Convergence Analysis of Iteratively Detected Time Hopping and DS-CDMA Ultrawide Bandwidth Systems by EXIT Charts

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Abstract— **This paper presents a novel analysis on the decoding convergence of Time Hopping (TH) and Direct Sequence (DS) Code-Division Multiple-Access (CDMA) Ultrawide Bandwidth (UWB) systems when communicating over multipath Nakagami channels. The analysis is based on the Extrinsic Information Transfer (EXIT) chart where the UWB systems are serially concatenated pulse-position modulated TH and code-synchronous DS-CDMA. It is shown from an EXIT chart analysis that the multipath diversity can yield a larger area under the EXIT curve of the inner detector. This area is related to the achievable rate of the system and it can be exploited with the aid of iterative detection. Simulation results of the iteratively detected TH and DS-CDMA UWB schemes verify the EXIT chart analysis.**

I. Introduction

Federal Communications Commission (FCC) ruling from 2002, of releasing the band from 3.1 to 10.6 GHz in USA for Ultrawide Bandwidth (UWB) forms the basis of the current era of UWB [1]. FCC also provided the definition of UWB at that particular stage. The aforesaid action precipitated establishment of Institute of Electrical and Electronic Engineering (IEEE) 802.15 high rate alternative Physical Layer (PHY) task group 3a for wireless personal networks. The task group endeavourred to define a universal standard that has best characteristics in all dimensions. All the proposals amalgamated into two: multiband and single band solutions [2], by the end of 2003. However, the proposals were withdrawn by both parties, and UWB was taken to the market without IEEE 802.15a standard [3, 4] on 19th of January 2006. This also resulted in dissolution of task group 15.3a work.

Heavy digital imaging and multimedia applications result in the requirements of high data rate wireless links. UWB has the capacity to fulfil the requirements of low cost and high speed digital indoor networks. 110 Mbps at a distance of 10m and 480 Mbps at a distance of 2m [5] can be provided through UWB. Even higher rates are foreseeing and coming. Both the Time Hopping (TH) Pulse Position Modulation (PPM) technology [6, 7] and Direct Sequence (DS) Spread Spectrum (SS) technology [8] have been used for the implementation of UWB system. In the former, the trains of time shifted pulses through PPM are used to transmit baseband or carrierless UWB system information. Transmission performance is enhanced by the usage of multiple pulses to transmit a single symbol. In the later, multiple chips with the chip duration equal to that of the basic time-domain signal are used to transmit data bits. Multiple access capability is enhanced by employing the conventional Code-Division Multiple-Access (CDMA) technology.

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TH-UWB and DS-UWB have their own merits and demerits. The former has the advantages of long battery life, because of low baseband pulses and low data rate supported [9], as well as multipath interference mitigating capability. The later has strong Multiuser Interference (MUI) mitigating capability [8]. Keeping energy per bit constant for the both systems, peak to average power and power spectral density in the DS-UWB are lower than those of the TH-UWB, because energy per pulse is higher for the TH-UWB than for the DS-UWB. DS-UWB also causes less in-band interference to other systems operated in the same frequence band of the UWB system.

As a powerful tool for convergence performance analysis of iterative decoding, the Extrinsic Information Transfer (EXIT) chart technique has been proposed by ten Brink [10]. EXIT chart predicts turbo cliff position and Bit Error Rate (BER) after an arbitrary number of iterations. In this paper, we analyse the convergence performance of iterative decoding for the aforesaid systems via the EXIT chart technique.

The paper is organised as follows. Section II introduces the system models consisting of TH-UWB and DS-UWB transmitted signals, respectively, as well as provides a brief discussion for the channel model and the Minimum Mean Square Error (MMSE) detector. Section III presents a comparative simulation study comprising of simulation configuration along with EXIT chart convergence performance analysis delibration. Our conclusions are offerred in Section IV.

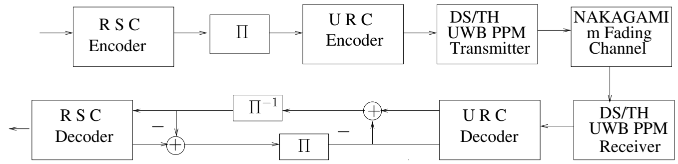

Fig. 1. System block diagram of the DS-CDMA/TH UWB system employing RSC as the outer encoder and URC as the inner encoder

II. System Models

Fig. 1 is the system block diagram of the DS-CDMA/TH UWB systems employing the Recursive Systematic Convolutional (RSC) code as the outer encoder and the Unity-Rate Code (URC) as the inner encoder. Specifically, we use the memory-two half-rate RSC code having a generator polynomial of $[1, (1+D^2)/(1+D+D^2)]$ and the memory-one recursive URC having a generator polynomial of $[1/(1+D)]$.

