On the MIMO Channel Capacity of Multi-Dimensional Signal Sets

S. X. Ng, Member, IEEE, and L. Hanzo, Fellow, IEEE.

Abstract—In this contribution, the capacity of Multi-Input Multi-Output (MIMO) systems using multi-dimensional phaseshift keying/quadratic-amplitude modulation signal sets is evaluated. It was shown that transmit diversity is capable of narrowing the gap between the capacity of the Rayleigh-fading channel and the AWGN channel. However, since this gap becomes narrower when the receiver diversity order is increased, for higherorder receiver diversity the performance advantage of transmit diversity diminishes. A MIMO system having full multiplexing gain has a higher achievable throughput than the corresponding MIMO system designed for full diversity gain, although this is attained at the cost of a higher complexity and a higher SNR. The tradeoffs between diversity gain, multiplexing gain, complexity and bandwidth are studied.

Index Terms— Capacity, diversity, Multiple-Input Multiple-Output (MIMO), multiplexing.

I. INTRODUCTION

The capacity, C, of a Single-Input Single-Output (SISO) AWGN channel was quantified by Shannon in 1948 [1], [2]. Since then, substantial research efforts have been invested in finding channel codes that would produce an arbitrarily low probability of error at a transmission rate close to $C^* = C/T$, where T is the symbol period. We note however that Shannon's channel capacity is only defined for Continuous-Input Continuous-Output Memoryless Channels (CCMC) [3], where the channel input is a continuous-amplitude, discretetime Gaussian-distributed signal and the capacity is only restricted either by the signalling energy or by the bandwidth. Therefore we will refer to the capacity of the CCMC as the *unrestricted bound*.

By contrast, in the context of discrete-amplitude QAM [4] and PSK [3] signals, we encounter a Discrete-Input Continuous-Output Memoryless Channel (DCMC) [3]. Therefore, the capacity of the DCMC is more pertinent in the design of channel coded modulation schemes. With the advent of powerful space-time coding schemes [5]–[7], the Multi-Input Multi-Output (MIMO) channel capacity is of immediate interest. Note that multiple antennas can be utilised for providing diversity gain and/or multiplexing gain [8]. Specifically, Space-Time Trellis Coding (STTC) [5] and Space-Time Block Coding (STBC) [6], [9] were designed for achieving diversity gains by conveying the same information through different paths over the MIMO channel in order to combat the channelinduced fading. By contrast, Bell Lab's Layered Space-Time (BLAST) [7] scheme transmits independent information in parallel over the MIMO channel for the sake of achieving multiplexing gain, hence increasing the attainable transmission rate. Furthermore, both STTC and STBC schemes are capable of achieving full transmit diversity¹ at the cost of providing no multiplexing gain, while the BLAST scheme was designed for achieving full multiplexing gain at the cost of having no transmit diversity gain. The tradeoffs associated with having partial diversity gain and partial multiplexing gain when communicating over MIMO channels was studied in [8].

Note however that the STTC scheme [5] is also capable of achieving temporal or time diversity gain, which is commonly referred to as coding gain. On the other hand, the BLAST scheme [7] is unable to provide spatial diversity or temporal diversity, since both of these have been utilised for achieving full multiplexing gain. The STTC scheme may be viewed as a rate- $1/N_t$ channel code, where N_t is the number of transmit antennas. By contrast, the BLAST scheme [7] may be viewed as a rate-1 channel code. Despite having different code rates, both the STTC and BLAST schemes share the same MIMO channel capacity. This is similar to the case, where two different-rate temporal domain channel codes share the same *M*-ary QAM SISO channel capacity, when transmitting *M*-ary QAM signals across the SISO channels. By contrast, the orthogonal STBC may be viewed as a rate- $1/N_t$ spatialdomain repetition coding scheme. The STBC scheme may also be viewed as a MIMO system, which employs an orthogonal spreading code in the spatial and temporal domains [10]. Note that the STBC scheme is unable to provide temporal diversity gain due to employing an orthogonal code. Hence, the capacity of the 'spatial-domain-spread' STBC MIMO scheme is lower than that of the non-spread MIMO scheme. Nonetheless, the code orthogonality of the STBC scheme facilitates a lowcomplexity 'de-spreading' detection compared to the highcomplexity ML detection employed by the STTC scheme. The BLAST scheme also achieves its best performance, when ML detection is invoked.

However, the MIMO channel's capacity was only found for the CCMC in [11]–[15]. Furthermore, only the SISO AWGN channel capacity was found for multi-dimensional signal sets, such as M-ary orthogonal signalling [3] and L-ary PSK based L-orthogonal signalling [16], [17]. More specifically, the L-

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The authors are with the School of Electronics and Computer Science, University of Southampton, SO17 1BJ, UK (e-mail:sxn@ecs.soton.ac.uk; lh@ecs.soton.ac.uk).

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 $^{^{1}}$ A system is said to have a full transmit diversity, when the transmit diversity order is identical to the number of transmit antennas [6].

orthogonal PSK signal [17], [18] is a hybrid form of M-ary orthogonal and PSK signalling, combining the benefits of powerefficient and error-resilient M-ary orthogonal signalling [3, p. 284] as well as bandwidth-efficient PSK signalling. At this stage we note that STTC and STBC schemes have so far been exclusively designed for complex-valued (two-dimensional) PSK/QAM signal sets, but not for multi-dimensional signal sets. Against this background, the novel contribution of this treatise is that we derive channel capacity formulae applicable to MIMO systems employing multi-dimensional signal sets, in the quest for more error-resilient, power-efficient and bandwidth-efficient MIMO channel coding schemes.

The paper is organised as follows. In Section II the multidimensional signal set is described. In Sections III and IV, the channel capacity formulae are derived for the specific orthogonal STBC based MIMO system and the general MIMO system, respectively. In Section V the capacity and bandwidth efficiency of the MIMO channel are investigated. Finally, our conclusions are offered in Section VI.

