.
- [4] V. Tarokh, N. Seshadri and A. R. Calderbank, "Space-time Codes for high rate wireless communication: performance analysis and code construction," *IEEE Transactions on Information Theory*, vol. 44, pp. 744–765, March 1998.
- [5] M. Tao and R. S. Cheng, "Diagonal block space-time code design for diversity and coding advantage over flat Rayleigh fading channels," *IEEE Transactions on Signal Processing*, pp. 1012–1020, April 2004.
- [6] L. Hanzo, P.J. Cherriman and J. Street, *Wireless video communications: second to third generation systems and beyond*. NJ, USA : IEEE Press., 2001.
- [7] ITU-T, *Recommendation H.263: Video Coding for Low Bitrate Communication*, March 1996.
- [8] International Standards Organization, *ISO/IEC CD 13818 MPEG 2 International Standard, Information Technology, Generic Coding of Moving Video and Associated Audio Information, Parts 1–3*.
- [9] R. Bauer, J. Hagenauer, "On variable length codes for iterative source/channel decoding," in *IEEE Data Compression Conference*, (UT, USA), pp. 273–282, 27–29 March 2001.
- [10] S. X. Ng, F. Guo, J. Wang, L-L. Yang and L. Hanzo, "Joint source-coding, channel-coding and modulation schemes for AWGN and Rayleigh fading channels," *IEE Electronics Letters*, vol. 39, pp. 1259–1261, August 2003.
- [11] S. X. Ng, F. Guo and L. Hanzo, "Iterative detection of diagonal block space time trellis codes, TCM and reversible variable length codes for transmission over Rayleigh fading channels," in *IEEE Vehicular Technology Conference*, (Los Angeles, USA), 26–29 September 2004.
- [12] S. Lin, A. Stefanov, and Y. Wang, "Joint source and space-time block coding for MIMO video communications," in *Proc. of IEEE VTC-Fall*, Sept 2004.
- [13] T. Holliday and A. Goldsmith, "Joint source and channel coding for MIMO systems," in *Proc. of Allerton Conference on Comm., Control, and Computing*, Oct 2004.
- [14] A. F. Naguib, V. Tarokh, N. Seshadri and A. R. Calderbank, "A Space-Time Coding Modem for High-Data-Rate Wireless Communications," *IEEE Journal on Selected Areas in Communications*, vol. 16, pp. 1459 – 1478, October 1998.
- [15] G. D. Golden, C. J. Foschini, R. A. Valenzuela and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *IEE Electronics Letters*, vol. 35, pp. 14–16, January 1999.