A. TH-UWB

The transmitted TH-UWB signal conveying the M-ary symbol is generated by invoking several frames. Let one symbol invokes L number of frames. Then the transmitted singal for the kth user can be given by:

$$u^{(k)}(t) = \sum_{m=0}^{\infty} \sum_{l=0}^{L} \varphi(t - mT_s - lT_f - s_m^{(k)}(l)T_h) \quad (1)$$

where $\varphi(t)$ is the time domain UWB pulse, $T_s = NT_f$ is the symbol duration, T_f is the frame duration, T_h denotes the time slot duration and $s_m^{(k)}(l)$ is the kth user mth symbol TH code generated by:

$$\begin{aligned} \mathbf{S}_m^{(k)} &= \left[s_m^{(k)}(1), s_m^{(k)}(2), ..., s_m^{(k)}(L)\right] \\ &= \mathbf{X}^{(k)}.\mathbf{1} \oplus \mathbf{t}_m^{(k)} \end{aligned} \quad (2)$$

where $\mathbf{1}$ is the unit vector of Length L and $\oplus$ denotes the modulo additon operation in Galois Field having Q elements, $\mathbf{X}^{(k)}$ is M-ary transmitted signal and $\mathbf{t_m^{(k)}}$ is the TH address code of user k given by:

$$\mathbf{t_m^{(k)}} = \left[t_m^{(k)}(1), t_m^{(k)}(2), ..., t_m^{(k)}(L)\right] \quad (3)$$

Furthermore, each user k has a specific TH address code defined as above.

B. DS-UWB

A DS-SS transmitted UWB signal can be expressed as [11]

$$u^{(k)}(t) = \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} \varphi(t - mT_s - nT_c) c_n^{(k)} d_n^{(k)} \quad (4)$$

where we assume that a block of data consisting of M bits is transmitted within the block time duration of $0 < t \leq MT_b = T_s$, T_b is the bit duration, $d_n^{(k)}$ corresponds to the data information, T_c is the chip duration and $c_n^{(k)}$ is the n^{th} chip of the spreading code for user k. The DS-SS waveform $c^{(k)}(t)$ for user k consists of a periodic Pseudo Noise (PN) sequence having a period of N and can be expressed as

$$c^{(k)}(t) = \sum_{n=0}^{N-1} c_n^{(k)} \psi(t - nT_c) \quad (5)$$

Here $c_n^{(k)}$ assumes a value of +1 or -1, while $\varphi(t)$ is the chip waveform or the UWB pulse, which controls the bandwidth of the UWB signal.

C. Channel Model

In this paper, TH-UWB and DS-UWB both are evaluated using UWB multipath channel model based on indoor channel measurements between 3.1 GHZ to 10.6 GHz over a range of less than 10 meters. The model accepted by IEEE 802.15.3 and considered can be expressed as [12]

$$h(t) = \sum_{u=1}^{U} h_u e^{j\phi_u} \delta(t - uT_\psi) \quad (6)$$

where U represents the number of resolvable multipaths, h_u and ϕ_u are the gain and phase of the uth resolvable multipath component. While uT_ψ represent the corresponding delay of the uth multipath component.

As shown in [13], the measured data shows that the UWB channels follow lognormal or Nakagami distribution, which has been validated by using the Kolmogorov-Smirnov testing with a significance level of 1 percent. The fading gain amplitude h_u comply the independent Nakagami-m distribution for our analysis with a probability density function (PDF) of the envelop r is given by [14]

$$P_{h_u}(r) = \frac{2m_u^{m_u} r^{2m_u - 1}}{\Gamma(m_u)\Omega_u^{m_u}} e^{\frac{-m_u r^2}{\Omega_u}}, \qquad r > 0 \quad (7)$$

where m_u is the fading parameter defined as:

$$m_u = \frac{E^2\left[r^2\right]}{var(r^2)} \quad (8)$$

The parameter Ω_u is

$$\Omega_u = E\left[r^2\right] \quad (9)$$

Further the moments of random variable r is given by:

$$E\left[r^v\right] = \frac{\Gamma\left(m_u + \frac{v}{2}\right)}{\Gamma(m_u)} \left(\frac{\Omega}{m_u}\right)^{\frac{v}{2}} \quad (10)$$

Finally $\Gamma(\cdot)$ is the gamma function defined as

$$\Gamma(m_u) = \int_0^{\infty} x^{m_u - 1} e^{(-x)} dx \quad (11)$$

We assume in our analysis that the phase rotation due to fading channel is uniformly distributed in $[0, 2\pi]$. Let us now consider the receiver structure.