II. MULTIDIMENSIONAL SIGNAL SET

The dimensionality of a time- and band-limited signal is defined as [19, pp. 348-351]:

$$D = 2WT,\tag{1}$$

where W is the bandwidth and T is the signalling period of the finite-energy signalling waveform. In an L-orthogonal PSK signal set [16], [17], there are V = WT independent Lary PSK subsets. The total number of waveforms is M =VL and the number of dimensions is D = 2V, which is independent of L. Specifically, an L-orthogonal PSK signal requires splitting the original PSK symbol period into V number of proportionately shortened PSK symbol periods and hence necessitates V times the bandwidth of PSK signalling, in order to transmit $\log_2(M)$ bits. The vector representation of L-orthogonal PSK signalling may be formulated as:

$$\mathbf{x}_m = \mathbf{x}_l^{LPSK} \phi_k, \ m = 1, \dots, M \ , \tag{2}$$

where l = ((m-1)%L) + 1 and a%b is the remainder of a/b, while $k = (\lfloor \frac{m-1}{L} \rfloor + 1)$ and \mathbf{x}_l^{LPSK} is the classic two-dimensional *L*-ary PSK signal. Furthermore, the orthonormal basis function $\phi_k = (\phi_k[1], \dots, \phi_k[v], \dots, \phi_k[V])$ is a vector of *V* elements, which may be constructed from nonoverlapping signalling pulses as follows:

$$\phi_k[i] = \begin{cases} 1, & i=k, \\ 0, & i\neq k. \end{cases}$$
(3)

Figure 1 illustrates an example of L = 8-orthogonal PSK signalling splitting the original signalling interval into V = 2subintervals at the cost of doubling the required bandwidth. The total number of waveforms is M = VL = 16 and the number of dimensions is D = 2V = 4. Note that only one of the V = 2 timeslots of duration T_p is active during the symbol period of $T_s = VT_p$. Therefore, *L*-orthogonal PSK signalling achieves $\log_2(V)$ bits higher capacity at the cost of *V* times lower bandwidth efficiency, than that of classic



Fig. 1. An *L*-orthogonal PSK example conveying 4 bits per symbol using L = 8-ary PSK subset, where the total symbol period T_s consists of V = 2 LPSK subset's signalling durations T_p .

L-ary PSK signalling. As we can see from Figure 1, there are *V* subsets of *L* phasors and each subset is assigned to one of the *V* orthonormal basis functions ϕ_k , hence, each subset of phasors is orthogonal to each other. However, the *L* phasors assigned to the same ϕ_k behave as in ordinary *L*-ary PSK signalling. Hence *L*-orthogonal PSK signalling and PSK signalling. For V = 1, *L*-orthogonal PSK signalling. As a further contribution to the current state-of-the-art, we extended the concept of *L*-orthogonal PSK signalling to *L*-orthogonal QAM signalling and we will quantify the achievable capacity of *L*-orthogonal QAM in Figures 3 to 8.

To elaborate a little further, the D = 2V-dimensional Lorthogonal PSK/QAM scheme conveys $\log_2(M)$ bits using V timeslots and orthogonal transmissions, where the total throughput is $\log_2(M)/V$ bits per timeslot. Hence a V-fold bandwidth expansion occurred compared to the D = 2dimensional PSK/QAM scheme, which conveys $\log_2(M)$ bits per timeslot. However, if a 2V-dimensional PSK/QAM scheme conveys $V \log_2(M)$ bits using V timeslots, then the total throughput will be $V \log_2(M)/V = \log_2(M)$ bits per timeslot, which is similar to that of the two-dimensional PSK/QAM scheme. Hence, there is no bandwidth expansion. The multidimensional lattice code [20] belongs to the family of nonorthogonal multi-dimensional PSK/QAM schemes, where an effective throughput of $\log_2(M)$ bits per timeslot is attained, regardless of the signal dimensionality of D = 2V. Hence the bit/s/Hz bandwidth efficiency of the non-orthogonal multidimensional PSK/QAM scheme is the same as that of the twodimensional PSK/QAM scheme, when communicating over SISO or MIMO channels. Therefore, the capacity of the nonorthogonal multi-dimensional PSK/QAM scheme is simply V times the bandwidth efficiency of the two-dimensional PSK/QAM scheme. For this reason, in this paper we mainly focus our attention on the capacity of the multi-dimensional L-orthogonal PSK/QAM scheme. Note however that the orthogonality of the L-orthogonal PSK/QAM scheme is not exploited for achieving diversity or multiplexing gain, but only for attaining a higher error-resilience in a fashion similar to that of the classic M-ary orthogonal scheme [21].

III. THE SPECIFIC MIMO CHANNEL CAPACITY OF THE ORTHOGONAL STBC SYSTEM

When classic D = 2-dimensional PSK/QAM is employed, the received signal at receiver *i* of Alamouti's orthogonal STBC [6] having $N_t = 2$ transmit antennas and N_r receive antennas can be transformed into [22]:

$$\mathbf{y}_{i} = \sum_{j=1}^{N_{t}} |\mathbf{h}_{i,j}|^{2} \mathbf{x} + \mathbf{\Omega}_{i} = \chi_{2N_{t},i}^{2} \mathbf{x} + \mathbf{\Omega}_{i} , \ i = \{1, \dots, N_{r}\}$$
(4)

where we define $\vec{\mathbf{y}} = (\mathbf{y}_1, \dots, \mathbf{y}_{N_r})^T$ as the N_r -element complex-valued received signal vector. Furthermore, \mathbf{x} is the complex-valued transmitted signal, $\mathbf{h}_{i,j}$ is the complex-valued Rayleigh fading coefficient between transmitter j and receiver $i, \chi_{2N_t,i}^2 = \sum_{j=1}^{N_t} |\mathbf{h}_{i,j}|^2$ represents the chi-squared distributed random variable having $2N_t$ degree of freedom at receiver i and Ω_i is the *i*th receiver's complex-valued AWGN after transformation, which has a zero mean and a variance of $\chi_{2N_t,i}^2 N_0/2$ per dimension, where $N_0/2$ is the original noise variance per dimension. More specifically, due to the code orthogonality of STBC, the MIMO channel was transformed into a Single-Input Multi-Output (SIMO) channel, where the equivalent Rayleigh fading coefficient between the transmitter and the *i*th receiver is given by $\chi_{2N_t,i}^2$ and the equivalent noise at the *i*th receiver is given by Ω_i .

It was shown in [23] that a full-rate, full-diversity orthogonal STBC also exists for $N_t > 2$. Let us now generalise Equation (4) for each component of the a D > 2-dimensional *L*-orthogonal PSK/QAM scheme as:

$$y_i[d] = \chi^2_{2N_t,i}[d]x[d] + \Omega_i[d] , \qquad (5)$$

where $\mathbf{y}_i = (y_i[1], \ldots, y_i[D])$, $\mathbf{x} = (x[1], \ldots, x[D])$ and $\mathbf{\Omega}_i = (\Omega_i[1], \ldots, \Omega_i[D])$. Note that when D > 2, we have D/2 number of different $\chi^2_{2N_t,i}$ values for the *D*-dimensional signals. Specifically, we have $\chi^2_{2N_t,i}[k] = \chi^2_{2N_t,i}[k+1]$ for $k \in \{1, 3, 5 \ldots\}$, since a complex channel has two dimensions. Furthermore, $\Omega_i[d]$ has a variance of $\chi^2_{2N_t,i}[d]N_0/2$ per each *D* dimensions.

The conditional probability of receiving a *D*-dimensional signal vector $\vec{\mathbf{y}}$ given that a *D*-dimensional *M*-ary signal \mathbf{x}_m , $m \in \{1, \ldots, M\}$, was transmitted over an AWGN channel is determined by the PDF of the noise, yielding:

$$p(\vec{\mathbf{y}}|\mathbf{x}_m) = \prod_{d=1}^{D} \prod_{i=1}^{N_r} \frac{1}{\sqrt{\pi N_0}} \exp\left(\frac{-(y_i[d] - x_m[d])^2}{N_0}\right) \quad (6)$$

where $N_0/2$ is the channel's noise variance. For the orthogonal STBC MIMO system of Equation (5), we have:

$$p(\vec{\mathbf{y}}|\mathbf{x}_m) = \left(\frac{1}{\sqrt{\pi N_0}}\right)^{N_r D} \cdot \\ \exp\left(\sum_{d=1}^D \sum_{i=1}^{N_r} \frac{-(y_i[d] - \chi^2_{2N_t,i}[d]x_m[d])^2}{\chi^2_{2N_t,i}[d]N_0}\right) .$$
(7)

The channel capacity for the STBC MIMO system using Ddimensional M-ary signalling over the DCMC can be derived from that of the Discrete Memoryless Channel (DMC) [24] as:

$$C_{\text{DCMC}}^{\text{STBC}} = \max_{p(\mathbf{x}_1)\dots p(\mathbf{x}_M)} \sum_{m=1}^{M} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} p(\mathbf{y}|\mathbf{x}_m) p(\mathbf{x}_m) \cdot \\ \log_2 \left(\frac{p(\mathbf{y}|\mathbf{x}_m)}{\sum_{n=1}^{M} p(\mathbf{y}|\mathbf{x}_n) p(\mathbf{x}_n)} \right) d\mathbf{y} \text{ [bit/sym], (8)}$$

where $p(\mathbf{x}_m)$ is the probability of occurrence for the transmitted signal \mathbf{x}_m . It was shown in [24, p. 94] that for a symmetric DMC, the full capacity may only be achieved by using equiprobable inputs. Hence, the right hand side of Equation (8) is maximised, when the transmitted symbols are equiprobably distributed, i.e. when we have $p(\mathbf{x}_m) = 1/M$ for $m \in \{1, \ldots, M\}$. Hence, we arrive at:

$$\log_{2} \left(\frac{p(\vec{\mathbf{y}} | \mathbf{x}_{m})}{\sum_{n=1}^{M} p(\vec{\mathbf{y}} | \mathbf{x}_{n}) p(\mathbf{x}_{m})} \right)$$

= $-\log_{2} \left(\frac{1}{M} \sum_{n=1}^{M} \frac{p(\vec{\mathbf{y}} | \mathbf{x}_{n})}{p(\vec{\mathbf{y}} | \mathbf{x}_{m})} \right),$
= $\log_{2}(M) - \log_{2} \sum_{n=1}^{M} \exp(\Psi_{m,n}),$ (9)

where the term $\Psi_{m,n}$ is given by:

$$\Psi_{m,n} = \sum_{d=1}^{D} \sum_{i=1}^{N_r} \left(\frac{-\left(y_i[d] - \chi^2_{2N_t,i}[d]x_n[d]\right)^2}{\chi^2_{2N_t,i}[d]N_0} + \frac{\left(y_i[d] - \chi^2_{2N_t,i}[d]x_m[d]\right)^2}{\chi^2_{2N_t,i}[d]N_0} \right) ,$$

$$= \sum_{d=1}^{D} \sum_{i=1}^{N_r} \left(-\frac{\left(\chi^2_{2N_t,i}[d](x_m[d] - x_n[d]) + \Omega_i[d]\right)^2}{\chi^2_{2N_t,i}[d]N_0} + \frac{\left(\Omega_i[d]\right)^2}{\chi^2_{2N_t,i}[d]N_0} \right).$$
(10)

By substituting Equation (9) and $p(\mathbf{x}_m) = 1/M$ into Equation (8) we have:

$$C_{\text{DCMC}}^{\text{STBC}} = \frac{\log_2(M)}{M} \sum_{m=1}^{M} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} p(\vec{\mathbf{y}} | \mathbf{x}_m) \ d\vec{\mathbf{y}} - \sum_{\text{D-fold}}^{\infty} p(\vec{\mathbf{y}} | \mathbf{x}_m) \ d\vec{\mathbf{y}} - \frac{1}{2} \sum_{m=1}^{\infty} p(\vec{\mathbf{y}} | \mathbf{x}_m) \ d\vec{\mathbf{y}} + \frac{1}{2} \sum_{m=1}^{\infty} p(\vec{\mathbf{y} | \mathbf{x}_m) \ d\vec{\mathbf{y}} + \frac{1}{2} \sum_{m=1}^{\infty} p(\vec{\mathbf{x}_m} + \frac{1}{2} \sum_{m=1}^{\infty} p(\vec{\mathbf{x}_m} + \frac{1}{2} \sum_{m=1}^{\infty} p$$

$$\frac{1}{M} \sum_{m=1}^{M} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} p(\vec{\mathbf{y}} | \mathbf{x}_{m}) \log_{2} \sum_{n=1}^{M} \exp(\Psi_{m,n}) d\vec{\mathbf{y}}$$
$$= \log_{2}(M) - \frac{1}{M} \sum_{m=1}^{M} E\left[\log_{2} \sum_{n=1}^{M} \exp(\Psi_{m,n}) \mid \mathbf{x}_{m}\right] [\text{bit/sym}$$
(11)

where $E[A|\mathbf{x}_m]$ is the expectation of A conditioned on \mathbf{x}_m and the expectation in Equation (11) is taken over $\chi^2_{2N_t,i}[d]$ and $\Omega_i[d]$ for $i = \{1, \ldots, N_r\}$. This expected value can be estimated using the Monte Carlo averaging method. More specifically, Equation (11) represents the capacity of the MIMO DCMC, when employing STBC for achieving full diversity gain for D-dimensional, M-ary PSK/QAM signals with the aid of N_t number of transmit antennas and N_r number of receive antennas.