D. Minimum Mean Square Error Detector

For both the TH-UWB and DS-UWB systems, the received signal vector can be expressed by [15]

$$\mathbf{y} = \mathbf{J}\mathbf{b} + \mathbf{n}, \quad (12)$$

where $\mathbf{J}$ is the overall system matrix, $\mathbf{b}$ is the information symbol vector. while $\mathbf{n}$ denotes the channel Additive White Gaussian Noise (AWGN) vector having $E[\boldsymbol{n}\boldsymbol{n}^H] = 2\sigma_n^2 \boldsymbol{I}$ with $\boldsymbol{I}$ being the identity matrix of appropriate dimension. The MMSE detector is a linear multiuser detector $\mathbf{P}$ [15] and can be expressed as:

$$\begin{aligned} \mathbf{P} &= \mathbf{R}_b \mathbf{J}^H (\mathbf{J}\mathbf{R}_b\mathbf{J}^H + \mathbf{R}_n)^{-1} \\ &= (\mathbf{J}^H \mathbf{R}_n^{-1} \mathbf{J} + \mathbf{R}_b^{-1})^{-1} \mathbf{J}^H \mathbf{R}_n^{-1} \end{aligned} \quad (13)$$

where $\mathbf{R}_b$ is the covarience matrix of information symbols and $\mathbf{R}_n$ is the covarience matrix of the noise.

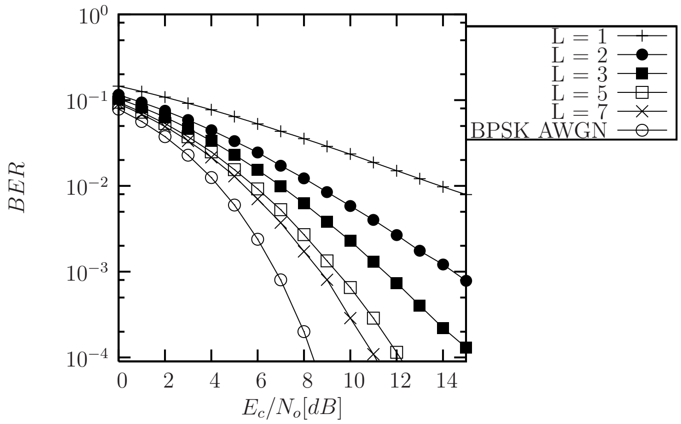

Fig. 2. BER versus $E_c/N_o[dB]$ performance for the DS/TH UWB systems over multipath Rayleigh ($m = 1$) fading channels upon varying the diversity order L, in the single user senario with a bit-duration to chip-duration ratio $N = 10$.

III. Comparative Simulation Study

A. Simulation Configuration

According to the simulation block diagram Fig. 1, the simulation configuration is summarised as follows:

1) For the both UWB systems, i.e. TH and DS-CDMA, the bit-duration to chip-duration ratio or the spreading factor is fixed to 10 for all the simulations.
2) The URC as depicted in Fig. 1 is used.
3) The RSC encoding and decoding schemes are used with RSC-2-(7,5) specifications.
4) To achieve an independent interleaver, length of 1000 000 bits are simulated.
5) The Nakagami-m distribution is reduced to the Rayleigh distribution by fixing the fading parameter m to $m = 1$.
6) Single, seven and fifteen reslovable multipaths have been simulated within the single user scenario.
7) We use the Binary Phase Shift Keying (BPSK) modulation and a half rate $R = 0.5$ RSC code. The relationship between the Singal to Noise Ratio (SNR) and the SNR per bit is given by

$$E_c/N_o = SNR/bps \tag{14}$$

where bps is the information bits per BPSK modulated symbol in our system.