Note that in a SISO AWGN channel we have $\chi^2_{2N_t,i}[d] = N_t = 1$ and hence the noise variance of $\Omega_i[d]$ is $N_0/2$ per each dimension. For D = 2-dimensional signalling, Equation (10) can be simplified to: $\Psi_{m,n} = \sum_{i=1}^{N_r} \frac{-|\chi^2_{2N_t,i}(\mathbf{x}_m - \mathbf{x}_n) + \Omega_i|^2 + |\Omega_i|^2}{\chi^2_{2N_t,i}N_0}$, where we have $\mathbf{x}_k = x_k[1] + jx_k[2]$ and $\Omega_i = \Omega_i[1] + j\Omega_i[2]$. It is reassuring to note that in the simplified case of SISO AWGN channels, Equations (10) and (11) agree with the results of [25]. The average SNR can be determined from [16], [25] as:

$$SNR = \frac{\frac{1}{M} \sum_{m=1}^{M} \sum_{d=1}^{D} |x_m[d]|^2}{\sum_{d=1}^{D} E[(\Omega_i[d])^2]} = \frac{E_s}{DN_0/2},$$
 (12)

where E_s is the average energy of the *D*-dimensional *M*ary symbol \mathbf{x}_m and $D\frac{N_0}{2}$ is the average energy of the *D*dimensional AWGN. Additionally, the energy of the signal sets is further normalised by $\sqrt{N_t}$. More specifically, we have $x_k[d] = \tilde{x}_k[d]/\sqrt{N_t}$, where $\tilde{x}_k[d]$ is the *k*th modulated signal, $k = \{1, \ldots, M\}$, of dimension *d* in the case of $N_t = 1$. In an AWGN channel, the channel capacity is not expected to increase, when N_t is increased. However, if the transmitter knows the complex Rayleigh-distributed channel coefficient of each of the MIMO links, the transmitted power to be assigned to the various transmit antennas can be distributed according to the "water-filling" principle [12], [15] in order to increase the achievable capacity.

The capacity formula of Equations (10) and (11) can also be applied to real-valued signal sets, such as M-ary orthogonal signals, as well as to amplitude-modulated signals following straightforward adjustments of the signalling space dimensionality, the channel fading and the noise. The MIMO CCMC capacity (unrestricted bound) of the STBC scheme designed for achieving full diversity gain can be derived based on the equivalent SIMO channel of Equation (4) as:

$$C_{\text{CCMC}}^{\text{STBC}} = E \left[WT \log_2(1 + \sum_{i=1}^{N_r} \chi_{2N_t,i}^2 \frac{SNR}{N_t}) \right] \text{ [bit/sym]}$$
$$E \left[\frac{D}{2} \log_2(1 + 2\sum_{i=1}^{N_r} \chi_{2N_t,i}^2 \frac{SNR}{N_t}) \right] \text{ [bit/sym]}$$

$$= E \left[\frac{D}{2} \log_2(1 + \chi_{2N}^2 \frac{SNR}{N_t}) \right] \text{ [bit/sym]} \quad (13)$$

where $\chi^2_{2N} = \sum_{i=1}^{N_r} \chi^2_{2N_t,i} = \sum_{i=1}^{N_r} \sum_{j=1}^{N_t} |\mathbf{h}_{i,j}|^2$ and the expectation is taken over χ^2_{2N} . Again, the achievable capacity

can be further enhanced by distributing the transmitted power according to the "water-filling" principle, when the channel knowledge is available at the transmitter [12], [15].

IV. THE GENERAL MIMO CHANNEL CAPACITY

In a two-dimensional MIMO system there are $M = L^{N_t}$ number of possible L-ary PSK/QAM phasor combinations in the transmitted signal vector $\vec{\mathbf{x}} = (\mathbf{x}_1, \dots, \mathbf{x}_{N_t})^T$, where \mathbf{x}_j is the two-dimensional L-ary PSK/QAM signal emitted from antenna j. The STTC scheme of [5] designed for attaining transmit diversity and coding gain may be viewed as a rate- $1/N_t$ channel code, where there are only $L^1 = L$ legitimate space-time codewords out of the L^{N_t} possible phasor combinations during each transmission period. By contrast, the BLAST scheme [7] designed for attaining multiplexing gain may be viewed as a rate-1 channel code, where all L^{N_t} phasor combinations are legitimate during each transmission period. In the case of the SISO system, the higher the temporal diversity (coding gain) the lower the coding rate and hence a lower throughput is resulted. Similarly, in the case of the MIMO system, the higher the transmit diversity (a maximum of order N_t) the lower the coding rate (multiplexing gain) and hence a lower throughput is vielded.

Let us consider a general MIMO system, which invokes N_t transmit antennas and N_r transmit antennas. We will refer to this general MIMO system as the ML-detected MIMO system for the sake of differentiating it from the orthogonal STBC based MIMO system of Section III. When D = 2-dimensional *L*-ary PSK/QAM is employed, the received signal vector of the MIMO system is given by:

$$\vec{\mathbf{y}} = \mathbf{H}\vec{\mathbf{x}} + \vec{\mathbf{n}} , \qquad (14)$$

where $\vec{\mathbf{y}} = (\mathbf{y}_1, \dots, \mathbf{y}_{N_r})^T$ is an N_r -element vector of the received signals, \mathbf{H} is an $N_r \times N_t$ channel matrix, $\vec{\mathbf{x}} = (\mathbf{x}_1, \dots, \mathbf{x}_{N_t})^T$ is an N_t -element vector of the transmitted signals and $\vec{\mathbf{n}} = (\mathbf{n}_1, \dots, \mathbf{n}_{N_r})^T$ is an N_r -element noise vector, where each elements in $\vec{\mathbf{n}}$ is an AWGN having a zero mean and a variance of $N_0/2$ per dimension. The two-dimensional signalling based Equation (14) can be generalised for D = 2V-dimensional signals as:

$$\vec{\mathbf{y}}[v] = \mathbf{H}[v]\vec{\mathbf{x}}[v] + \vec{\mathbf{n}}[v] , \qquad (15)$$

where $\vec{\mathbf{Y}} = (\vec{\mathbf{y}}[1], \dots, \vec{\mathbf{y}}[V])$ is defined as the 2V-dimensional received signal vector, $\vec{\mathbf{X}} = (\vec{\mathbf{x}}[1], \dots, \vec{\mathbf{x}}[V])$ is defined as the 2V-dimensional transmitted signal vector, $\mathbf{H}[v]$ is the vth element of the 2V-dimensional channel matrix and $\vec{\mathbf{n}}[v]$ is the vth element of the 2V-dimensional AWGN vector. As we have seen in Figure 1, there are (V - 1) number of subsets that are orthogonal to a particular subset in 2Vdimensional *L*-orthogonal PSK/QAM signalling. There are a total of V^{N_t} number of possible transmitted phasor constellation combinations in the 2V-dimensional *L*-orthogonal PSK/QAM signalling. However, from the V^{N_t} number of constellation combinations only (V - 1) are orthogonal to a particular phasor constellation, since the dimensionality is still D = 2V.

To elaborate a little further, let us define a set of basis functions, $\vec{\Phi}_k$, which are not necessarily orthogonal to each

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N_t			
1	$\phi_{1,2}[1] = 1 \; ; \mathbf{x}_2^L$	$\phi_{1,2}[2] = 0$	_
_	$\phi_{1,1}[1] = 1$; \mathbf{x}_1^L	$\phi_{1,1}[2] = 0$	– எ.
 N.	$\longrightarrow V$	1	Ψ_1
	$\phi_{2,2}[1] = 1 \; ; \mathbf{x}_2^L$	$\phi_{2,2}[2] = 0$	_
_	$\phi_{2,1}[1] = 0$	$\phi_{2,1}[2] = 1 \; ; \; \mathbf{x}_1^L$	- đ.
 N.	$\longrightarrow V$	· · ·	± 2
1 -	$\phi_{3,2}[1] = 0$	$\phi_{3,2}[2] = 1 \ ; \mathbf{x}_2^L$	_
_	$\phi_{3,1}[1] = 1$; \mathbf{x}_1^L	$\phi_{3,1}[2] = 0$	- .
N.	> V		±3
1 _	$\phi_{4,2}[1] = 0$	$\phi_{4,2}[2] = 1 \ ; \mathbf{x}_2^L$	_
_	$\phi_{4,1}[1] = 0$	$\phi_{4,1}[2] = 1 \; ; \; \mathbf{x}_1^L$	- .
	$\longrightarrow V$	1	Ψ_4

Fig. 2. The V^{N_t} =4 phasor constellation combinations for the D=2V=4-dimensional N_t =2 ML-detected L-orthogonal PSK/QAM signalling scheme.

other, for representing the V^{N_t} possible transmitted phasor constellation combinations, where $k \in \{1, \ldots, V^{N_t}\}$. More specifically, the above-mentioned basis function $\vec{\Phi}_k$ may be described by an $N_t \times V$ matrix, which can be constructed from non-overlapping signalling pulses for each of the rows, where there is only a single '1' in each of the N_t rows. Explicitly, we have $\vec{\Phi}_k = (\vec{\phi}_k[1], \ldots, \vec{\phi}_k[v], \ldots, \vec{\phi}_k[V])$, where $\vec{\phi}_k[v] = (\phi_{k,1}[v], \ldots, \phi_{k,j}[v], \ldots, \phi_{k,N_t}[v])^T$ is an N_t -element column vector and $\phi_{k,j}[v] \in \{0, 1\}$. The relationship between the transmitted N_t -element vector $\vec{\mathbf{x}}[v]$, the *L*-ary PSK/QAM signal \mathbf{x}_j^L transmitted from the *j*th transmit antenna and the *v*th column vector of the basis function $\vec{\phi}_k[v]$ may be formulated as:

$$\vec{\mathbf{x}}[v] = \vec{\mathbf{x}}^L \cdot \vec{\phi}_k[v] = \left(\mathbf{x}_1^L \phi_{k,1}[v], \dots, \mathbf{x}_j^L \phi_{k,j}[v], \dots, \mathbf{x}_{N_t}^L \phi_{k,N_t}[v]\right)^T (16)$$

where $\vec{\mathbf{x}}^L = (\mathbf{x}_1^L, \dots, \mathbf{x}_j^L, \dots, \mathbf{x}_{N_t}^L)^T$ is the N_t -element column vector representing the N_t L-ary PSK/QAM phasors transmitted from N_t transmitters. Again, there are V^{N_t} number of transmitted phasor constellation combinations in a general MIMO system and each constellation combination can host L^{N_t} number of L-ary PSK/QAM phasor combinations. Hence, the total number of possible combinations for $\vec{\mathbf{X}}$ is given by:

$$M = (V L)^{N_t} . (17)$$

Figure 2 portrays the $V^{N_t}=2^{2}=4$ legitimate phasor constellation combinations for the $N_t=2$ MIMO D=2V=4-dimensional *L*-orthogonal PSK/QAM signalling scheme. As shown in Figure 2, we can always find (V-1)=1 orthogonal phasor constellation combination for each of the $V^{N_t}=4$ possible phasor constellation combination. In other words, the vectors $(\vec{\Phi}_1, \vec{\Phi}_2)$ and $(\vec{\Phi}_4, \vec{\Phi}_3)$ constitute the V=2 orthogonal basis functions for this system.