B. Simulation Results

To commence explicating the simulation results, we start with Fig. 2, which shows the conventional graph of diversity effect. It depicts the BER as a function of the average received SNR per bit for both the DS-CDMA/TH UWB systems in multipath Rayleigh fading environment $(m = 1)$. The curves in Fig. 2 are parameterised by the diversity order $L = 1, 3, 5, 7, 15$. Both the DS/TH curves are indistinguishable. It can be easily visualised from Fig. 2 that the BER improves as the diversity order increases, as expected.

Figs. 3, 4 and 5 depict the EXIT charts and iterative decoding trajectories for the TH serial concatenated system of a URC with the RSC-2-(7,5) under various SNR conditions for the BPSK modulated transmissions over Rayleigh fading channels of $L = 1$, $L = 7$ and $L = 10$, respectively. The iterative decoding trajectories correspond to an I=1000 000 bit interleaver. A critical point to observe in these graphs are the areas under the detector EXIT chart curves corresponding to the SNR value of zero dB. It can be seen from Figs. 3 to 5 that this area increases as the multipath channel diversity order L increases. Thus, the system's capacity increases with the increase of diversity gain. The critical performance indicator of the iterative system can also be observed from another angle by examining the SNR threshold value that allows a narrow tunnel to begin opening up between the detector EXIT chart curve and the channel decoding EXIT chart curve. It can be seen from Figs. 3 to 5 that the E_c/N_o threshold values are approximately -0.2, -0.6 and -1.1 dB for the diversity orders $L = 1$, $L = 7$ and $L = 10$, respectively. This signifies the decoding convergence (turbo cliff) at a lower SNR value as the diversity order increases.

To correlate the same scenarios for the DS-UWB system, Figs. 6, 7 and 8 illustrate the EXIT charts and iterative decoding trajectories for the DS serial concatenated system of a URC with the RSC-2-(7,5) under various SNR conditions

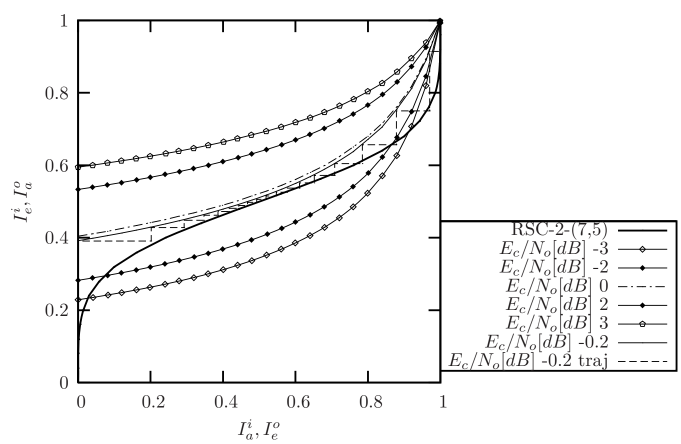

Fig. 3. EXIT charts and iterative decoding trajectory for the TH serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 1$ path.

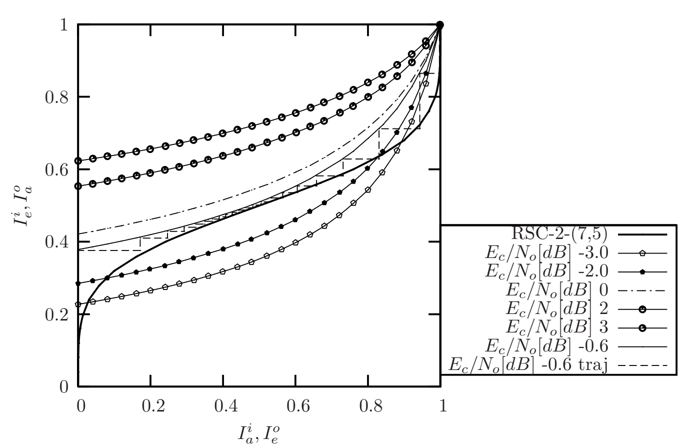

Fig. 4. EXIT charts and iterative decoding trajectory for the TH serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 7$ paths.