The conditional probability of receiving a 2V-dimensional signal vector $\vec{\mathbf{Y}}$ given that a 2V-dimensional *M*-ary signal vector $\vec{\mathbf{X}}_m$, $m \in \{1, \ldots, M\}$, was transmitted over Rayleigh

fading channels is determined by the PDF of the noise, yielding:

$$p(\vec{\mathbf{Y}}|\vec{\mathbf{X}}_m) = \prod_{v=1}^{V} \frac{1}{(\pi N_0)^{N_r}} \exp\left(\frac{-||\vec{\mathbf{y}}[v] - \mathbf{H}[v]\vec{\mathbf{x}}_m[v]||^2}{N_0}\right),$$
$$= \frac{1}{(\pi N_0)^{N_r V}} \exp\left(\sum_{v=1}^{V} \frac{-||\vec{\mathbf{y}}[v] - \mathbf{H}[v]\vec{\mathbf{x}}_m[v]||^2}{N_0}\right).$$
(18)

The channel capacity of the ML-detected MIMO system using 2V-dimensional M-ary signalling over the DCMC can be written as:

$$C_{\text{DCMC}}^{\text{ML}} = \max_{p(\vec{\mathbf{x}}_{1})\dots p(\vec{\mathbf{x}}_{M})} \sum_{m=1}^{M} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} p(\vec{\mathbf{Y}} | \vec{\mathbf{X}}_{m}) p(\vec{\mathbf{X}}_{m})$$

$$\cdot \log_{2} \left(\frac{p(\vec{\mathbf{Y}} | \vec{\mathbf{X}}_{m})}{\sum_{n=1}^{M} p(\vec{\mathbf{Y}} | \vec{\mathbf{X}}_{n}) p(\vec{\mathbf{X}}_{n})} \right) d\vec{\mathbf{Y}} \text{ [bit/sym]}$$
(19)

where the right hand side of Equation (19) is maximised, when we have $p(\vec{\mathbf{X}}_m) = 1/M$ for $m \in \{1, \ldots, M\}$. Hence, Equation (19) can be simplified to:

$$C_{\text{DCMC}}^{\text{ML}} = \log_2(M) - \frac{1}{M} \sum_{m=1}^M E\left[\log_2 \sum_{n=1}^M \exp(\Psi_{m,n}) \mid \vec{\mathbf{X}}_m\right]$$

[bit/sym] (20)

where $E[A|\vec{\mathbf{X}}_m]$ is the expectation of A conditioned on $\vec{\mathbf{X}}_m$ and the expectation in Equation (20) is taken over $\mathbf{H}[v]$ and $\vec{\mathbf{n}}[v]$, while $\Psi_{m,n}$ is given by:

$$\Psi_{m,n} = \sum_{v=1}^{V} \frac{-||\mathbf{H}[v](\vec{\mathbf{x}}_{m}[v] - \vec{\mathbf{x}}_{n}[v]) + \vec{\mathbf{n}}[v]||^{2} + ||\vec{\mathbf{n}}[v]||^{2}}{N_{0}} ,$$
$$= \sum_{v=1}^{V} \sum_{i=1}^{N_{r}} \frac{-\left|\vec{\mathbf{h}}_{i}[v](\vec{\mathbf{x}}_{m}[v] - \vec{\mathbf{x}}_{n}[v]) + \mathbf{n}_{i}[v]\right|^{2} + |\mathbf{n}_{i}[v]|^{2}}{N_{0}}$$
(21)

where $\vec{\mathbf{h}}_i[v]$ is the *i*th row of $\mathbf{H}[v]$ and $\mathbf{n}_i[v]$ is the AWGN at the *i*th receiver.

It was shown in [12], [15] that the MIMO capacity of the CCMC can be expressed as:

$$C_{\text{CCMC}}^{\text{ML}} = E\left[WT\sum_{i=1}^{r}\log_2\left(1+\lambda_i\frac{SNR}{N_t}\right)\right] , (22)$$

where r is the rank of \mathbf{Q} , which is defined as $\mathbf{Q} = \mathbf{H}^H \mathbf{H}$ for $N_r \ge N_t$ or $\mathbf{Q} = \mathbf{H}\mathbf{H}^H$ for $N_r < N_t$. Furthermore, λ_i is the *i*th eigenvalue of the matrix \mathbf{Q} . The extension of Equation (22) to D-dimensional signalling can be carried out by noting that WT = D/2 = V.

When communicating over AWGN channels and assuming that there is no path loss, we have $\mathbf{h}_{i,j} = 1$ for all *i* and *j* in the channel matrix **H**, hence the rank of **Q** becomes unity and the only non-zero eigenvalue is given by $\lambda_1 = N_r \times N_t$ [15]. The capacity of the AWGN CCMC becomes identical to that of the orthogonal STBC scenario characterised in Equation (13), where $\chi_{2N}^2/N_t = N_r$. Therefore, no multiplexing or transmit diversity gain may be attained in an AWGN CCMC. On the other hand, we have $\mathbf{H}\vec{\mathbf{x}} = N_r \sum_{j=1}^{N_t} \mathbf{x}_j$, when communicating over an AWGN DCMC. More explicitly, the signals transmitted from the N_t transmit antennas may cancel out each other and result in a severe interference. Hence, no multiplexing or transmit diversity gain may be attained in the AWGN DCMC and its capacity is also the same as that of the orthogonal STBC scheme quantified by Equations (10) and (11), where $\chi_{2N_t,i}^2[d] = N_t$.

Note that the closed-form evaluation of the MIMO CCMC capacity in Equation (22) has been given by [13, Equation (40)] and in [14]. A closed-form evaluation of the channel capacity for the MIMO CCMC when employing STBC in Equation (13) may also be derived based on [13], [14]. However, a closed-form evaluation of the MIMO DCMC channel capacity in Equations (11) and (19) is computationally complex due to the existence of the 'summation over M exponential functions' in the multidimensional integral. In this case, the Monte Carlo averaging method is the most efficient approximation technique of computing the expectation terms.

V. NUMERICAL RESULTS

In this section, we will evaluate both the capacity and bandwidth efficiency of MIMO channels for the scenario, when the transmitter does not have any channel knowledge. Explicitly, the bandwidth efficiency is computed by normalising the channel capacity, as it transpires from Equations (11), (13), (19) and (22), with respect to the product of the bandwidth W and the signalling period T:

$$\eta = \frac{C}{WT} = \frac{C}{D/2} \text{ [bit/s/Hz]}.$$
(23)

The bandwidth efficiently is typically plotted against the SNR per bit given by: $E_b/N_0 = SNR/\eta$. We denote the 'L = 16-orthogonal QAM scheme having V = v' as '16QAM, V = v' for brevity. Again, L = 16-orthogonal PSK/QAM signalling having V = 1 represents classic two-dimensional L-ary PSK/QAM signalling.