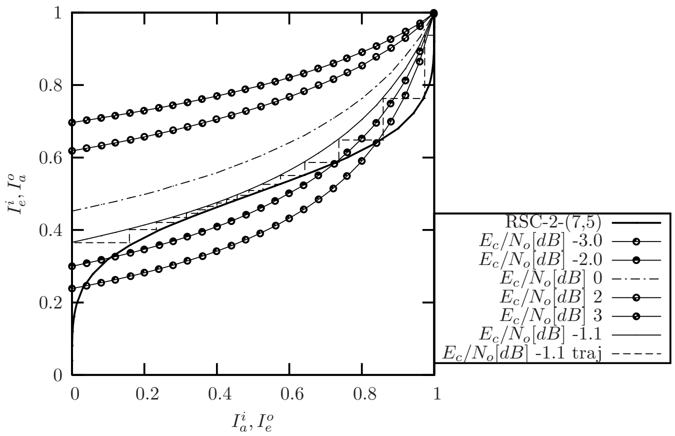

Fig. 5. EXIT charts and iterative decoding trajectory for the TH serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 10$ paths.

for the BPSK modulated transmissions over Rayleigh fading channels of $L = 1$, $L = 7$ and $L = 10$, respectively. The iterative decoding trajectories again correspond to an I=1000 000 bit interleaver. Similar patterns to those of the TH based scheme can be observed from these EXIT chart graphs. Specifically, the system's capacity increases with the increase of diversity gain, and a lower E_c/N_o threshold value is achieved to obtained a convergence tunnel as the diversity order increases. However, by carefully comparing Figs. 6 to 8 with Figs. 3 to 5, it can be concluded that the performance of the iterative TH-UWB system is slightly better than that of the iterative DS-UWB system, under the single-user senario. In general, the area under the EXIT chart curve of the amalgamated detector and the URC decoder is related to the achievable rate of the system. The larger this area the higher the attainable rate. At the E_c/N_o value of -1.0 dB and for the diversity order of $L = 10$, for instance, the areas under the EXIT chart curves of the TH-based detector and the DS-based detector are $A_E^{\mathrm{TH}} = 0.57$ and $A_E^{\mathrm{DS}} = 0.56$, respectively. Since $A_E^{\mathrm{TH}} > A_E^{\mathrm{DS}}$, the iterative TH-based system can attain a convergence tunnel at a lower SNR value.

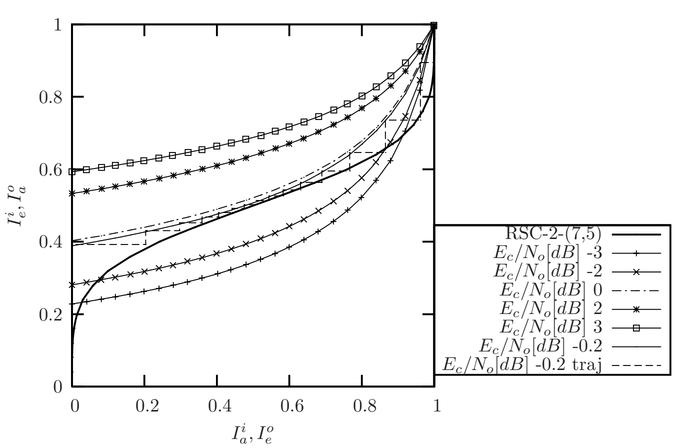

Fig. 6. EXIT charts and iterative decoding trajectory for the DS serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 1$ path.

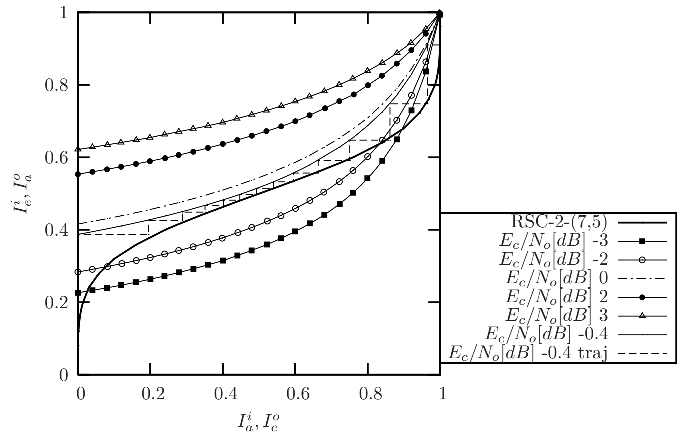

Fig. 7. EXIT charts and iterative decoding trajectory for the DS serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 7$ paths.

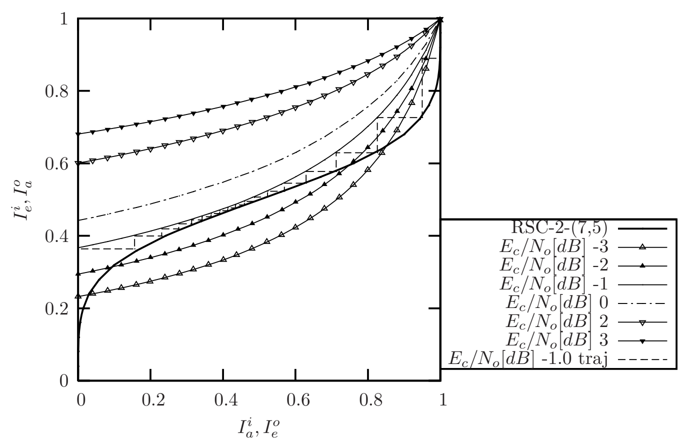

Fig. 8. EXIT charts and iterative decoding trajectory for the DS serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 10$ paths.

Figs. 9 and 10 show the BER iterative decoding convergence performance for the TH-UWB and DS-UWB schemes under the single-user senario with BPSK modulated transmission, respectively, when communicating over the Rayleigh fading channel having a diversity order of $L = 10$. An interleaver length of I=1000 000 is used again.

IV. Conclusions

Iterative detection for the TH and DS-CDMA UWB systems, when communicating over multipath fading channels, has been investigated with the aid of EXIT chart analysis. It has been confirmed in this study that a higher diversity order increases the attainable rate of the system. From the results obtained, it can be concurred that, under the single-user senario, the scheme based on the iterative TH-UWB system achieves a higher attainable rate, compared with the scheme based on the iterative DS-UWB system. Our future study will extend the analysis to the multiuser scenario.

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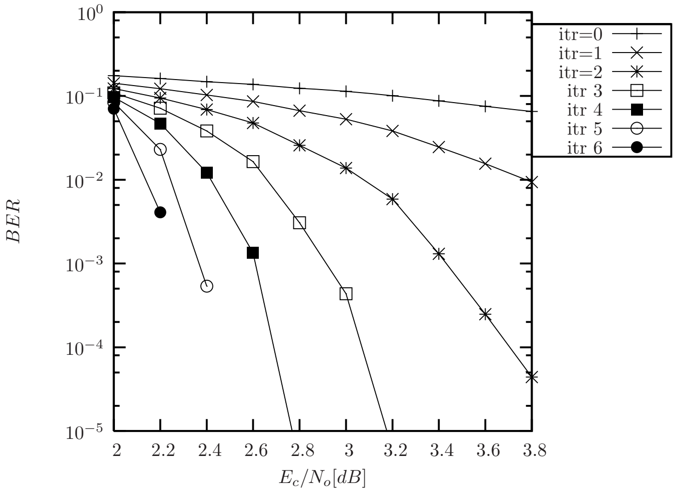

Fig. 9. Bit error rate iterative decoding convergence performance for the TH serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 10$ paths.

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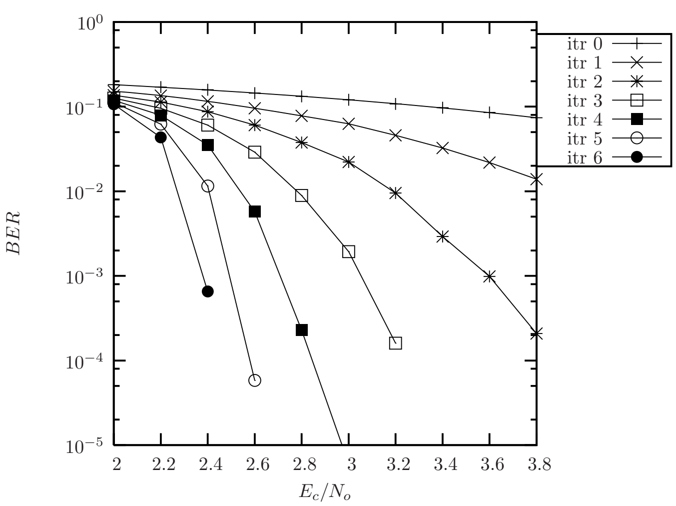

Fig. 10. Bit error rate iterative decoding convergence performance for the DS serial concatenated system of a URC with the RSC-2-(7,5) under the various E_c/N_o[dB] conditions for the single-user BPSK modulated transmission over the Rayleigh fading channel of $L = 10$ paths.

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