Fig. 3. The capacity of the orthogonal STBC MIMO uncorrelated Rayleighfading channel and AWGN channel for 16QAM having V = 1 (M = 16, D = 2) and V = 2 (M = 32, D = 4).



Fig. 4. The bandwidth efficiency of the orthogonal STBC MIMO uncorrelated Rayleigh-fading channel and AWGN channel for 16QAM having V = 1 (M = 16, D = 2) and V = 2 (M = 32, D = 4).

Figure 3 illustrates the achievable capacity C of both the uncorrelated MIMO Rayleigh-fading channel and that of the AWGN channel for 16QAM signalling having both V = 1and V = 2, when aiming for a full diversity gain using an orthogonal STBC scheme. As shown in Figure 3, the achievable capacity of the Rayleigh-fading channel increases, as the number of transmit antennas N_t increases from 1 to 4, approaching the capacity of the AWGN channel, which is independent of N_t . Figure 4 depicts the bandwidth efficiency η of both the uncorrelated MIMO Rayleigh-fading channel and that of the AWGN channel for 16QAM signalling having both V = 1 and V = 2, when aiming for a full diversity gain using an orthogonal STBC scheme. It is shown in Figure 4 that as N_r increases, the bandwidth efficiency of the AWGN channel also improves, hence the corresponding performance over Rayleigh-fading channels follows the same trend. However, the attainable extra transmit diversity gain of the Rayleighfading channel reduces, as N_r increases, since a near-AWGN performance is achieved by the high-order receiver diversity. As seen by comparing Figures 3 and 4 for the systems having $N_r = 1$, the achievable channel capacity increases as the signal dimensionality D increases, although this is attained at a reduced bandwidth efficiency. However, the error-resilience of the power-efficient multi-dimensional orthogonal signals also improves as the dimensionality increases [3]. As evidenced in Figure 4, at low E_b/N_0 the bandwidth efficiency η of 16QAM attained in conjunction with both V = 1 and V = 2 converges to the unrestricted bound. Note that the unrestricted bound is independent of the signal dimensionality.

Let us now compare the achievable capacity of the orthogonal STBC MIMO system to that of the general (MLdetected) MIMO system in Figures 5 and 6, where the number of receivers is $N_r = 1$ and $N_r = 2$, respectively. As we can see from Figure 5, the Rayleigh fading CCMC capacity (unrestricted bound) of the ML-detected MIMO system is higher than that of the orthogonal STBC system by a constant margin, when we have $N_r = 1$ and $N_t = 2$. However, the Rayleigh fading DCMC capacity of the ML-detected MIMO system is identical to that of the orthogonal STBC system,



Fig. 5. The capacity of the MIMO uncorrelated Rayleigh-fading channel and AWGN channel for 16QAM having V = 1 (M = 16, D = 2) and V = 2 (M = 32, D = 4) when $N_r = 1$.



Fig. 6. The capacity of the MIMO uncorrelated Rayleigh-fading channel and AWGN channel for 16QAM having V = 1 (M = 16, D = 2) and V = 2 (M = 32, D = 4) when $N_r = 2$.

when the channel SNR is low. When the number of receivers is increased to $N_r = 2$, the gap between the Rayleigh fading CCMC/DCMC capacity of the ML-detected MIMO system and that of the orthogonal STBC system increases as the SNR increases, which is depicted in Figure 6. Hence, the capacity loss of the orthogonal STBC MIMO system increases, as N_r and the SNR increases. However, the ML-detected MIMO system, which has $(VL)^{N_t}$ number of possible transmitted signals, imposes a higher detection complexity compared to that of the orthogonal STBC system, which has only VL number of possible transmitted signals. Hence, it is more beneficial to employ the orthogonal STBC system when invoking a low-rate channel coding scheme, which results in a low throughput, since a lower detection complexity is required compared to that of the ML-detected MIMO system, especially when we have $N_r = 1$. By contrast, a higher capacity can be attained with the aid of the ML-detected MIMO system at the cost of a higher complexity and a higher SNR.

Let us now compare the Rayleigh fading MIMO channel capacity of the STBC, STTC and BLAST MIMO schemes at

 $N_t = N_r = 2$ and V = 1 in Figure 6. At a throughput of C = 4 bit/symbol the required SNRs for the STTC/BLAST and STBC schemes are approximately 7.0 dB ($E_b/N_0 = 1$ dB) and 14.5 dB ($E_b/N_0 = 8.5$ dB), respectively. Since both the STBC and STTC schemes achieve a full transmit diversity gain, the gap between the capacity curves of STBC and STTC quantifies the attainable temporal diversity gain (or coding gain) for the STTC scheme. Hence the STTC scheme is capable of achieving an additional coding gain of 7.5 dB compared to the STBC scheme at a throughput of 4 bit/symbol. Similarly, with the aid of a rate $R_o = 1/2$ outer channel code, the BLAST scheme is capable of benefitting from the coding gain of the outer channel code and hence achieve a similar performance to the STTC scheme at a throughput of $8R_o = 4$ bit/symbol. However, the BLAST scheme by itself requires an SNR of approximately 27.0 dB (or $E_b/N_0 = 18.0$ dB) in order to achieve a full multiplexing gain of 8 bit/symbol. Hence, when aiming for a near error-free performance, the BLAST scheme which exhibits a full transmit multiplexing gain is 18.0 - 8.5 = 9.5 dB inferior in terms of the required E_b/N_0 in comparison to the orthogonal STBC scheme, which exhibits a full transmit diversity gain. In other words, the full spatial diversity offers an achievable gain of 9.5 dB in this MIMO system. Furthermore, the BLAST scheme is 18.0 - 1.0 = 17.0 dB inferior in terms of the required E_b/N_0 compared to the STTC scheme, which exhibits a full transmit diversity gain plus a coding gain. Hence, a total of 17.0 dB E_b/N_0 reduction was offered by the spatial and temporal diversity. In the same way, the tradeoffs associated with having partial transmit multiplexing and transmit diversity gain may also be quantified based on the corresponding MIMO DCMC channel capacity curves. Note further that the capacity of the Rayleigh fading MIMO channel of the ML-detected system is higher than that of the AWGN MIMO channel. However, the capacity of the Rayleigh fading MIMO channel of the orthogonal STBC system is lower than that of the AWGN MIMO channel.

The asymptotic capacity of a DCMC system is given by $\log_2(M)$ bit/symbol, where we have $M = (VL)^{N_t}$ for a ML-detected MIMO system and M = VL for an orthogonal STBC system. Hence, a variety of different systems may be designed for achieving a given M by changing the values of V, L and N_t . As we can see from Figure 7, where we have M = 256 for all schemes, the different system designs achieve a different performance, despite having the same asymptotic capacity of 8 bit/symbol. Again, neither the BLAST nor the STBC schemes achieve a coding gain, unless an outer code is employed. However, for achieving the same asymptotic capacity using V = 1, the full diversity based orthogonal STBC MIMO system has to employ the higher-order modulation scheme of 256QAM compared to the 16QAM arrangement used by the BLAST scheme. As can be seen from Figure 7, the full diversity advantage of STBC cannot compensate for the minimum Euclidean distance loss imposed by employing 256QAM. This observation is also applicable for higher dimensionality signalling, where the BLAST MIMO system having L=4 and V=4 performs better than the orthogonal STBC system having L=64 and V=4. In the context of the ML-detected MIMO system, a

12 STBC N_r=2 11 256QAM, V=1, Nt=2 64QAM, V=4, N+=2 10 9 8 C (bit/symbol) 7 6 5 4 3 ML: 2 16QAM, V=1, N_t=2 4QAM, V=4, Nt=2 V=1 4QAM, V=1, N=4 0 . -10 0 10 20 30 SNR (dB)

Fig. 7. The capacity of the MIMO uncorrelated Rayleigh-fading channel for D = 2 (V = 1) and D = 8 (V = 4) dimensional signalling, when aiming for a throughput of 8 bit/symbol.



Fig. 8. The bandwidth efficiency of the MIMO uncorrelated Rayleigh-fading channel for D = 2 (V = 1) and D = 8 (V = 4) dimensional signalling, when aiming for a throughput of 8 bit/symbol.

scheme that employs a lower L and a higher N_t (L=4, N_t =4, V=1) may yield a higher capacity compared to a scheme that invokes a higher L and a lower N_t (L=16, N_t =2, V=1), when aiming for the same M, albeit this is achieved at the cost of a higher hardware complexity. When the signal dimensionality is increased from two (V=1) to eight (V=4), the achievable capacity also increases at the cost of a higher bandwidth requirement. The bandwidth efficiency of the M=256-based schemes characterised in Figure 7 is shown in Figure 8. As we can see from Figure 8, the bandwidth efficiency of the eightdimensional scheme is poorer than that of the two-dimensional scheme. The performance difference between the ML-detected MIMO system and the orthogonal STBC system is also more apparent in terms of their bandwidth efficiency. Again, the ML-detected scheme having L=4, N_t =4 and V=1 is more bandwidth efficient than the ML-detected arrangement having $L=16, N_t=2 \text{ and } V=1.$

VI. CONCLUSIONS

The capacity formulae of DCMC were derived for a specific orthogonal STBC MIMO system and for a general MIMO system, when employing multidimensional signal sets. The orthogonal STBC MIMO system was found to have a lower capacity, since its code orthogonality prevents it from achieving temporal diversity. Furthermore, STTC is a specific MIMO system, which attains full transmit diversity and a coding gain, whereas BLAST is a specific MIMO system, which achieves only the full transmit multiplexing gain. It was shown that transmit diversity is capable of narrowing the gap between the capacity of the Rayleigh-fading channel and the AWGN channel. However, the transmit diversity advantage becomes modest, when the receiver diversity order is increased since the remaining capacity gap becomes narrower. Hence, it is better to utilise temporal diversity for enhancing the error resilience while employing multiple transmitters for attaining transmit multiplexing gain, when sufficient receiver diversity is achieved. When aiming for a similar asymptotic capacity, the highest bandwidth efficiency is attained, when employing a two-dimensional ML-detected MIMO system having the lowest L and the highest N_t , at the cost of a higher hardware complexity. By contrast, the highest capacity was achieved at a given asymptotic capacity, when a ML-detected MIMO system having the highest dimensions, a low L and a high N_t was employed, although this was achieved at the cost of a higher bandwidth requirement.

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papers in this field.



Soon Xin Ng (M'03) received a first-class B.Eng. degree in Electronics Engineering and the Ph.D. degree in mobile communications from the University of Southampton, U.K, in July 1999 and December 2002, respectively.

He is currently continuing his research as a postdoctoral research fellow at the University of Southampton, U.K. His research interests are mainly in adaptive coded modulation, channel coding, turbo coding, space-time coding, joint source and channel coding and MIMO systems. He published numerous

Lajos Hanzo (M'91-SM'92-F'04) received his Master degree in electronics in 1976 and his doctorate in 1983. In 2004 he was awarded the Doctor of Sciences (DSc) degree by the University Southampton, UK.

During his 28-year career in telecommunications he has held various research and academic posts in Hungary, Germany and the UK. Since 1986 he has been with the Department of Electronics and Computer Science, University of Southampton, UK, where he holds the chair in telecommunications. He has co-authored 11 John Wiley/IEEE Press books

totalling about 9000 pages on mobile radio communications, published in excess of 500 research papers, organised and chaired conference sessions, presented overview lectures and has been awarded a number of distinctions. He is a non-executive director of the Virtual Center of Excellence (VCE), Basingstoke, U.K., and an enthusiastic supporter of the industrial liaison. He also offers a range of industrial esearch overview courses. He has coauthored 12 John Wiley/IEEE Press books on mobile radio communications, totaling about 9000 pages, published in excess of 600 research papers, organized and chaired conference sessions, and presented overview lectures. Further information on research in progress and associated publications may be found at http://www-mobile.ecs.soton.ac.uk.

Dr. Hanzo is Fellow of the Royal Academy of Engineering (FREng) and IEE. He is an IEEE Distinguished Lecturer of both the Communications Society and the Vehicular Technology Society. He has been awarded a number of distinctions. He is also the Governor of the IEEE Vehicular Technology Society and the Editor of the PROCEEDINGS OF THE IEEE.